Soluzioni

(1) $r > R_3$ $\quad \vec{E}_0 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}\hat{r}$

$$V(\vec{r}) = -\int_{\infty}^{r} \vec{E}_0 \cdot \vec{dl} + V(\infty) = \frac{Q}{4\pi\varepsilon_0 r}$$

$r = R_3$ $\quad V(R_3) = \dfrac{Q}{4\pi\varepsilon_0 R_3}$

che vale in $R_3 \geq r \geq R_2$

$R_1 < r < R_2$

$$V(\vec{r}) = -\int_{R_2}^{r} \vec{E}_0 \cdot \vec{dl} + V(R_2) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R_2}\right) + \frac{Q}{4\pi\varepsilon_0 R_3}$$

$$r = R_1 \Rightarrow V(R_1) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$r < R_1$: $\quad \vec{E} = \dfrac{Q}{4\pi\varepsilon R_1^3}\vec{r}$

$$V(\vec{r}) = -\int_{R_1}^{r} \frac{Q}{4\pi\varepsilon_0 R_1^3}\vec{r} \cdot \vec{dl} + V(R_1) = \frac{Q}{8\pi\varepsilon R_1^3}(R_1^2 - r^2) + V(R_1)$$

(2) Il campo può essere pensato prodotto dalle due correnti

$$j = \frac{I}{\pi(b^2-a^2)}$$ in tutto il cilindro di raggio b.

$-j$ nel cilindro di raggio a

Quindi al centro del foro si ha:

$$\oint \vec{H}\cdot d\vec{l} = H\, 2\pi d = \int_S \vec{j}\cdot \hat{n}\, dS = \frac{I}{\pi(b^2-a^2)}\pi d^2$$

$$H = \frac{I d}{2\pi(b^2-a^2)} = 25 \ A/m$$

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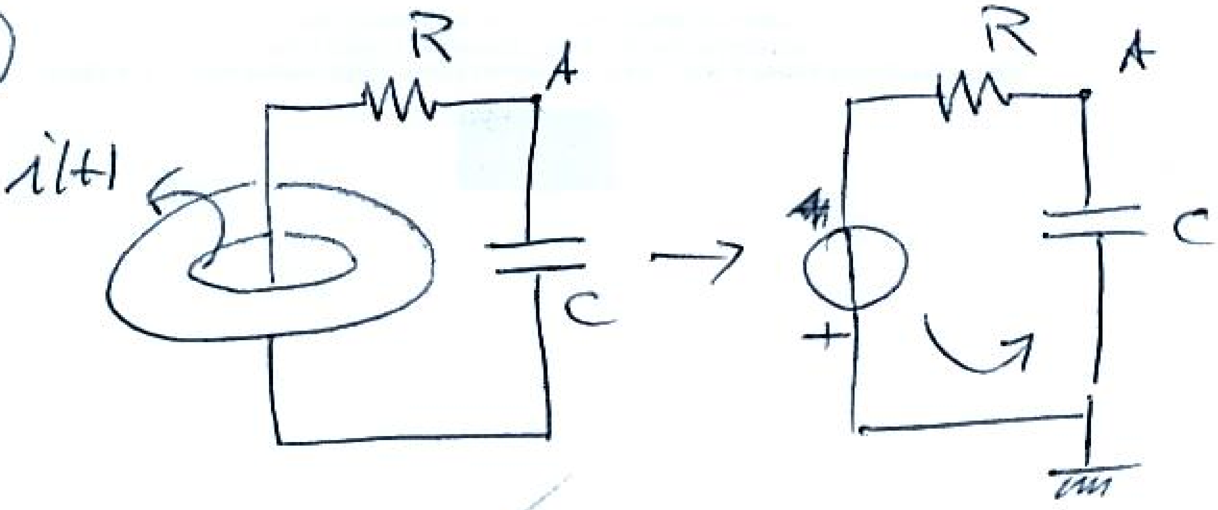
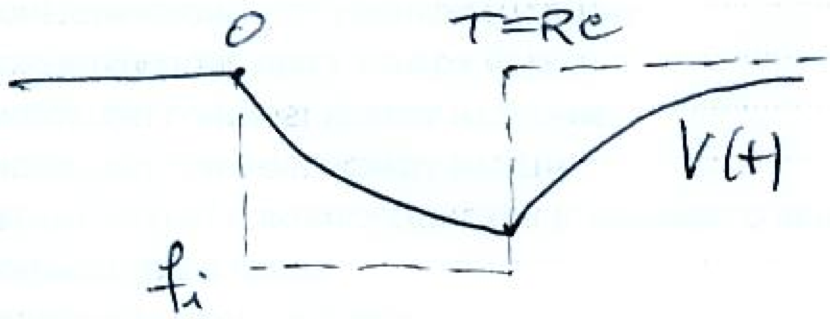


$$NI = \frac{e}{\mu S}\Phi \Rightarrow \Phi = \frac{\mu S N I}{e}$$

$$f_i = -\frac{d\Phi}{dt} = -\frac{d}{dt}\frac{\mu N k t S}{e} = -\frac{\mu N k S}{e}$$

$$V(t) = -\frac{\mu N k S}{e}\left(1 - e^{-t/\tau}\right)$$



④

$$q = \int_0^\infty i\,dt = \int_0^\infty -\frac{d\Phi/dt}{R} = -\frac{1}{R}\int_{\Phi_{iniziale}}^{\Phi_{finale}} d\Phi =$$

$$= -\frac{\Phi_{finale} - \Phi_{iniziale}}{R} = \frac{\Phi_{iniziale}}{R}$$

$$R = \rho\frac{\ell}{S} = \rho\frac{2\pi r}{S}$$

$$\Phi_{iniziale} = B\pi r^2$$

$$\rightarrow q = \frac{B\pi r^2}{2\rho\pi r}S = \frac{BrS}{2\rho} = 1{,}47\,C$$

(5)

[Formule di Werner: $2\cos\alpha\cos\beta = \cos(\alpha+\beta)+\cos(\alpha-\beta)$]

$$\vec{E}_1 = E_m \cos(kx-\omega t)\,\hat{j}$$

$$\vec{E}_2 = E_m \cos(kx+\omega t)\,\hat{j}$$

$$\vec{E}_T = \vec{E}_1 + \vec{E}_2 = 2E_m \cos(kx)\cos(\omega t)\,\hat{j}$$

$$\vec{J}_{sp} = \varepsilon_0 \frac{\partial \vec{E}_T}{\partial t} = -2\varepsilon_0 \omega E_m \cos(kx_0)\,\mathrm{sen}(\omega t)\,\hat{j}$$

$$E_m = \sqrt{2Z_0 I}$$

$$|\vec{J}_{sp}| = 2\varepsilon_0 \omega E_m = 2\varepsilon_0 2\pi f \sqrt{2Z_0 I} =$$

$$= 2\cdot\varepsilon_0 2\pi \frac{c}{\lambda}\sqrt{2Z_0 I} =$$

$$= 2\cdot 8{,}854\cdot 10^{-12}\left[\frac{F}{m}\right]\cdot 2\pi 3\cdot 10^{9}\left[\frac{1}{s}\right]\sqrt{2377{,}1}\left[\frac{V}{m}\right] =$$

$$= 2{,}9\cdot \left[\frac{A}{m^2}\right]$$

* $\cos(kx_0) = \cos\left(\frac{2\pi}{\lambda}x_0\right) \overset{x_0 = 1\,m}{=} \cos\left(\frac{2\pi}{\lambda}\right) = \cos[10(2\pi)]$

$$= 1$$