Dirichlet problem for the Laplacian and the Bilaplacian in Lipschitz Domains

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SUMMARY

We are interested here in questions related to the **maximal regularity** of solutions of **elliptic** problems with **Dirichlet** boundary condition (see ([1]). For the last 40 years, many works have been concerned with questions when Ω is a **Lipschitz domain**. Some of them contain incorrect results that are corrected in the present work.

We give here new proofs and some complements for the case of the **Laplacian** (see [3]), the **Bilaplacian** ([2] and [6]) and the operator div $(A\nabla)$ (see ([5]), when **A** is a matrix or a function. And we extend this study to obtain other regularity results for domains having an adequate regularity. We give also new results for the **Dirichlet-to-Neumann** operator for Laplacian and Bilaplacian.

Using the duality method, we can then revisit the work of Lions-Magenes [4], concerning the so-called **very weak solutions**, when the data are less regular.

Keywords: Elliptic problems, Lipschitz domains, maximal regularity, Steklov Poincaré operator.

AMS Classification: 35C15, 35J25, 35J40

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