Indecomposable hypergraphs and 1-factorizations.

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(based on a joint work with Gloria Rinaldi)

In this seminar we deal with the existence of indecomposable hypergraphs. In particular, we show old and new results about the existence of indecomposable 1-factorizations of the complete multigraph λK_{2n}.

A 1-factorization of λK_{2n} is indecomposable if it does not contain a 1-factorization of $\lambda' K_{2n}$, with $\lambda' < \lambda$. Indecomposable 1-factorizations of the complete multigraph correspond to a particular class of indecomposable hypergraphs.

We recall that a hypergraph H is a pair $H = (V(H), E(H))$, where V is a finite set, whose elements are called vertices, and $E(H)$ is a collection of subsets of V, whose elements are called hyperedges. If every hyperedge has cardinality k, then H is said to be k-uniform. H is said to be simple, if $E(H)$ does not contain the same set more than once. The degree of a vertex $x \in V$ is defined as the number of hyperedges containing x. If all vertices of H have the same degree d, then H is said to be d-regular. A spanning subhypergraph of H is a hypergraph $K = (V(K), E(K))$ such that $V(K) = V(H)$ and $E(K) \subseteq E(H)$. If the hypergraph H possesses no proper non-empty regular spanning subhypergraph, then H is said to be indecomposable. An indecomposable 1-factorization of the complete multigraph λK_{2n} provides a λ-regular indecomposable n-uniform hypergraph H with vertex-set of cardinality $n(2n-1)$. The vertices of H are the edges of λK_{2n}, each hyperedge is a 1-factor of λK_{2n}.

The problem of determining the exact value of $D(m)$ and $D(m,k)$, where $D(m) = \max\{d$: there exists a d-regular indecomposable hypergraph on m vertices $\}$, $D(m,k) = \max\{d$: there exists a d-regular indecomposable k-uniform hypergraph on m vertices $\}$, is widely studied since it has applications in Game Theory. We review some results estimating these parameters and show some open problems related to the existence of simple and indecomposable 1-factorization of λK_{2n} for small values of n.