# Divergence Elliptic Equations in Lipschitz and in $\mathcal{C}^1$ Domains

**Chérif Amrouche, Pau University (France) and IIT Kanpur (India)**

**SUMMARY**

We are interested here in questions related to the study of some divergence elliptic equations in bounded Lipschitz or $\mathcal{C}^1$ domains:

$$-\text{div}\,(a\nabla u) + bu = f \quad \text{in } \Omega, \tag{1}$$

with **Dirichlet** or **Neumann** boundary condition. We will consider three different cases.

**Case 1.** We assume $a = 1$ and $b = 0$, corresponding to the **Laplace equation**. We will give some new results on the **traces** of non smooth functions, harmonic or non-harmonic. Using in particular the interpolation theory, we are going to study the questions of existence and **maximal regularity** of solutions in **fractional Sobolev** spaces or with **weights** associated with the **distance to the boundary**.

**Case 2.** We assume that $b = 0$ and $a$ satisfies the classical condition to ensure the ellipticity of the operator - div($a$ grad). We will concentrate on the case of generalized solutions in $W^{1,p}(\Omega)$ with $1 < p < \infty$.

**Case 3.** We will finally consider the following problem:

$$-\text{div}\,(\varrho^\alpha \nabla u) + k\frac{u}{\varrho^\beta} = f \quad \text{in } \Omega, \tag{2}$$

with or without boundary condition and where $k$ is a non negative constant and $\alpha$ and $\beta$ belong to the interval $[0, 1]$.

**Keywords:** Elliptic problems, Lipschitz and $\mathcal{C}^1$ domains, maximal regularity, traces, fractional and weighted Sobolev spaces.

**AMS Classification:** 35C15, 35J25, 35J40