Conferences

Monday 13th

10.15 **Bruno Franchi** Macroscopic models of Alzheimer's Disease

11.30 **Roberto Verzicco** A multi-physics computational model for the human heart

12.15 **Paolo Podio Guidugli** From interaction potentials to distance stress

14.30 **Nicola Fusco** Regularity of capillarity droplets with obstacle

15.15 **Domenico Marinucci** The geometry of random spherical eigenfunctions

16.30 **Flavia Smarrazzo** Noncoercive diffusion equations with Radon measures as initial data

17.15 **Marcos Solera Diana** Variational and diffusion problems in random walk spaces

17.40 **Riccardo Durastanti** Spreading phenomena under mildly singular potentials

Tuesday 14th

09.30 **Henry Berestycki** Modeling the propagation of Covid-19

10.15 **Günther Grün** Free boundary propagation and noise: some recent results on stochastic degenerate parabolic equations

11.30 **Luca Rossi** Asymptotic one-dimensional symmetry for reaction-diffusion equations

12.15 **Alessio Porretta** Traveling waves in mean-field models of knowledge diffusion

14.30 **Hans van Duijn** Non-classical shocks for the Buckley-Leverett equation: The effect of hysteresis and dynamic capillarity

15.15 **Danielle Hilhorst** Convergence to a self-similar solution for a one-phase one dimensional Stefan problem arising in corrosion theory

16.30 **Andrea Tosin** The contribution of kinetic theory to the modelling of Alzheimer's disease

17.15 **Victor Hissink Muller** Well-posedness and regularity of singular-degenerate porous medium type equations and application to biofilm models

17.40 **Giulia Ciavolella** Reaction-diffusion membrane problems: existence of weak solutions

Wednesday 15th

09.30 **Sigurd Angenent** Dynamics of convex mean curvature flow

10.15 **Carlo Nitsch** To be (insulated), or not to be, that is the question

11.30 **Adriano Pisante** Torus-like solutions for the LdG model

12.15 **Joost Hulshof** Work is (in) progress

Abstracts

WE 09.30 **Sigurd Angenent** **Dynamics of convex mean curvature flow**

For compact convex surfaces Huisken proved in 1985 that Mean Curvature Flow (MCF) deforms them into "round points", i.e. after appropriate rescaling they converge to a sphere. On the other hand, solutions with noncompact convex initial data can converge after rescaling to cylinders. In a rescaled version of MCF these cylinders appear as fixed points. In this talk I will report on ongoing work with Daskalopoulos, Sesum, Bourni, Langford, and Nguyen on the solutions of rescaled MCF that connect these fixed points, and on what they mean for MCF.

TU 09.30 **Henry Berestycki** **Modeling the propagation of Covid-19**

I will present a new model in epidemiology that generalizes the classical SIR model. Considering heterogeneity, variability and social diffusion of behaviors we are led to a system of reaction-diffusion equations. I will show that this model yields a rich dynamical structure that reproduces outstanding features of epidemics such as the formation of plateaus, "shoulders" and rebounds. I will discuss the use of this model to understand observations from Wastewater Based Epidemiology (WBE) in France.

I report here on joint work with B. Desjardins, J-M. Oury and J. Weitz[1].

TU 17.15 **Giorgia Ciavolella** **Reaction-diffusion membrane problems: existence of weak solutions**

Partial differential equations describing the movement of cells or particles through a membrane are of high interest in physics, biology or medical sciences. We focus on the biology of cancer, aiming to characterise the invasive process. This lead to a system of reaction-diffusion equations in two sub-domains separated by a zero-thickness membrane with selective permeability to specific populations. The corresponding boundary conditions, describing the flow through the membrane, are called Kedem-Katchalsky conditions.

A huge literature deeply analyses usual reaction-diffusion systems. A specific challenge has been to describe systems with the natural L^1-regularity and with high order nonlinearities, which are the common properties of many biological systems, see works by Pierre and his collaborators. In this talk, we show the extension of this L^1-existence theory to membrane problems. This is a joint work with B. Perthame.

[1] - https://www.nature.com/articles/s41598-021-97077-x
- Quels sont les effets des comportements individuels dans la propagation des épidémies ? (in French) https://www.youtube.com/watch?v=dU7L3F77nU0

Abstracts

TU 14.30 **Hans van Duijn** **Non-classical shocks for the Buckley-Leverett equation: The effect of hysteresis and dynamic capillarity**

In the presentation a problem is discussed that arises in two-phase flow (say, of water and oil) in porous media. The particular focus is on the consequence of taking into account the effect of hysteresis (i.e. the difference between drainage and imbibition) and the effect of non-dynamic capillary pressure.

In the first part of the talk a sketch is given of the background of the problem. This results in a transport equation in which the nonlinear convective term is of convex-concave type. The (pseudo) parabolic term contains a small parameter. When this term is absent, the resulting first order equation is called the Buckley-Leverett (BL) equation.

In the second (main) part, a travelling wave analysis is presented. When the small parameter vanishes, such waves yield so-called viscous shocks for the corresponding (BL) equation. Due to hysteresis and non-dynamic behaviour, shocks may arise that violate the classical Oleinik condition.

In the third part, some numerical results are shown that support our theoretical findings. Moreover a related, gravity-segregation, problem will briefly be discussed.

MO 17.40 **Riccardo Durastanti** **Spreading phenomena under mildly singular potentials**

We look at spreading phenomena under the action of mildly singular potentials. We mainly discuss the static case: depending on the form of the potential, the macroscopic profile of equilibrium configurations can be either droplet-like or pancake-like, with a transition profile between the two at zero spreading coefficient. These results generalize, complete, and give mathematical rigor to de Gennes' formal discussion of spreading equilibria. If time permits, we will briefly discuss the dynamics. This is a joint work with Lorenzo Giacomelli.

MO 10.15 **Bruno Franchi** **Macroscopic models of Alzheimer's Disease**

In this talk we formulate a mathematical model in the case of a specific problem in an early stage of Alzheimer Disease, namely the propagation of pathological τ protein from the entorhinal cortex to the hippocampal region. The main feature of this model consists in the representation of the brain through two superposed finite graphs, which have the same vertices (that, roughly speaking, can be thought as parcels of a brain atlas), but different edges. We call these graphs "proximity graph" and "connectivity graph", respectively. The edges of the first graph take into account the distances of the vertices and the heterogeneity of the cerebral parenchyma, whereas the edges of the second graph represent the connections by white-matter fiber pathways between different structures. The diffusion of the proteins Aβ and τ are described through the Laplace operators on the graphs, whereas the phenomenon of aggregation of the proteins leading ultimately to senile plaques and neuro-fibrillar tangles (as already observed by A. Alzheimer in 1907) is modelled by means of the classical Smoluchowski aggregation system.

MO 14.30 **Nicola Fusco** **Regularity of capillarity droplets with obstacle**

I will present a recent regularity result for Λ-minimizers of the capillarity energy in a half space with the wet part constrained to be confined inside a given planar region. Applications to a model for nanowire growth are also provided.

Abstracts

TU 10.15 **Günther Grün** **Free boundary propagation and noise: some recent results on stochastic degenerate parabolic equations**

Finite speed of propagation and occurrence of waiting time phenomena are characteristic features of degenerate parabolic equations. During the last fifty years, much effort has been invested to develop mathematical theories – first for second-order equations like porous-medium or parabolic p-Laplace equations and more recently for fourth-order equations like the thin-film equation. In this talk, we are mostly interested in stochastic versions of porous-medium equations and of thin-film equations. The latter ones have been suggested to model the effect of thermal fluctuations on very thin liquid films.

For the former equations, we study the question how various versions of multiplicative noise influence expected values for propagation rates and for the size of waiting times. Our qualitative results are based on rigorous SPDE-analysis, the quantitative results are due to Monte-Carlo simulations using convergent discretization schemes. For stochastic thin-film equations, we focus on existence and regularity results of martingale solutions in one and two space dimensions – discussing in particular different analytical approaches depending on the smoothness of the degeneracy.

TU 15.15 **Danielle Hilhorst** **Convergence to a self-similar solution for a one-phase one dimensional Stefan problem arising in corrosion theory**

Steel corrosion plays a central role in different technological fields. In this paper, we consider a simple case of a corrosion phenomenon which describes a pure iron dissolution in sodium chloride. This article is devoted to prove rigorously that under rather general hypotheses on the initial data, the solution of this iron dissolution model converges to a self-similar profile as t tends to infinity. We will do so for an equivalent formulation as presented in the book of Avner Friedman about parabolic equations. In order to prove the convergence result, we apply a comparison principle together with suitable upper and lower solutions. This is joint work with Meriem Bouguezzi, Yasuhito Miyamoto and Jean-François Scheid.

TU 17.15 **Victor Hissink Muller** **Well-posedness and regularity of singular-degenerate porous medium type equations and application to biofilm models**

In this talk we consider a quasi-linear reaction-diffusion equation that appears in models for biofilm growth. The solution-dependent diffusion coefficient vanishes when the solution is zero and becomes singular as the solution tends to a certain maximal value. In particular, the equation is degenerate and it has a singularity, which makes it challenging and very interesting to study. I will discuss the existence, uniqueness and continuous dependence on initial data of solutions of initial- / boundary value problems with mixed Dirichlet-Neumann boundary conditions. These results are based on methods used to study elliptic-parabolic equations and they involve energy estimates and an L^1-contraction result. Further, I will discuss Hölder continuity of solutions, which is shown by intrinsic scaling methods known from the analysis of the porous medium equation and the p-Laplace equation.

Abstracts

WE 12.15 **Joost Hulshof** — **Work is (in) progress**

In honour of Michiel Bertsch, in loving memory of friends.

MO 15.15 **Domenico Marinucci** — **The geometry of random spherical eigenfunctions**

A lot of efforts have been devoted in the last decade to the investigation of the high-frequency behaviour of geometric functionals for the excursion sets of random spherical harmonics, i.e., Gaussian eigenfunctions for the spherical Laplacian. In this talk we shall review some of these results, with particular reference to the asymptotic behaviour of variances, phase transitions in the nodal case (the Berry's Cancellation Phenomenon), the distribution of the fluctuations around the expected values, and the asymptotic correlation among different functionals.

WE 10.15 **Carlo Nitsch** — **To be (insulated), or not to be, that is the question**

A well known dilemma in thermal insulation is how much insulator is worth using to insulate a given conductor. A careful estimate is necessary to avoid ineffective or self-defeating consequences. From the mathematical point of view a good insulation is achieved when one manages to decrease the boundary integral of the temperature (the heat loss rate) in spite of an unavoidable increase of the perimeter. For given volumes of both the conductor and the insulator, we shall look for optimal shapes which provide the ideal insulation. We set up and solve the (double) free boundary problem, which allows us to compare any configuration with a (completely solvable) symmetric one.

WE 11.30 **Adriano Pisante** — **Torus-like solutions for the LdG model**

We report on some recent progress about the study of global minimizers of a continuum Landau-de Gennes energy functional for nematic liquid crystals in three-dimensional domains. First, we discuss absence of singularities for minimizing configurations under norm constraint, as well as absence of the isotropic phase for the unconstrained minimizers, together with the related biaxial escape phenomenon. Then, under suitable assumptions on the topology of the domain and on the Dirichlet boundary condition, we show that smoothness of energy minimizing configurations yields the emergence of nontrivial topological structure in their biaxiality level sets. Then, we discuss the previous properties under both the norm constraint and an axial symmetry constraint, showing that in this case only partial regularity is available, away from a finite set located on the symmetry axis. In addition, we show that singularities may appear due to energy efficiency and we describe precisely the asymptotic profile around singular points. Finally, in an appropriate class of domains and boundary data we obtain qualitative properties of the biaxial surfaces, showing that smooth minimizers exhibit torus structure, as predicted in numerical simulations.

MO 12.15 **Paolo Podio-Guidugli** — **From interaction potentials to distance stress**

Discrete-to-continuum coarsening of matter description may imply the construction of a continuum notion of stress-like surface interactions accounting for discrete distance interactions between adjacent body parts. If time allows, I shall hint at certain connections with nonlocal geometry of surfaces and fractional diffusion.

Abstracts

TU 12.15 **Alessio Porretta** **Traveling waves in mean-field models of knowledge diffusion**

The Fisher-KPP equation is used in economics to describe endogenous growth models where production is based on technological skill and knowledge diffusion. We discuss here a mean-field model of this kind introduced by Nobel Laureate B.E. Lucas with S. Moll. This results into a PDE system where a backward Hamilton-Jacobi-Bellman equation is coupled with a forward KPP-type equation with nonlocal reaction term. In a joint work with Luca Rossi (Rome La Sapienza), we study the existence of critical traveling waves which yield balanced growth paths for the described economy, supposed to be the expected stable growth in the long run. Our result stands on a new analysis of forced speed traveling waves for KPP equations of nonlocal-type, and several interesting questions remain open.

TU 11.30 **Luca Rossi** **Asymptotic one-dimensional symmetry for reaction-diffusion equations**

The symmetry of solutions of elliptic equations is a classical and challenging problem in PDEs, strictly linked with stability. We consider in this talk parabolic equations and we ask whether the 1-dimensional symmetry eventually emerges in the long time, for solutions which are initially non-symmetric. We will present a satisfactory answer in the case of the Fisher-KPP equation, together with some counter-examples and open questions. This topic is the object of a joint work with F. Hamel.

MO 17.00 **Flavia Smarrazzo** **Noncoercive diffusion equations with Radon measures as initial data**

Initial-boundary value problems for the nonlinear parabolic equation $u_t = \Delta\phi(u)$, having Radon measures as initial data, have been widely investigated in the last decades, looking for solutions which for positive times take values in some *function* space. On the other hand, if the diffusivity degenerates too fast at infinity, it is well known that *function-valued* solutions may not exist and it looks very natural to consider *measure-valued* solutions, *i.e.* mappings taking values in the space of the finite Radon measures, which satisfy the problem in a suitable weak sense.

In this talk, I will discuss existence and partial uniqueness results of *measure-valued* solutions to the Cauchy-Dirichlet problem for the equation $u_t = \Delta\phi(u)$, for a continuous, nondecreasing, at most power-like ϕ. The notion of solution is natural, since it is obtained by a suitable approximation procedure which also relies on regularization of the initial measure. Moreover, some qualitative properties of the *constructed solutions* will be discussed, concerning in particular *persistence* or *regularization* of initial singularities.

Joint work with M. M. Porzio and A. Tesei.

Abstracts

MO 17.15 **Marcos Solera** **Variational and diffusion problems in random walk spaces**

The digital world has brought with it many different kinds of data of increasing size and complexity. Therefore, the study and treatment of big data sets has become of great interest and value. In this respect, weighted discrete graphs provide a natural and flexible workspace in which to represent the data. As a consequence, research of partial differential equations on graphs has become of primary interest. On another note, nonlocal models arising from a variety of scientific fields like biology, particle systems, coagulation models, nonlocal anisotropic models for phase transition, mathematical finances using optimal control theory, etc. have brought special attention to the study of partial differential equations in nonlocal settings.

The aim of this talk is to present a framework in which to unify into a broad framework the study of the previously mentioned problems. In doing so we study the heat flow, the total variation flow, nonlinear diffusion problems of p-Laplacian type with nonlinear boundary conditions, and more.

TU 17.00 **Andrea Tosin** **The contribution of kinetic theory to the modelling of Alzheimer's disease**

The onset and spreading of Alzheimer's disease in the cerebral tissue is a macroscopic outcome of cellular and subcellular chemical processes happening to single neurons. In recent years, the mathematical modelling of Alzheimer's disease has gained a lot of momentum in an attempt to help understand its fundamentals so far known mostly at a descriptive level and often with substantial disagreement among different neurobiological schools. Due to the intrinsic multiple-scale nature of this complex phenomenon, most models tend to focus on single biochemical aspects taking place at a specific micro-scale. More comprehensive modelling requires instead non-trivial multi-scale mathematical theories and tools. In this talk, I will show how the Boltzmann-type paradigm of the classical kinetic theory may be revisited to pass from a statistical description of networked interactions among the neurons to an aggregate one of macroscopic disease waves spreading in the brain. In particular, this requires elaborating on concepts such as integro-differential kinetic equations and their asymptotic and embedding a statistical description of the networked structure of the cerebral tissue into a Boltzmann-type description of particle interactions.

The work presented in this talk is part of a broader research line[2] that I am honoured to have been introduced to and share with Michiel, Bruno Franchi and Maria Carla Tesi.

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[2] M. Bertsch, B. Franchi, N. Marcello, M. C. Tesi, A. Tosin. Alzheimer's disease: a mathematical model for onset and progression, Math. Med. Biol., 34(2):193-214, 2017

M. Bertsch, B. Franchi, M. C. Tesi, A. Tosin. Microscopic and macroscopic models for the onset and progression of Alzheimer's disease, J. Phys. A: Math. Theor., 50(41):414003/1-22, 2017

M. Bertsch, B. Franchi, M. C. Tesi, A. Tosin. Well-posedness of a mathematical model for Alzheimer's disease, SIAM J. Math. Anal., 50(3):2362-2388, 2018

M. Bertsch, B. Franchi, V. Meschini, M. C. Tesi, A. Tosin. A sensitivity analysis of a mathematical model for the synergistic interplay of amyloid beta and tau on the dynamics of Alzheimer's disease, Brain Multiphysics, 2:100020/1-13, 2021

Abstracts

MO 11.30 **Roberto Verzicco** **A multi-physics computational model for the human heart**

Clinical trials are key for advancing cardiovascular research although they entail long and costly processes for the recruitment of representative cohorts of volunteers. The problems are exacerbated for rare pathologies or if multiple concurrent requirements have to be met; in fact, uneven sampling and missing data might result in biased cohorts which, in turn, yield incomplete or misleading results. On the other hand, a digital twin of the heart (or a virtual physical model), once fed by appropriate input parameters, can be resorted to surrogate real patients provided its outcome is reliable and cost effective. However, creating a virtual model of the whole heart is a formidable task since it involves the complex deforming biological tissues, the transitional and turbulent hemodynamics, the myocardium electrophysiology the strong multi-way interaction of all these systems and the heart connection with the main arteries and veins. Furthermore, in order for the digital twin to be predictive, hundreds of million degrees of freedom are necessary and, even on supercomputers, they require simulation times of weeks or months, thus preventing the clinical use of these models: overcoming such limitation has huge cardiovascular potential and this motivates the present work.

In this study we present a high-fidelity computational model of the whole human heart, relying on the latest GPU-acceleration technologies, fully replicating the cardiac dynamics within a few hours. The predictive capability of the model combined with the unprecedented computational speed-up open the way to simulation campaigns to study the response of synthetic cohorts to pathologies, novel medical devices and surgical procedures. This innovative approach makes possible a systematic use of digital twins in cardiovascular research, thus reducing the extensive use of in–vivo experiments with their economical and ethical implications. This study is a major step towards in-silico clinical trials in the era of digital medicine.

This is a joint work with G. Del Corso and F. Viola.