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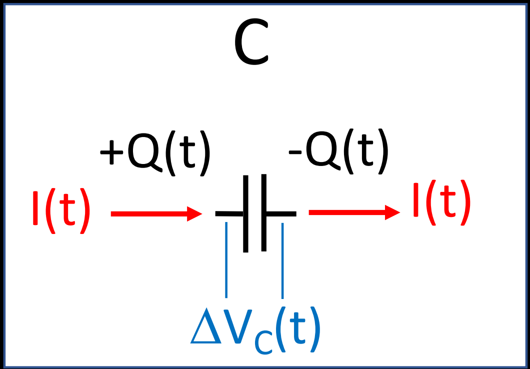
circuiti elettrici

circuiti (R e C) in condizioni quasi stazionarie

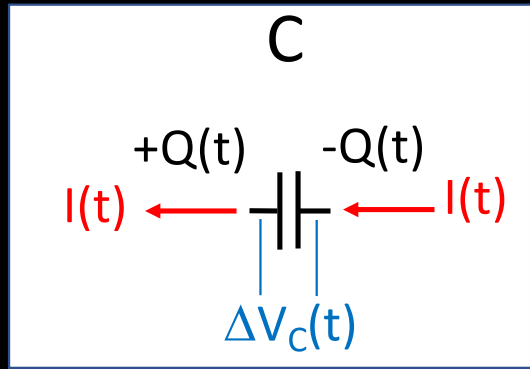
CONDIZIONI QUASI STAZIONARIE
CORRENTI LENTAMENTE VARIABILI



la corrente aumenta la carica positiva del condensatore (CARICA)

$$I(t) = \frac{dq}{dt} = \frac{dQ(t)}{dt}$$

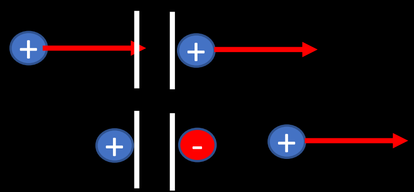
$$Q(t+dt) = Q(t) + I(t)\,dt$$



la corrente diminuisce la carica positiva del condensatore (SCARICA)

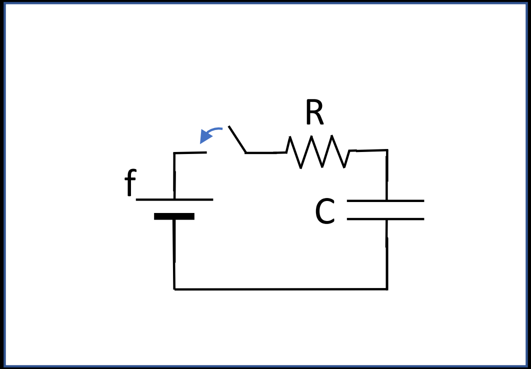
$$I(t) = \frac{dq}{dt} = \frac{-dQ(t)}{dt}$$

$$Q(t+dt) = Q(t) - I(t)\,dt$$



fra le armature non c'è corrente di conduzione ma corrente di spostamento dovuta alla variazione di **E**

CORRENTI LENTAMENTE VARIABILI



$$\Delta V_C(t) = \frac{Q(t)}{C} = \frac{Q_0 + \int_0^t I(t')dt'}{C}$$

$$t < 0 \quad I(t) = 0 \quad \Delta V_C(t) = \frac{Q_0}{C}$$

prima della commutazione dell'interruttore
il circuito è in condizioni stazionarie

$$t > 0 \quad V_0 + f - R\,I(t) - \frac{Q(t)}{C} = V_0$$

$$t = 0^+ \quad V_0 + f - R\,I(0^+) - \frac{Q(0^+)}{C} = V_0$$

$$\Delta V_C(0^+) = \frac{Q(0^+)}{C} = \frac{Q_0 + I(0^+)\cancel{dt}}{C} = \frac{Q_0}{C}$$

subito dopo la commutazione, la corrente non ha tempo sufficiente per variare significativamente la carica

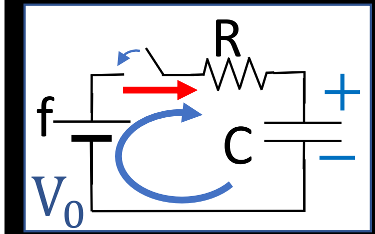
la carica di un condensatore non cambia istantaneamente

CONDIZIONI QUASI STAZIONARIE

CORRENTI LENTAMENTE VARIABILI

CARICA DEL CONDENSATORE

(inizialmente scarico)



$$\Delta V_C(0^+) = \Delta V_C(0^-) = \frac{Q(0^-)}{C} = \frac{Q_0}{C} = 0$$

$$I(t) = \frac{dq}{dt} = \frac{dQ(t)}{dt}$$

$$t > 0 \quad V_0 + f - R\,I(t) - \frac{Q(t)}{C} = V_0$$

$$f = R\,I(t) + \frac{Q(t)}{C}$$

$$f = R\frac{dQ(t)}{dt} + \frac{Q(t)}{C}$$

$$f\,C = RC\frac{dQ(t)}{dt} + Q(t)$$

$$f\,C - Q(t) = RC\frac{dQ(t)}{dt}$$

$$\frac{dt}{RC} = \frac{dQ(t)}{f\,C - Q(t)}$$

$$\int_0^t \frac{dt}{RC} = \int_0^{Q(t)} \frac{dQ(t)}{f\,C - Q(t)} = \int_0^{Q(t)} \frac{-d[f\,C - Q(t)]}{f\,C - Q(t)}$$

$$= -\int_0^{Q(t)} d\ln[f\,C - Q(t)]$$

$$\rightarrow \frac{t}{RC} = -\ln\left(\frac{f\,C - Q(t)}{f\,C - 0}\right)$$

$$-\frac{t}{RC} = \ln\left(\frac{f\,C - Q(t)}{f\,C}\right)$$

$$e^{-\frac{t}{RC}} = \frac{f\,C - Q(t)}{f\,C}$$

$$f\,C\,e^{-\frac{t}{RC}} = f\,C - Q(t)$$

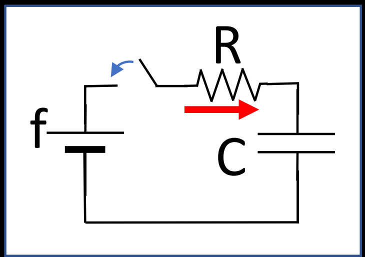
$$\boxed{Q(t) = f\,C\,(1 - e^{-\frac{t}{RC}})}$$

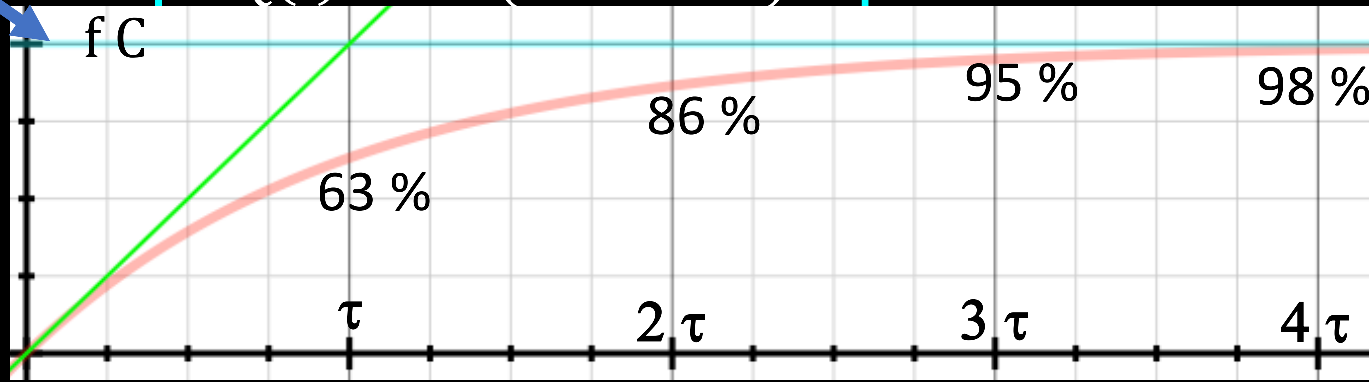
CONDIZIONI QUASI STAZIONARIE
CORRENTI LENTAMENTE VARIABILI

CARICA DEL CONDENSATORE
(inizialmente scarico)

valore asintotico

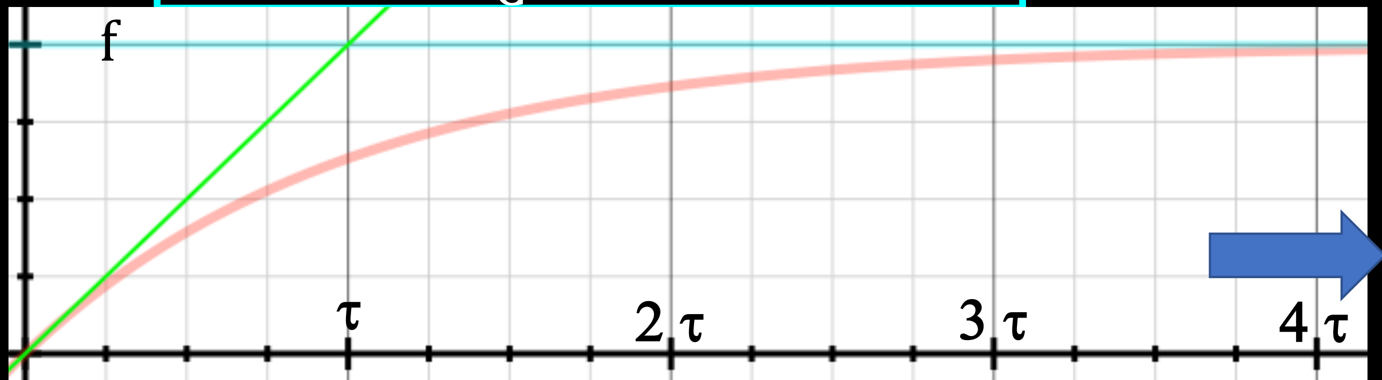
$$Q(t) = f\,C\,(1 - e^{-\frac{t}{RC}})$$





tangente nell'origine

$$\Delta V_C(t) = \frac{Q(t)}{C} = f\,(1 - e^{-\frac{t}{RC}})$$



RC = τ
costante di tempo del circuito

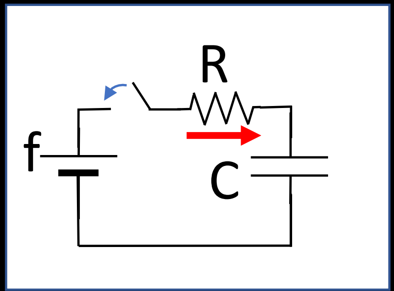
CONDIZIONI QUASI STAZIONARIE
CORRENTI LENTAMENTE VARIABILI

CARICA DEL CONDENSATORE
(inizialmente scarico)

$$Q(t) = f\,C\,(1 - e^{-\frac{t}{RC}})$$

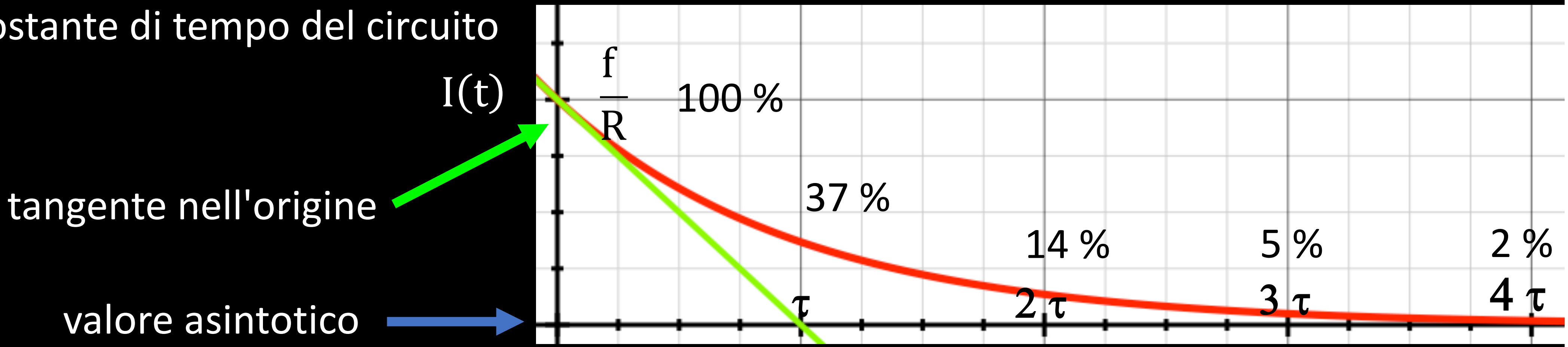
$$I(t) = \frac{dq}{dt} = \frac{dQ(t)}{dt}$$

$$I(t) = \frac{dQ(t)}{dt} = \frac{f\,C}{RC}\,e^{-\frac{t}{RC}} = \frac{f}{R}\,e^{-\frac{t}{RC}}$$



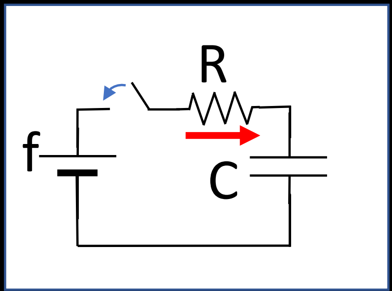
RC = τ

costante di tempo del circuito



CONDIZIONI QUASI STAZIONARIE
CORRENTI LENTAMENTE VARIABILI

CARICA DEL CONDENSATORE
(inizialmente scarico)



$$Q(t) = f\,C\,(1 - e^{-\frac{t}{RC}})$$

$$I(t) = \frac{dQ(t)}{dt} = \frac{f\,C}{RC} e^{-\frac{t}{RC}} = \frac{f}{R} e^{-\frac{t}{RC}}$$

più direttamente

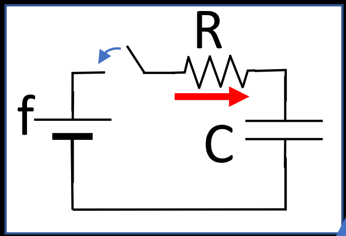
$$I(t) = \frac{\Delta V_R(t)}{R} = \frac{f - \frac{Q(t)}{C}}{R} = \frac{f}{R} - \frac{Q(t)}{RC} \quad \rightarrow \frac{dI(t)}{dt} = 0 - \frac{I(t)}{RC} \quad \rightarrow \frac{dI(t)}{I(t)} = -\frac{dt}{RC}$$

$$\int_{I_0}^{I(t)} d\ln I(t) = \int_0^t -\frac{dt}{RC} \quad \rightarrow \ln\frac{I(t)}{I_0} = -\frac{t}{RC} \quad \rightarrow \frac{I(t)}{I_0} = e^{-\frac{t}{RC}} \quad \rightarrow I(t) = I_0\, e^{-\frac{t}{RC}}$$

$$I(0^+) = \frac{f - \frac{Q(0^+)}{C}}{R} = \frac{f}{R} - \frac{0}{RC} = \frac{f}{R} \quad \rightarrow I(t) = \frac{f}{R} e^{-\frac{t}{RC}}$$

CORRENTI LENTAMENTE VARIABILI: POTENZA

CARICA DEL CONDENSATORE (inizialmente scarico)



$$P_G = f\, I(t) = f\,\frac{f}{R}\,e^{-\frac{t}{RC}} = \frac{f^2}{R}\,e^{-\frac{t}{RC}}$$

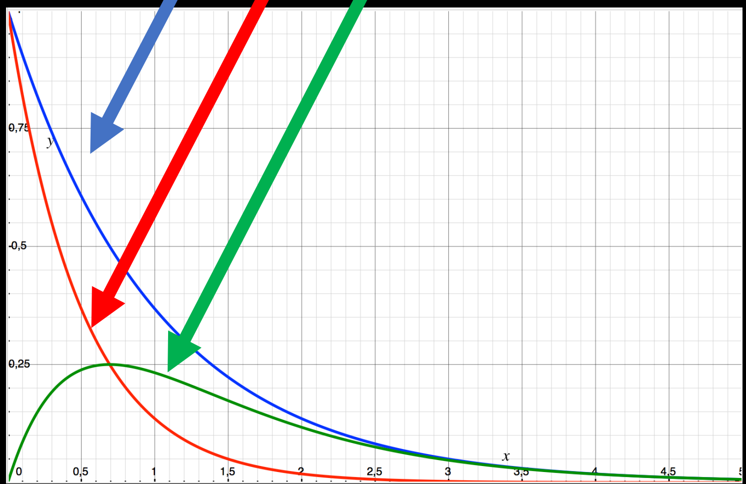
$$\Delta V_C(t) = f\,(1 - e^{-\frac{t}{RC}})$$

$$I(t) = \frac{f}{R}e^{-\frac{t}{RC}}$$

$$P_R = R\, I^2(t) = R\,\frac{f^2}{R^2}\,e^{-\frac{2t}{RC}} = \frac{f^2}{R}\,e^{-\frac{2t}{RC}}$$

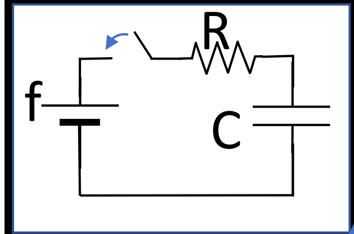
$$P_C = \frac{dU_C(t)}{dt} = \frac{d\left(\frac{1}{2}\,C\,\Delta V_C(t)^2\right)}{dt} = \frac{1}{2}\,C\,2\,\Delta V_C(t)\,\frac{d\,\Delta V_C(t)}{dt} > 0$$

$$= C\,f\,(1 - e^{-\frac{t}{RC}})\,\frac{f}{RC}e^{-\frac{t}{RC}} = \frac{f^2}{R}\,(e^{-\frac{t}{RC}} - e^{-\frac{2t}{RC}})$$



CORRENTI LENTAMENTE VARIABILI: ENERGIA

CARICA DEL CONDENSATORE
(inizialmente scarico)



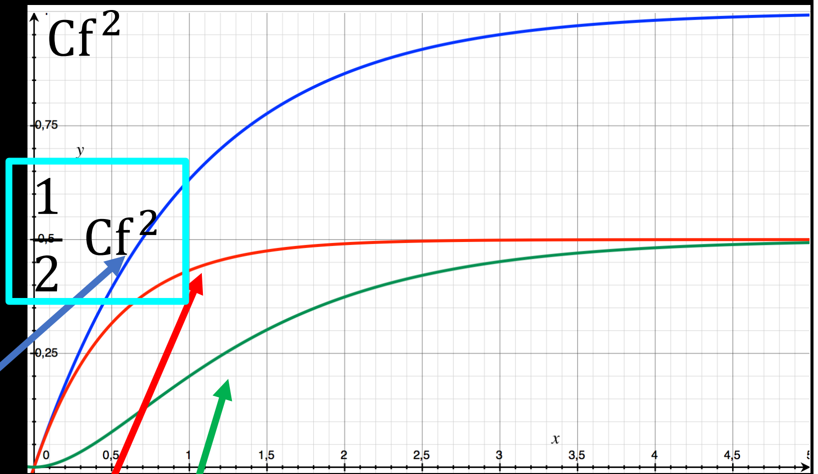
$$\Delta V_C(t) = f\left(1 - e^{-\frac{t}{RC}}\right)$$

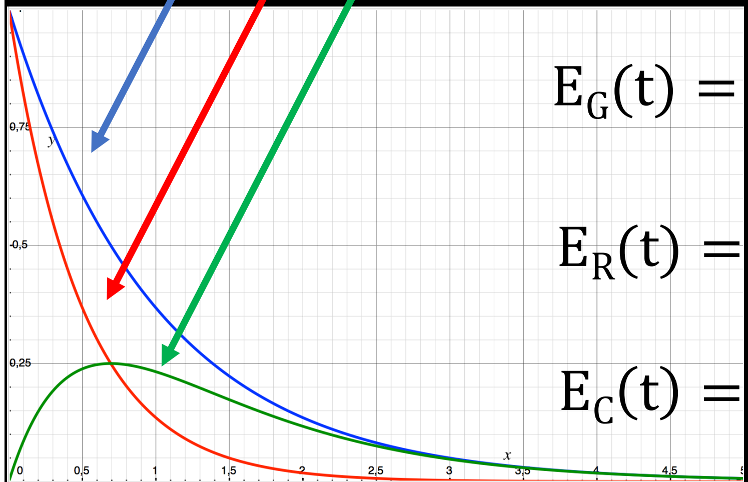
$$I(t) = \frac{f}{R} e^{-\frac{t}{RC}}$$

$$P_G = f\, I(t) = \frac{f^2}{R} e^{-\frac{t}{RC}}$$

$$P_R = R\, I^2(t) = \frac{f^2}{R} e^{-\frac{2t}{RC}}$$

$$P_C = \frac{dU_C(t)}{dt} = \frac{f^2}{R}\left(e^{-\frac{t}{RC}} - e^{-\frac{2t}{RC}}\right)$$





$$E_G(t) = \int_0^t f\, I(t)\, dt = \int_0^t \frac{f^2}{R} e^{-\frac{t}{RC}} dt = Cf^2\left(1 - e^{-\frac{t}{RC}}\right)$$

$$E_R(t) = \int_0^t R\, I^2(t)\, dt = \int_0^t \frac{f^2}{R} e^{-\frac{2t}{RC}} dt = \frac{1}{2} Cf^2\left(1 - e^{-\frac{2t}{RC}}\right)$$

$$E_C(t) = \frac{1}{2} C\, \Delta V_C(t)^2 = \frac{1}{2} Cf^2\left(1 - e^{-\frac{t}{RC}}\right)^2$$

ESONERO sabato 23 ore 9

Meet: fwd-ssxa-yfz

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