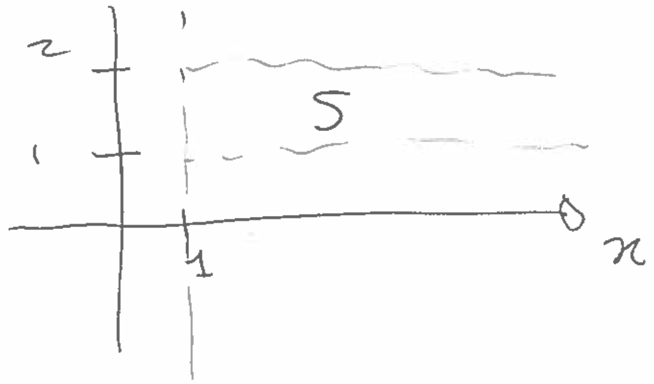
Es. 3



$$f(x,y) = \begin{cases} x^{-2} & (x,y) \in S \\ 0 & \text{altrove} \end{cases}$$

$$f_X(x) = \begin{cases} 0 & x < 1 \\ x^{-2} & x \geq 1 \end{cases}$$

$1 < x$

$$f_X(x) = \int_{\mathbb{R}} f(x,y)\, dy = \int_1^2 x^{-2}\, dy = x^{-2}$$

~~X e Y sono indipendenti~~

$f_Y(y) = 0 \quad \forall y \notin (1,2)$

$1 < y < 2 \quad f_Y(y) = \int_{\mathbb{R}} f(x,y)\, dx = \int_1^{+\infty} x^{-2}\, dx$

$$= -x^{-1}\Big|_1^{+\infty} = 1 \quad \Rightarrow Y \sim \mathcal{U}_{(1,2)}$$

X e Y sono indipendenti siccome

$\forall (x,y) \in S \quad f(x,y) = x^{-2} = f_X(x) f_Y(y)$

$\forall (x,y) \notin S \quad f(x,y) = 0$ e $f_X(x)$ o $f_Y(y)$ è nullo

Q è quadrato di vertici
$(1,1), (1,2), (2,1), (2,2)$

$$P((X,Y) \in Q) = P(1 \leq X \leq 2)$$

$$= \int_1^2 x^{-2}\, dx = -x^{-1}\Big|_1^2 = 1 - \frac{1}{2} = 1/2$$

$$\mathbb{E}(X) = \int_{\mathbb{R}} x f_X(x)\, dx = \int_1^{+\infty} x \frac{1}{x^2}\, dx$$

$$= (\ln x)\Big|_1^{+\infty} = +\infty$$

$\mathbb{E}(\sqrt{X}\, Y) = \mathbb{E}(\sqrt{X})\,\mathbb{E}(Y)$ dall'indipendenza di X e Y

$\mathbb{E}(Y) = 3/2$ siccome $Y \sim Unif(1,2)$

$$\mathbb{E}(\sqrt{X}) = \int_{\mathbb{R}} \sqrt{x}\, f_X(x)\, dx = \int_1^{+\infty} \sqrt{x}\, \frac{1}{x^2}\, dx$$

$$= \int_1^{+\infty} x^{-3/2}\, dx = -x^{-1/2}\Big|_1^{+\infty} = 1$$

$$\Rightarrow \mathbb{E}(\sqrt{X}\, Y) = \mathbb{E}(\sqrt{X})\,\mathbb{E}(Y) = 1 \cdot \frac{3}{2} = 3/2$$

Es. 4

X assume valori nell' intervallo $[5, 10]$

$$f_X(x) = F_X'(x)$$

$$= \begin{cases} 0 & x < 0 \text{ opp } x \geq 10 \\ x/25 & 0 \leq x < 5 \\ -\frac{x}{25} + \frac{2}{5} & 5 \leq x < 10 \end{cases}$$

$$\mathbb{E}(X) = \int_{\mathbb{R}} x f_X(x)\, dx =$$

$$= \int_0^5 x \cdot \frac{x}{25}\, dx + \int_5^{10} x \cdot \left(-\frac{x}{25} + \frac{2}{5}\right) dx$$

$$= \frac{5^3}{75} + \frac{10^2 - 5^2}{5} - \frac{10^3 - 5^3}{75}$$