MATEMATICA PER L'INGEGNERIA

Corsi di Dottorato

M1 On diffusion phenomena and fractional time-derivatives (3 CFU),
Planned period: 02/05/2019 – 31/05/2019 (maybe delayed to June)
Prof. Masahiro Yamamoto, (The University of Tokyo)

M2 Four Lectures on Analysis (3 CFU), Planned period: May 2019
Prof. Vilmos Komornik, (Université de Strasbourg, France)

M3 The Total Variation Flow (3 CFU), Planned period: June 3-28, 2019
Prof. Jose M. Mazon, (University of Valencia, Spain)

M4 Lectures on Fractional Calculus and Singular Equations (4 CFU) Planned period: January -February 2019
Prof. Mirko D'Ovidio, Tommaso Leonori Francesco Petitta (SBAI, Sapienza)

M5 Lectures on Mean Field Games (2 CFU) Planned period: January -February 2019
Prof. Fabio Camilli (SBAI, Sapienza)

M6 Four Lectures on Homogenization (3 CFU), period to be defined
Prof. Claudia Timofte (University of Bucharest)

List of useful courses from the Master degree:

Master M1 Mathematical Methods for Information Engineering (3 CFU) (Corso di Laurea in Ingegneria delle Comunicazioni)
Prof. Paola Loreti (SBAI, Sapienza)
February-May 2019

Master M2 Metodi Matematici per l'Ingegneria (3 CFU) (Corso di Laurea in Ingegneria Meccanica)
Prof. Daniele Andreucci (SBAI, Sapienza)
November-December 2018

Master M3 Discrete Mathematics (3 CFU) (Corso di Laurea in Ingegneria Elettronica)
Prof. Stefano Capparelli (SBAI, Sapienza)
February-May 2019

Master M4 Metodi Numerici per l'Ingegneria Biomedica (4 CFU) (Corso di Laurea in Ingegneria Biomedica)
Prof. Francesca Pitolli (SBAI, Sapienza)
November-December 2018

M1 <u>On diffusion phenomena and fractional time-derivatives</u> (3 CFU),
Planned period: 02/05/2019 – 31/05/2019 (maybe delayed to June)
Prof. Masahiro Yamamoto, (The University of Tokyo)
(collaboratori: Paola Loreti, Daniela Sforza)

1. Introduction of fractional derivatives
2. Fractional calculus
3. Definition of fractional derivatives in Sobolev spaces and properties
4.-5. Unique existence of solution to the initial −boundary value problem
6. Asymptotic behavior, maximum principle
7. Non-homogeneous boundary value problems
8. Nonlinear equations
9. Optimal control problems
10.-12. Various inverse problems

$$\begin{cases} D^\alpha u &= Lu + f & \text{in} & \Omega^T, \\ u_{|\partial\Omega} &= 0 & \text{for} & t \in (0,T) \\ u_{|t=0} &= u_0 & \text{in} & \Omega, \end{cases}$$

where

$$Lu(x,t) = \sum_{i,j=1}^{N} \partial_i(a_{i,j}(x,t)\partial_j u(x,t)) + \sum_{j=1}^{N} b_j(x,t)\partial_j u(x,t) + c(x,t)u(x,t),$$

$\partial_i = \frac{\partial}{\partial x_i}$ for $i = 1, \dots, N$, and by D^α we denote the Caputo fractional time derivative, i.e.

$$D^\alpha u(x,t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\tau)^{-\alpha} [u(x,\tau) - u(x,0)] d\tau.$$

M2 Four Lectures on Analysis (3 CFU), Planned period: May 2019
Prof. Vilmos Komornik, (Université de Strasbourg, France)
(collaboratori: Paola Loreti)

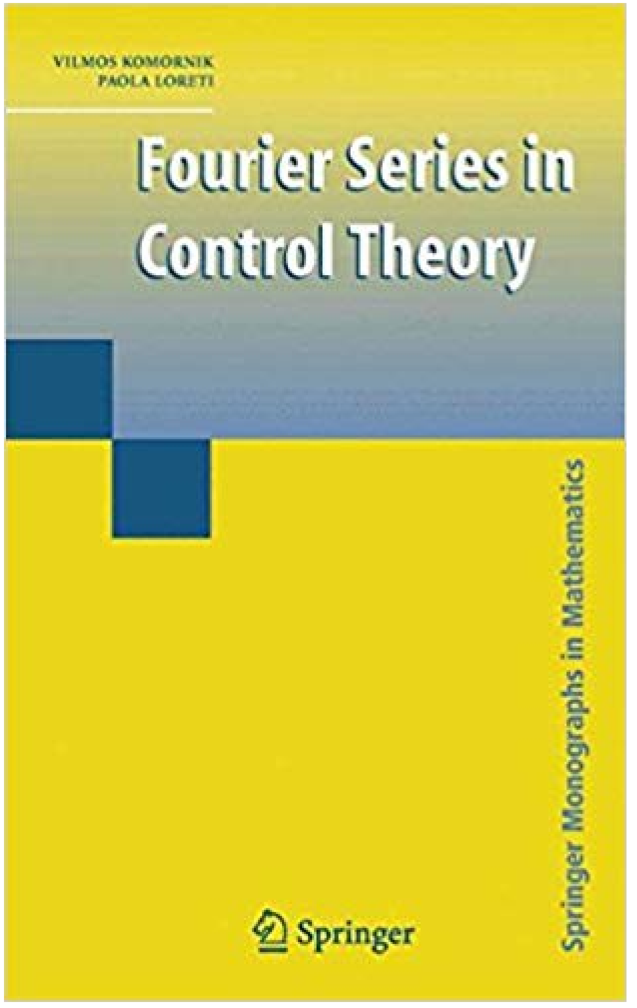
Program:

Research oriented lectures:

1. Fourier analysis in control theory
2. Combinatorial theory of numbers

Didactically oriented lectures:

3. A simplified introduction of the Lebesgue integral
4. Simple and not too well-known proofs of some important theorem of analysis



M3 The Total Variation Flow (3 CFU), Planned period: June 3-28, 2019
Prof. Jose M. Mazon, (University of Valencia, Spain)
(collaboratori: Virginia De Cicco)

Program:
We summarize in this lectures some of our results about the Minimizing Total Variation Flow, which have been mainly motivated by problems arising in Image Processing. First, we recall the role played by the Total Variation in Image Processing, in particular the variational formulation of the restoration problem. Next
we outline some of the tools we need: functions of bounded variation (Section 2), pairing between measures and bounded functions (Section 3) and gradient flows in Hilbert spaces (Section 4). Section 5 is devoted to the Neumann problem for the Total variation Flow. Finally, in Section 6 we study the Cauchy problem for the Total Variation Flow.

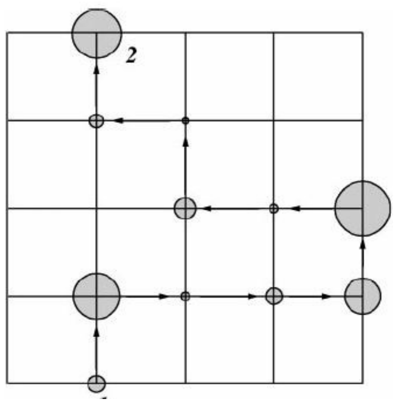
Given an image u_d=Ku+n, ove K é un operatore di convoluzione e n un disturbo, Rudin-Osher-Fatemi proposed the following model the recover the original image u

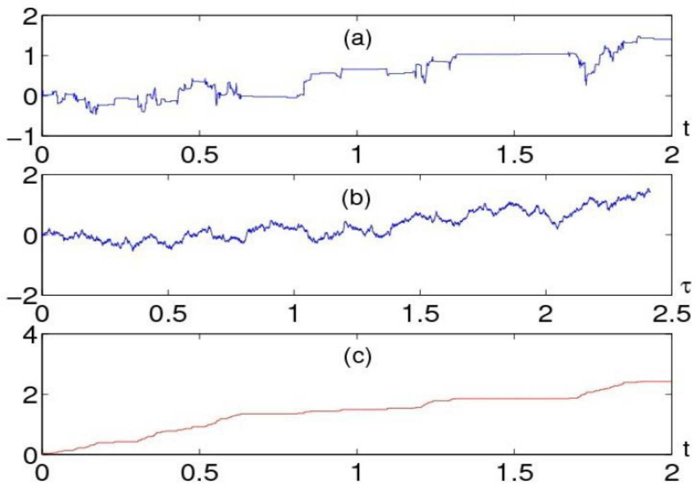
$$\text{Minimize} \int_D |Du|\, dx$$
$$\text{with} \quad \int_D Ku = \int_D u_d, \quad \int_D |Ku - u_d|^2\, dx = \sigma^2 |D|.$$

The first constraint corresponds to the assumption that the noise has zero mean, and the second that its standard deviation is σ. The constraints are a way to incorporate the image acquisition model given by ud. The problem of recovering u from ud is ill-posed and requires a detailed study of the theory of BV function.

M4 Lectures on Fractional Calculus and Singular Equations (4 CFU) Planned period: January -February 2019
Prof. Mirko D'Ovidio, Tommaso Leonori Francesco Petitta (SBAI, Sapienza)

The limit of a continuous time random walk (CTRW)





gives rises to the fractional diffusion equation

$$\partial^{\beta}_{(0,t]} p(x,t) = \partial^2_x p(x,t)$$

where

$$\partial^{\beta}_{(0,t]} p_\beta(x,t) = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{\partial_\tau p_\beta(x,\tau)}{(t-\tau)^\beta} d\tau$$

Mirko D'Ovidio: Time-changed Stochastic processes and Fractional partial differential equations

- Fractional diffusions and time-changed processes
  - Bernstein functions and Sonine kernels
  - Non-local (time-fractional) operators $\mathfrak{D}_t^\Phi$;
  - Non-local (space-fractional) operator $-\Phi(-\Delta)$;
  - The particular case $\Phi(z) = z^\alpha$, $\alpha \in (0,1)$:
    - $\mathfrak{D}_t^\Phi u = (\partial_t)^\alpha u - u_0 t^{-\alpha}/\Gamma(1-\alpha)$, the Caputo fractional derivative
    - $\Phi(-\Delta) = (-\Delta)^\alpha$, the fractional Laplacian
  - We focus on the problems:
    P1) $\mathfrak{D}_t^\Phi u = Au$, $u_0 \in D(A)$
    P2) $\partial_t u = -\Phi(-A)u$, $u_0 \in D(\Phi(-A)) \subset D(A)$
    where $(A, D(A))$ is the generator of a Markov process, Φ is defined above. We study the probabilistic representation of the solutions (time-change and boundary value theory)
  - Probabilistic representation of the solutions to

$$\Phi(-A)u = f \quad \text{and} \quad -Au = \Phi'(0)f.$$

F. Petitta e T.Leonori: Singular Equations

- Anomalous diffusions and Fractional Laplacian
  - The Fractional Laplacian: $(-\Delta)^s u(x)$, $s \in (0,1)$: introduction to the (several) definitions, differences and equivalences;
  - Motivation and framework;
  - Fractional Sobolev spaces;
  - Minimization in $H^s(\Omega)$;
  - Weak and energy formulation for boundary value problems of the type

$$\begin{cases} \mathcal{L}_s u = f & \text{in } \Omega \\ u = 0 & \text{in } \Omega^c \end{cases}$$

    for f in some Lebesgue space, being Ω a bounded open set and $\mathcal{L}$ a nonlocal operator of order s;
  - Elliptic equations involving Fractional Laplacian with Dirichlet and Neumann boundary conditions.

- Singular elliptic equations

Consider a singular elliptic boundary-value problem modeled by

$$\begin{cases} -\Delta u = \frac{f}{u^\gamma} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain of $\mathbb{R}^N$, $\gamma > 0$, and f is a datum. The problem is singular as one ask to u to be zero at the boundary of Ω. One is interested in various questions among which existence, uniqueness and regularity of the solution of BVP modeled by (1).

Physical motivations in the study of such problem arise from the study of thermo-conductivity where u^γ represents the resistivity of the material (Fulks - Maybee, Osaka J. Math. Soc '60). Also in signal transmissions (Nowosad '66), and in the theory non-Newtonian pseudoplastic fluids (Nachman-Callegari, '86).

A main issue that we will address is whether (1) possesses a finite energy solution i.e. $\int_\Omega |\nabla u|^2 < \infty$.

M5 Lectures on Mean Field Games (2 CFU) Planned period: January -February 2019
Prof. Fabio Camilli (SBAI, Sapienza)

The Mean Field Games (MFG) theory was proposed by Lasry-Lions, and independently by Huang-Malhamé-Caines, in 2006. Distinctive features of the model:

MFG theory is a model to describe interactions among a very large number of agents (crowd dynamics, financial market, management of exhaustible resources,...).

MFG shares some analogy with Statistical Mechanics, where an external field (usually a statistics of some given physical quantity) influences the behavior of the particles. But in MFG theory the agent is not a black-box, since it can decide a strategy based on a set of preferences.

The single agent by itself cannot influence the collective behavior, it can only optimize its own strategy given the environmental situation. The mean field is given by the collective behavior of the population.

The mathematical formulation of the problem gives a strongly coupled system of PDEs

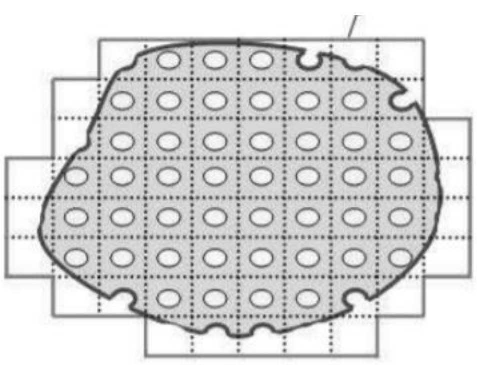
$$\begin{cases} -\nu\Delta v + H(Dv) + \lambda = V[m](x) \\ \nu\Delta m + \operatorname{div}\left(\frac{\partial H}{\partial p}(Dv)m\right) = 0 \\ \int v(x)dx = 0,\ \int m(x)dx = 1,\ m \geq 0 \end{cases}$$

M6 Four Lectures on Homogenization (3 CFU), period to be defined
Prof. Claudia Timofte (University of Bucharest)
(collaboratori: Micol Amar, Daniele Andreucci)

Homogenization appeared more than 100 years ago. It is an approach to study the macro-behavior of a medium by its micro-properties. Homogenization theory studies the limit of the sequences of the problems and its solutions when a parameter tends to zero. This parameter is regarded as the ratio of the characteristic size in the micro scale to that in the macro scale, hence it is considered is a sequence of problems in a fixed domain while the characteristic size in micro scale tends to zero. Aclassicall problem is

$$\begin{cases} -\nabla \cdot (A^\varepsilon(x)\nabla u^\varepsilon(x)) = f, & \text{in } \Omega, \\ u^\varepsilon = 0, & \text{on } \partial\Omega, \end{cases}$$

where $A^\varepsilon(x)=A(x/\varepsilon)$ A typical application is the homogenization of perforated domains



Giovanni Cerulli Irelli: Geometric methods in representation theory of finite dimensional algebras
(15 hours, January-April 2019)

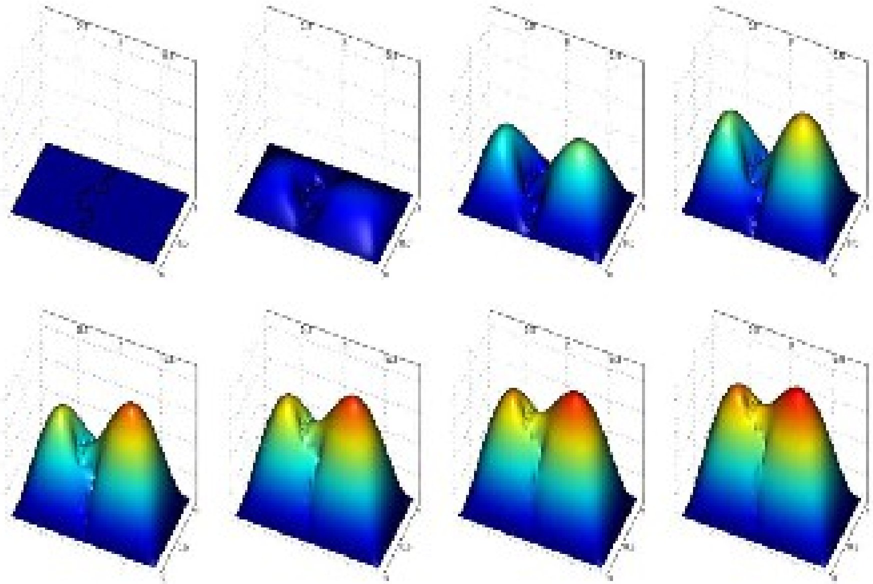
The course is an introduction to the representation theory of quivers and finite dimensional algebras with emphasis on the geometry of objects naturally arising in the context (representation varieties and quiver Grassmannians).

- Generalities on finite dimensional algebras and quivers
- Representations of quivers
- Auslander-Reiten theory
- Quivers of fintie type and Gabriel's theorem
- Quiver Grassmannians and relation to cluster algebras

Bibliography:

- I. Assem, D. Simson, A. Skowronski, “Elements of the representation theory of finite dimensional algebras 1”.
- R. Schiffler: “Quiver Representations”
- W. Crawley-Boevey: “Lectures on Representations of quivers”
- C. M. Ringel: “Tame algebras and integral quadratic forms”

Heat diffusion across fractal interfaces



Temperature at different time steps

DESCRIPTION AND APPLICATIONS

The research is concerned with the rigorous study of a model and its numerical approximation for heat diffusion problems across highly conductive fractal interfaces

- Fractal interfaces, compared with flat interfaces enhance the heat exchange with the surrounding enviroment, they have many applications :
  - Cooling of electronic devices
  - Solar Panels
  - Catalic Converters
  - More efficient hydrophopic surfaces can be obtained by considering fractal interfaces made by nanostructured materials

THE MATHEMATICAL MODEL AND METHODS

- The mathematical model is described by a parabolic PDE (eventually non linear) with dynamical BCS
- The numerical approximation is carried on by a FEM scheme in space and a finite difference scheme in time

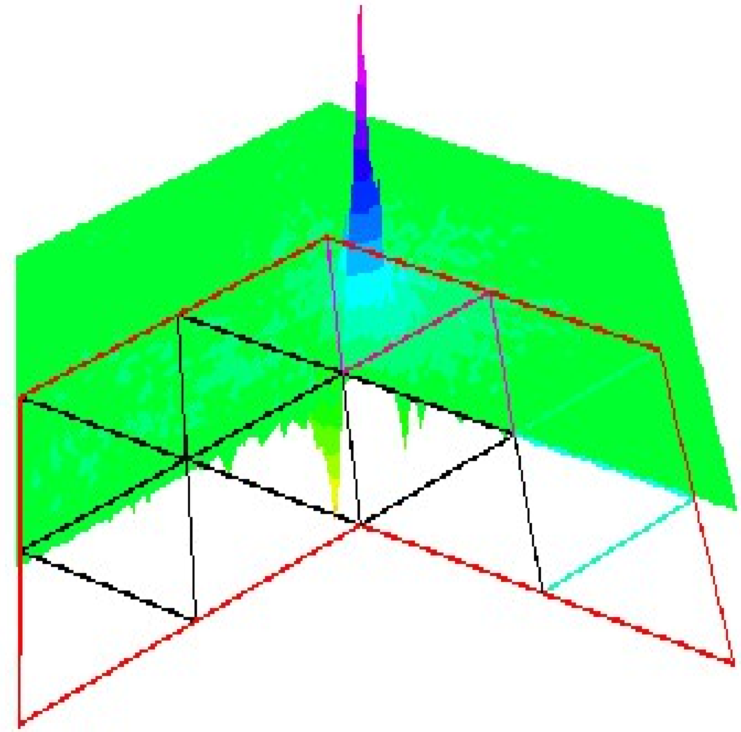
This is part of a general project:
http://www.sbai.uniroma1.it/Fast_diffusion_across_fractal_interfaces

M.R.Lancia

$$\text{Find } u\in\mathcal{K} : \int_\Omega (k^2+|\nabla u|^2)^{\frac{p-2}{2}} \nabla u \, \nabla(v-u)\, dx - \int_\Omega f(v-u)\, dx \geq 0, \; \forall v\in\mathcal{K}$$

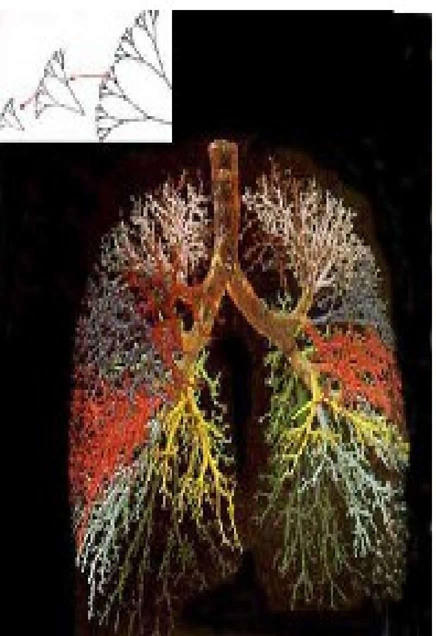
where:

- Ω is a bad domain (e.g. pre-factal or fractal) and $k\in\mathbb{R}$;
- $\mathcal{K}$ is a convex subset of $W^{1,p}(\Omega)$.



Motivations:

- p-Laplacian because it provides a model for many applied problems;
- Fractal and pre-fractal because they describe in a better way the objects of Nature.



Results achieved:

- Existence, uniqueness and regularity of the solutions;
- Optimal error estimates for FEM-solutions.

Perspectives:

- Uniform estimates for the norm of the pre-fractal solutions;
- Uniqueness results for the corresponding problem for $p=\infty$.

R. Capitanelli, S. Fragapane, M. A. Vivaldi