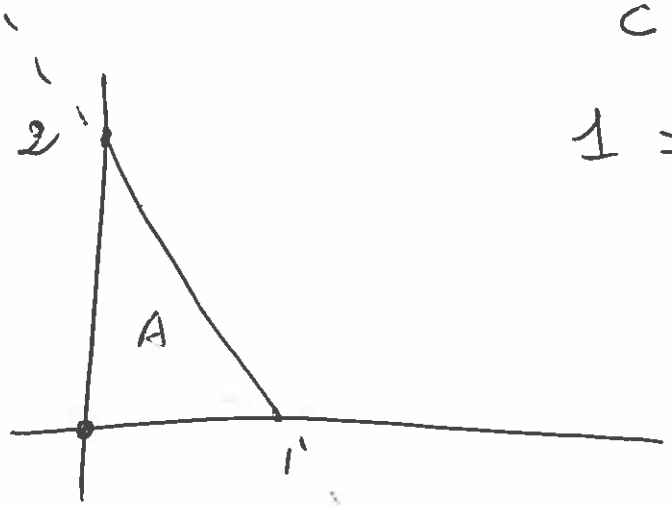
1) $$f(x,y) = \begin{cases} c\,x^2 y & (x,y) \in A \\ 0 & \text{altrove} \end{cases}$$




$c > 0$

$$1 = \int_{\mathbb{R}^2} f(x,y)\,dy\,dx = \int_0^1 \int_0^{2-2x} c\,x^2 y\,dy\,dx$$

$y = 2 - 2x$

$$= \int_0^1 c\,x^2 \frac{(2-2x)^2}{2}\,dx = \int_0^1 c\,x^2\,2\,(1-x)^2\,dx = 2c\,\frac{2!\,2!}{5!} = \frac{c}{15}$$

$$\Rightarrow c = 15$$

$$f_X(x) = \begin{cases} 0 & x < 0 \text{ opp } x \geq 1 \\ ? & 0 \leq x < 1 \end{cases}$$

$0 < x < 1$

$$f_X(x) = \int_0^{2-2x} 15\,x^2 y\,dy = \frac{15}{2}\,x^2 \cdot 2\,(1-x)^2 = 30\,x^2(1-x)^2$$

$$F_Y(y) = \begin{cases} 0 & y<0 \\ ? & 0\le y<2 \\ 1 & y\ge 2 \end{cases}$$

$0 \le y < 2$

$y = 2 - 2x \Rightarrow 2x = 2-y \Rightarrow x = 1 - y/2$

$$F_Y(y) = \int_0^y \int_0^{1-t/2} 15x^2 t \, dx \, dt$$

$$= \int_0^y \frac{15}{3}\left(1-t/2\right)^3 t \, dt$$

$$= \int_0^y \frac{5}{8}(2-t)^3 t \, dt$$

$$= \int_0^{y/2} \frac{5}{8}(2-2z)^3 2z \cdot 2\,dz \qquad z = t/2$$

$$= \int_0^{y/2} 20(1-z)^3 z \, dz =$$

$$= \int_0^{y/2} 20(1-z^3+3z^2-3z)z \, dz = \int_0^{y/2} 20(z-z^4+3z^3-3z^2)dz$$

$$= 20\left(\frac{y^2}{8} - \frac{1}{5}\frac{y^5}{32} + \frac{3}{4}\frac{y^4}{16} - \frac{y^3}{8}\right)$$

$$Cov(X,Y) = \mathbb{E}(X \cdot Y) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$\mathbb{E}(X \cdot Y) = \int_0^1 \int_0^{2-2x} 15 x^3 y^2 \, dy \, dx =$$

$$= \int_0^1 15 x^3 \, \frac{(2-2x)^3}{3} \, dx$$

$$= \int_0^1 5 x^3 \, 8 (1-x)^3 dx = 40 \, \frac{6 \cdot 6}{7!}$$

$$\mathbb{E}(X) = \int_0^1 30 x^3 (1-x)^2 dx = 30 \, \frac{6 \cdot 2}{6!}$$

$$\mathbb{E}(Y) = \int_0^2 y \, f_Y(y) \, dy$$

dove

$f_Y(y) = F'(y)$ .

④

$$P(Y \le X \mid Y \le 1) = \frac{P(Y \le X,\ Y \le 1)}{P(Y \le 1)}$$

$$P(Y \le 1) = F_Y(1) = 20\left(\frac{1}{8} - \frac{1}{5}\frac{1}{32} + \frac{3}{4}\frac{1}{16} - \frac{1}{8}\right)$$

$$= 5\left(\frac{1}{4} - \frac{1}{5}\frac{1}{8} + \frac{3}{16} - \frac{1}{2}\right)$$

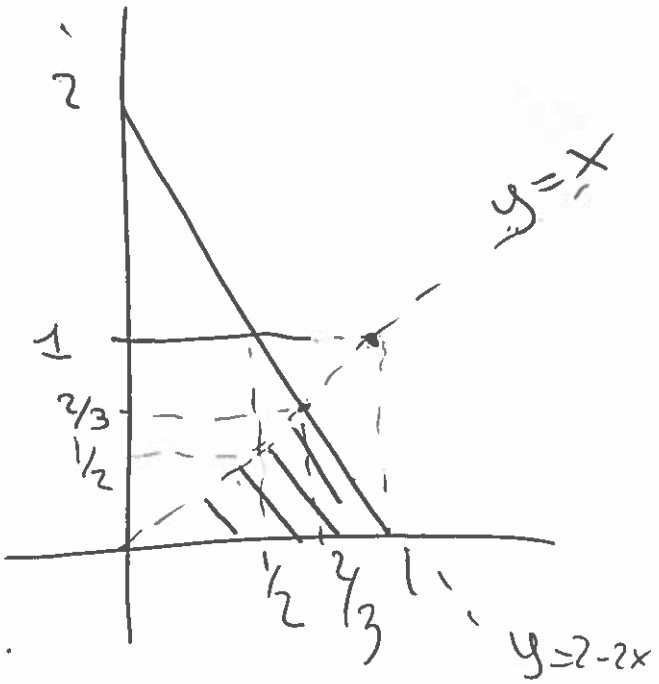
$$P(Y \le X,\ Y \le 1) =$$

$$\begin{cases} y = x \\ y = 2 - 2x \end{cases}$$

$$\Downarrow$$

$$x = 2 - 2x \Rightarrow x = 2/3$$

$$\Rightarrow y = 2/3$$




$$= \int_0^{2/3} \int_y^{\frac{2-2x}{2}} 15\, x^2 y \, dx\, dy$$

# Es 5.2

$$P(X=0) = \frac{8}{32} = 1/4$$

$$P(X=1) = 1/4$$

$$P(X=2) = 1/4$$

$$P(X=3) = 1/4$$

$$\mathbb{E}(X) = \sum_{k=0}^{3} k\, P(X=k) = \frac{1}{4}(0+1+2+3) = \frac{6}{4} = \frac{3}{2}$$

siccome $P(X=2, Y=2) = 0$

mentre $P(X=2) = 1/4$ e $P(Y=2) = \frac{5}{32}$

si ha $P(X=2, Y=2) \neq P(X=2)P(Y=2)$

$$\mathrm{cov}(X,Y) = \mathbb{E}(X \cdot Y) - \mathbb{E}(X)\mathbb{E}(Y)$$

$$\mathbb{E}(X \cdot Y) = 1 \cdot \frac{1}{8} + 1 \cdot 2 \cdot \frac{1}{16} + 2 \cdot 1 \cdot \frac{1}{8}$$

$$\mathbb{E}(Y) = 1 \cdot \frac{11}{32} + 2 \cdot \frac{5}{32} + 3 \cdot \frac{1}{32}$$

~~$Z = X \cdot Y$~~

$Z = Y - X$

$Z = 0 \equiv (X = Y)$

$P(X = Y) = 1/32 + \frac{1}{8}$

$Z = 1 \equiv (Y = X + 1)$

$P(Y = X+1) = \frac{3}{32} + \frac{1}{16}$

$Z = 2 \equiv (Y = X + 2)$

$P(Y = X+2) = \frac{3}{32}$

$P(Z = 3) = P(Y = 3, X = 0) = 1/32$

$P(Z = -1) = P(Y = -1 + X) = 1/16 + \frac{1}{8}$

$P(Z = -2) = P(Y = -2 + X) = \frac{1}{8} + 0$

$P(Z = -3) = P(X = 3, Y = 0) = 1/4$