Soluzioni scritto 18/Gennaio/2016

1

Simmetria + Gauss

$$E_r = \frac{1}{4\pi\varepsilon_0}\frac{Q(r)}{r^2}$$

$r<a$: $Q(r) = -\frac{3\sigma}{a}\frac{4}{3}\pi r^3$

$$E_r(r) = -\frac{\sigma}{\varepsilon_0}\frac{r}{a}$$

$r>a$: $Q(r) = 0$

$$E_r(r) = 0$$

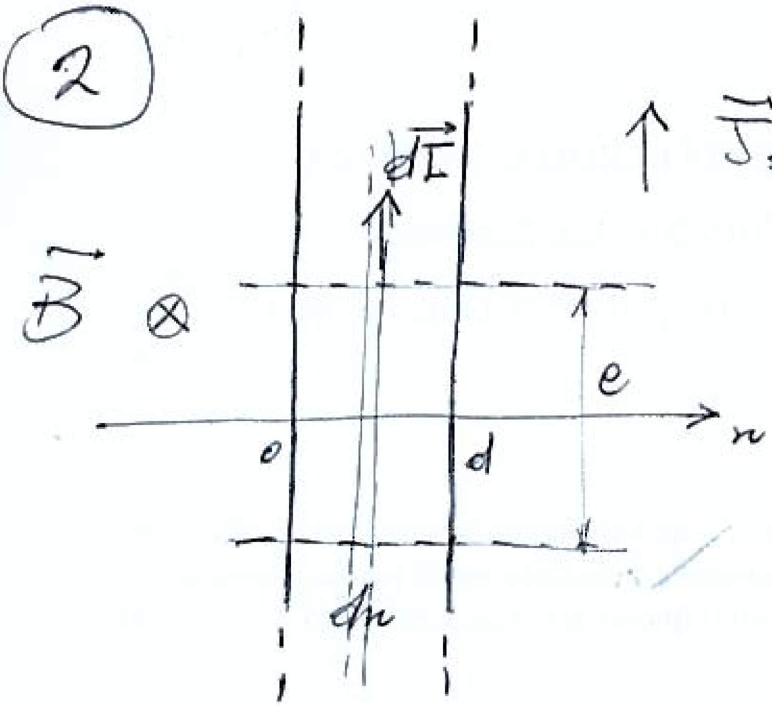
$\Rightarrow$ Per il potenziale: $V(r) = \cancel{V(\infty)}^{=0} + \int_r^\infty E_r\,dr'$

$r \geq a$ $V(r) = V(\infty) = 0$

$r \leq a$ $V(r) = V(a) - \int_r^a \frac{\sigma}{\varepsilon_0 a} r'\,dr'$

$$V(r) = -\frac{\sigma}{2\varepsilon_0 a}(a^2 - r^2)$$

(2)



$\uparrow \vec{J}_s = \vec{J}_{s0}(1+hx)$, $0<x<d$;

$$\underline{\underline{d\vec{I} = \vec{J}_s\,dx}}$$

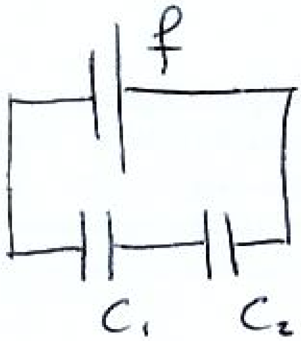
$$d\vec{F} = dI\,\vec{l}\times\vec{B}$$

$$dF_x = -dI\,l\,B = -J_s\,dx\,l\,B$$

$$F_x = -J_{s0}\,l\,B\int_0^d (1+hx)\,dx = -J_{s0}\,l\,B\left(d+h\frac{d^2}{2}\right)$$

(3)

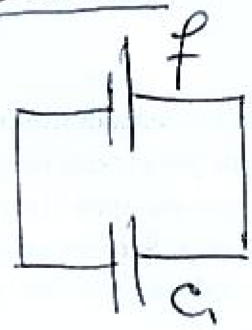
Iniziale



$$C_{eq} = \frac{C_1C_2}{C_1+C_2}$$

$$q_{in} = C_{eq}f$$

Finale



$$q_{fin} = C_1 f$$

$$U_{gen} = f\Delta q = f(q_{fin}-q_{in}) = f(C_1 f - C_{eq} f) = f^2\frac{C_1^2}{C_1+C_2}$$

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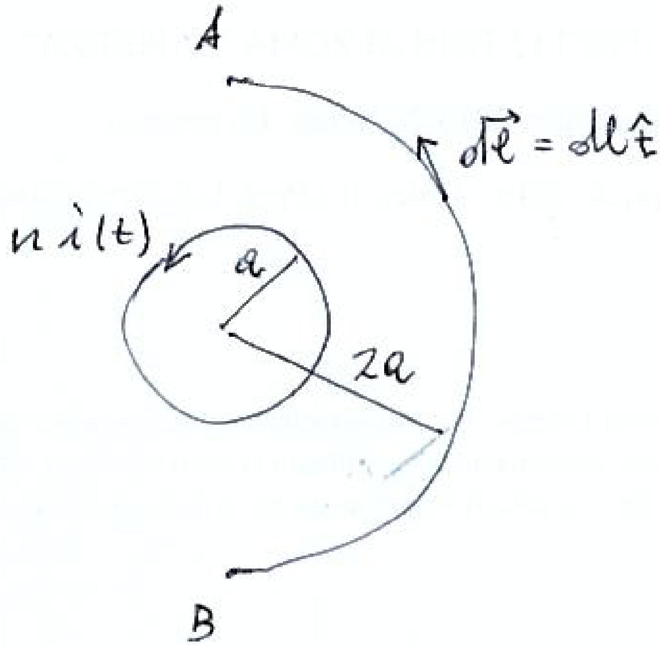

$$\oint \vec{E}\cdot d\vec{l} = E_t \cdot 2\pi r = -\frac{d\Phi}{dt} =$$

Simmetria

$$= -\frac{dB}{dt}\pi a^2 = -\pi a^2 \frac{d}{dt}\left[\mu_0 n\, i(t)\right] =$$

$$= -\pi a^2 \mu_0 n I \left(-e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right) =$$

$$= -\frac{\mu_0 n I \pi a^2}{\tau} e^{-t/\tau}$$

Per $r = 2a \Rightarrow E_t = -\frac{\mu_0 n I a}{4\tau} e^{-t/\tau}$

$$V_A - V_B = \int_A^B E_t\,\hat{t}\cdot d\vec{l} = E_t \cdot \pi 2a =$$

$$= -\frac{\mu_0 n I \pi a^2}{2\tau} e^{-t/\tau}$$

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$$f_{eff} = \frac{d\Phi_{eff}}{dt} = A\cos\alpha \left(\frac{dB}{dt}\right)_{eff} =$$

$$= A\cos\alpha\ \omega B_{eff} = \frac{A\cos\alpha\ 2\pi\nu\ E_{eff}}{c} =$$

$$= \frac{A\cos\alpha\ 2\pi\nu \sqrt{I Z_0}}{c}$$

$$\vec{B} = B\hat{z} = B_0 \cos(\omega t)\hat{z} = \left(\frac{E_0}{c}\right)\cos(\omega t)\hat{z}$$

avendo posto la spira nell'origine

e ricordato che:

$$I = \frac{E_0^2}{2Z} = \frac{E_{eff}^2}{Z}$$

$$f_{eff} \simeq \frac{10^{-2}\pi \cdot \cancel{10^{8}} \cdot 10^{-3} \cdot 20}{3 \cdot \cancel{10^{8}}} \simeq 200\mu V$$