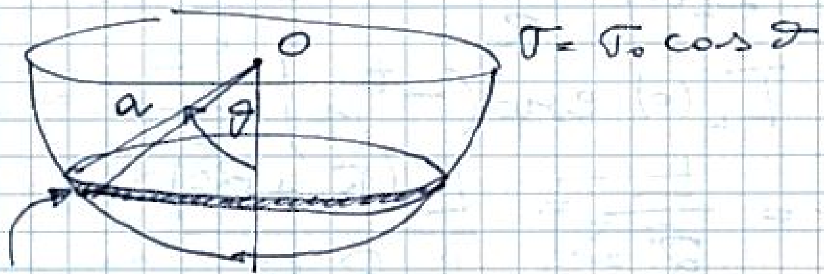
Soluzioni

(1)



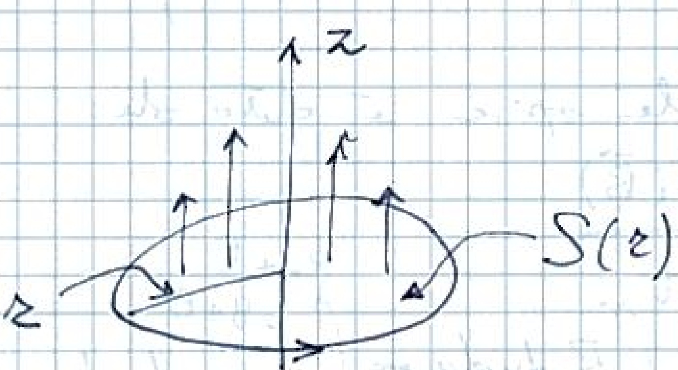
$2\pi a \sin\vartheta \cdot a\,d\vartheta \rightarrow$ area infinitesima.

$$dV(O)_{spira} = \frac{1}{4\pi\varepsilon_0} \frac{\sigma_0 \cos\vartheta \cdot 2\pi a \sin\vartheta \cdot a\,d\vartheta}{a}$$

$$V(O) = \frac{\sigma_0 a}{2\varepsilon_0} \int_0^{\pi/2} \sin\vartheta \cos\vartheta\, d\vartheta = \frac{\sigma_0 a}{2\varepsilon} \int_0^{\pi/2} d\left(\frac{\sin^2\vartheta}{2}\right)$$

$$V(O) = \frac{\sigma_0 a}{4\varepsilon_0}.$$

(2)



Per Ampere:

$$\underline{\underline{B \cdot 2\pi r}} = \mu_0 \int_{S(r)} \vec{J} \cdot d\vec{S} = \mu_0 \int_0^r J_0 e^{-kr'^2} \cdot 2\pi r' dr' =$$

$$= \mu_0 J_0 2\pi \int_0^r d\left(-\frac{1}{2k} e^{-kr'^2}\right) = \underline{\underline{\frac{\mu_0 J_0 \pi}{k}\left(1 - e^{-kr^2}\right)}}$$

(3)

$$I_L(t<0) = I_L(0) = \frac{f}{R_1}$$

$$I_L(t>0) = I_L(0)\,\exp(-t/\tau)$$

dove $\tau = \dfrac{L}{R_{par}} = \dfrac{L(R_2+R_3)}{R_2 R_3}$

$$\begin{cases} I_2 R_2 = I_3 R_3 \\ I_L = I_2 + I_3 \end{cases} \Rightarrow I_3 = I_L \frac{R_2}{R_2+R_3}$$

$$W_3 = R_3 I_3^2 = I_L^2 \frac{R_3 R_2^2}{(R_2+R_3)^2}$$

$$W_3 = \frac{R_3 R_2^2}{(R_2+R_3)^2}\,\frac{f^2}{R_1^2}\,\exp\left(-\frac{2t}{\tau}\right)$$

(4) La corrente nella spira è data da:

$$i = -\frac{1}{R}\frac{d\Phi(\vec{B})}{dt}$$

$$\Phi(\vec{B}) = \int_S \vec{B}\cdot d\vec{S} = \iint_S B\,dx\,dy = \int_{x_0}^{x_0+\ell}\int_{y}^{y+\ell} a y\,dx\,dy$$

$$\Phi(\vec{B}) = \frac{a\ell^2}{2}(2y+\ell) \;;$$

$$i = -\frac{a\ell^2}{R} v_0 = -1\times 10^{-5}\,A .$$

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$$P = 4\pi r^2 I$$

$$I = \frac{E_0^2}{2Z_0}$$

$$E_0^{min} \leq E_0 = \frac{1}{r}\sqrt{\frac{Z_0 P}{2\pi}}$$

$$\Downarrow$$

$$r \leq \frac{1}{E_0^{min}}\sqrt{\frac{Z_0 P}{2\pi}} .$$