Complementi di fisica generale

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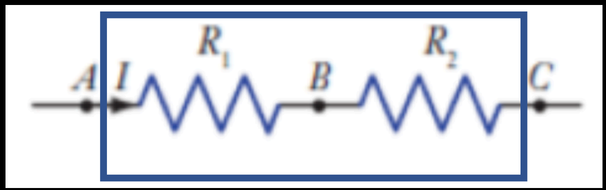
circuiti elettrici

esercitazione su:

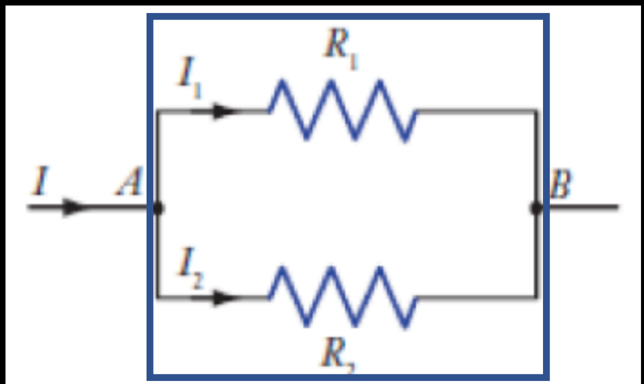
circuiti in condizioni quasi stazionarie

ELEMENTI IN SERIE/PARALLELO



$$R_S = \frac{\Delta V_{AC}}{I} = \frac{\Delta V_{AB} + \Delta V_{BC}}{I} = \frac{\Delta V_{AB}}{I} + \frac{\Delta V_{BC}}{I} = R_1 + R_2$$

due (o più) resistenze sono in serie se sono attraversate dalla stessa corrente



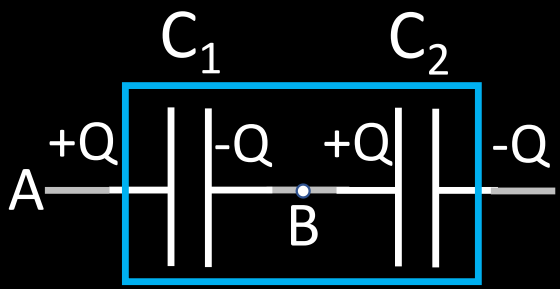
$$\frac{1}{R_p} = \frac{I}{\Delta V_{AB}} = \frac{I_1 + I_2}{\Delta V_{AB}} = \frac{I_1}{\Delta V_{AB}} + \frac{I_2}{\Delta V_{AB}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1\, R_2}$$

$$R_p = \frac{R_1\, R_2}{R_1 + R_2}$$

due (o più) resistenze sono in parallelo se hanno la stessa d.d.p.

ELEMENTI IN SERIE/PARALLELO

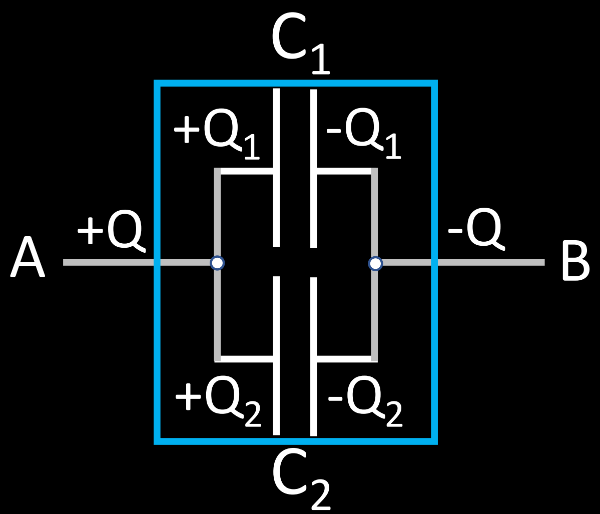
analogamente ma diversamente...



$$\frac{1}{C_S} = \frac{\Delta V_{AC}}{Q} = \frac{\Delta V_{AB} + \Delta V_{BC}}{Q} = \frac{\Delta V_{AB}}{Q} + \frac{\Delta V_{BC}}{Q} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1\, C_2}$$

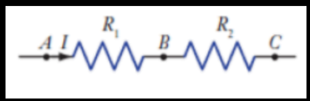
due (o più) capacità sono in serie se hanno la stessa carica

$$C_S = \frac{C_1\, C_2}{C_1 + C_2}$$

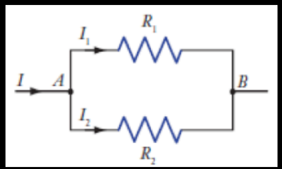


$$C_P = \frac{Q}{\Delta V_{AB}} = \frac{Q_1 + Q_2}{\Delta V_{AB}} = \frac{Q_1}{\Delta V_{AB}} + \frac{Q_2}{\Delta V_{AB}} = C_1 + C_2$$

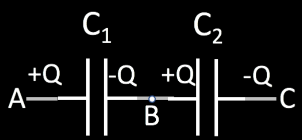
due (o più) capacità sono in parallelo se hanno la stessa d.d.p.



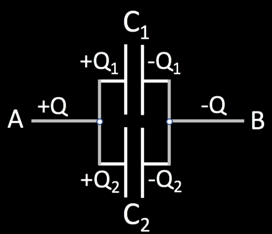
$$R_S = R_1 + R_2$$



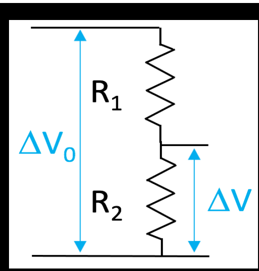
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \qquad R_p = \frac{R_1\,R_2}{R_1 + R_2}$$



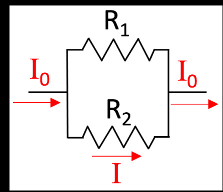
$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_S = \frac{C_1\,C_2}{C_1 + C_2}$$



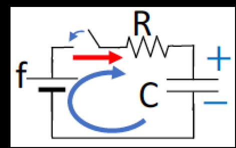
$$C_P = C_1 + C_2$$



$$\Delta V = \Delta V_0 \frac{R_2}{R_1 + R_2}$$



$$I = I_0 \frac{R_1}{R_1 + R_2}$$

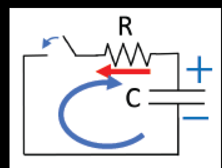


se $\Delta V_C(0) = Q(0)/C = 0$

$Q(t) = f\,C\,(1 - e^{-t/\tau})$

$I(t) = f/R\; e^{-t/\tau}$

$\Delta V_C(t) = f\,(1 - e^{-t/\tau})$

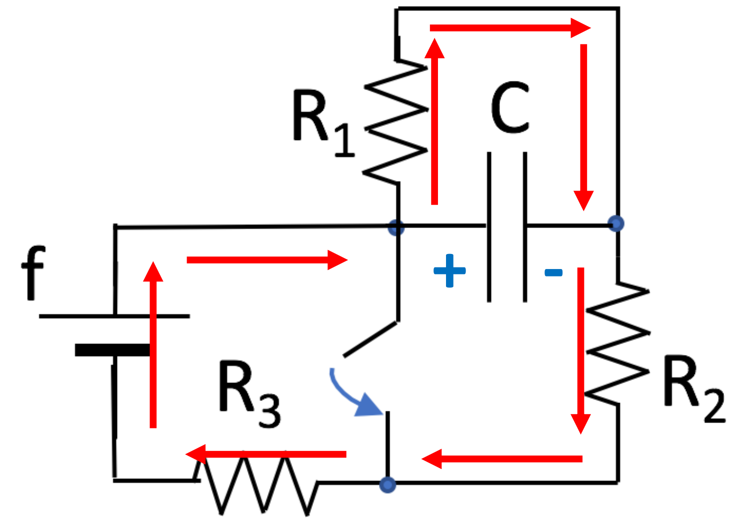


se $\Delta V_C(\infty) = Q(\infty)/C = 0$

$Q(t) = Q_0\, e^{-t/\tau}$

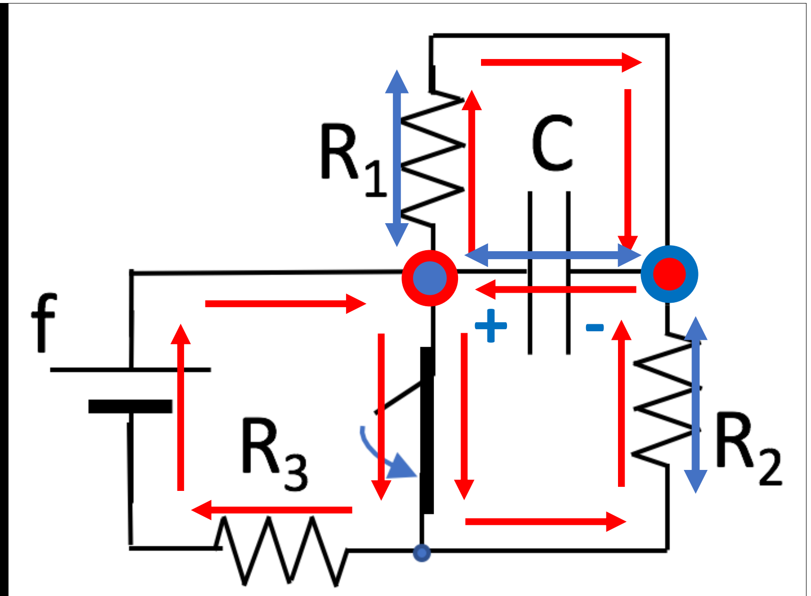
$I(t) = \Delta V_C(0)/R\; e^{-t/\tau}$

$\Delta V_C(t) = \Delta V_C(0)\, e^{-t/\tau}$

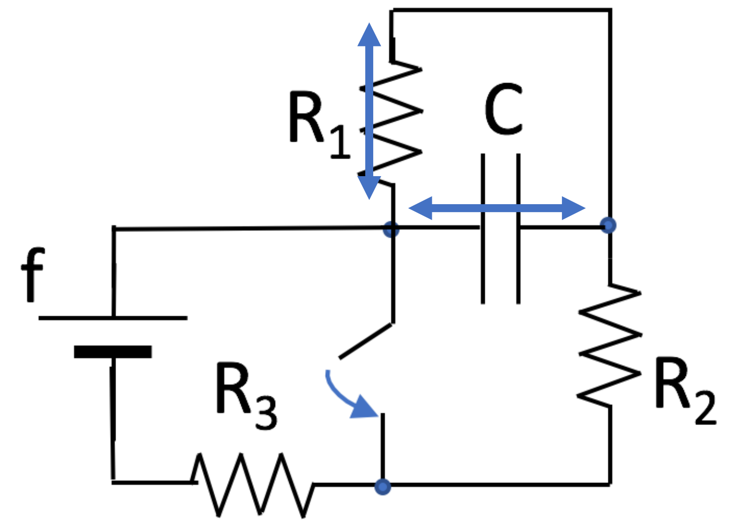


$I = f/(R_1 + R_2 + R_3)$

$I_G = f/R_3$



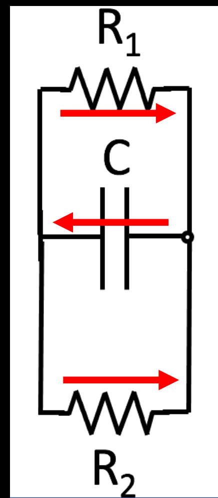
COMMUTAZIONE

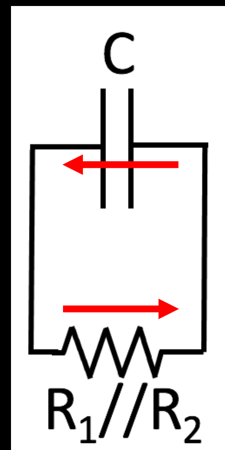


$\Delta V_C(0) = R_1\, I$

$= R_1\, f/(R_1 + R_2 + R_3)$

$\Delta V_C(t) = \Delta V_{R1}(t) = \Delta V_{R2}(t)$

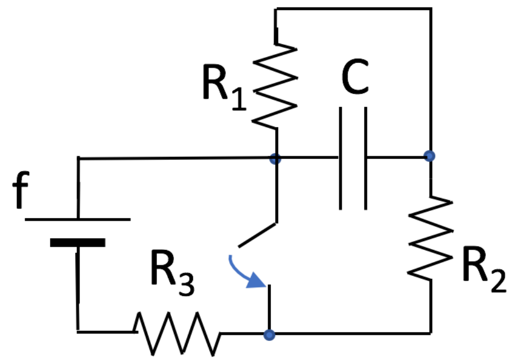


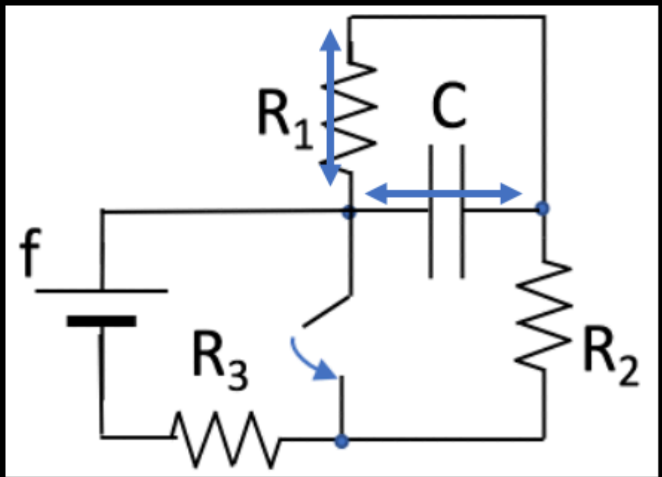


2) Il circuito in figura è in condizioni stazionarie quando, all'istante t= 0, viene chiuso l'interruttore. Determinare l'espressione della differenza di potenziale $\Delta V_C(t)$ ai capi del condensatore.

Dati: $R_1 = R_2 = R_3 = R$ con $R = 200\ \Omega$; $f = 15$ V; $C = 20$ nF.

>>> soluzione: $V_c(t) = (5\ V)\ e^{-t/2\mu s}$





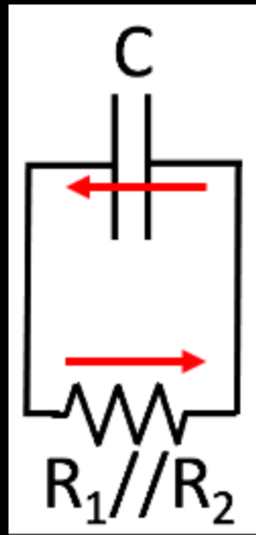
$V_c(2\ \mu s) = 1,8$ V
$V_c(4\ \mu s) = 0,7$ V
$V_c(6\ \mu s) = 0,2$ V

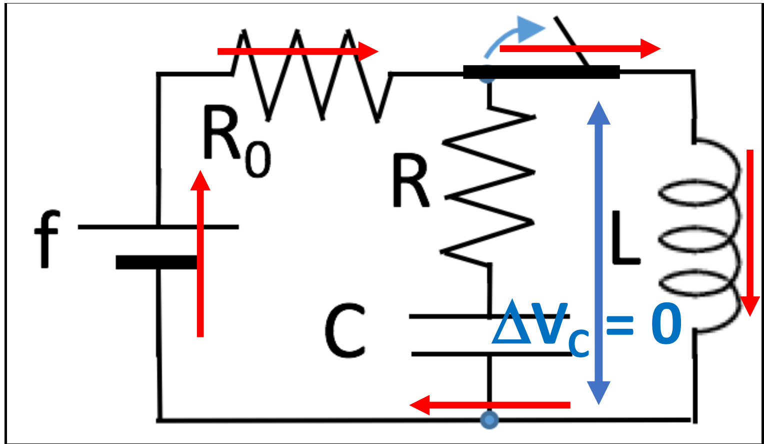
$\Delta V_C(0) = R_1 I$
$= R_1 f/(R_1 + R_2 + R_3) = f/3 = 5$ V

$\tau = (R_1 R_2)/(R_1+R_2)\ C$

$R_1//R_2 = R^2/2R = R/2 = 100\ \Omega$

$\tau = R/2\ C = 100 \times 20 \times 10^{-9}$ s $= 2\ \mu s$





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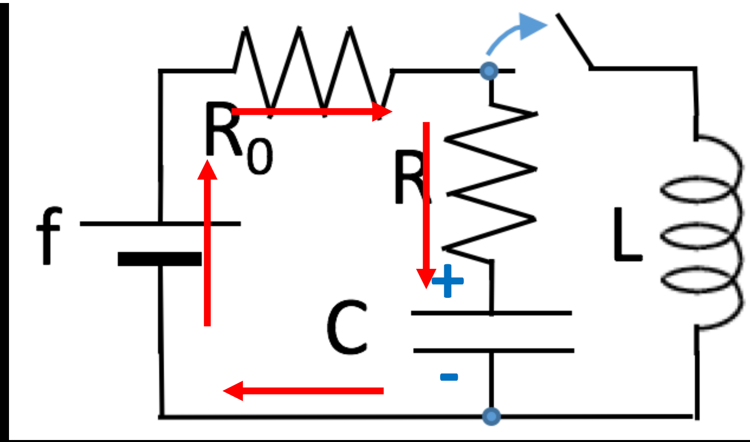


$I_C(0^-) = 0$

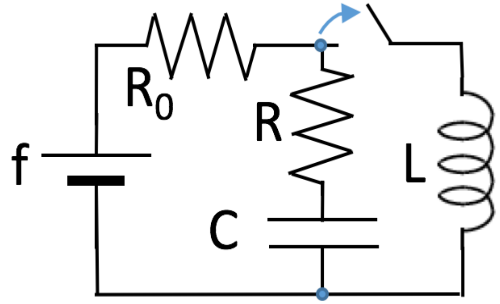
$I_G = f/R_0$

$\Delta V_C(0^-) = 0$

$\tau = (R_0+R)\,C$

$I(0^+) = f/(R_0+R)$

5) Il circuito in figura ($R_0 = R = 1\ k\Omega$; $C = 5\ nF$; $L = 10\ mH$) è a regime quando, all'istante $t = 0$, l'interruttore viene aperto. Calcolare dopo quanto tempo la tensione ai capi della resistenza R è uguale alla tensione ai capi del condensatore.



>>> soluzione: 4 μs

$\Delta V_R(t) = R\, I(t) = R\, I_0\, e^{-t/\tau}$

$\Delta V_C(t) = f\,(1 - e^{-t/\tau})$

$\Delta V_R(t^*) = \Delta V_C(t^*)$

$\tau = (R_0+R)\, C = 10\ \mu s$

$I(0^+) = f/(R_0+R)$

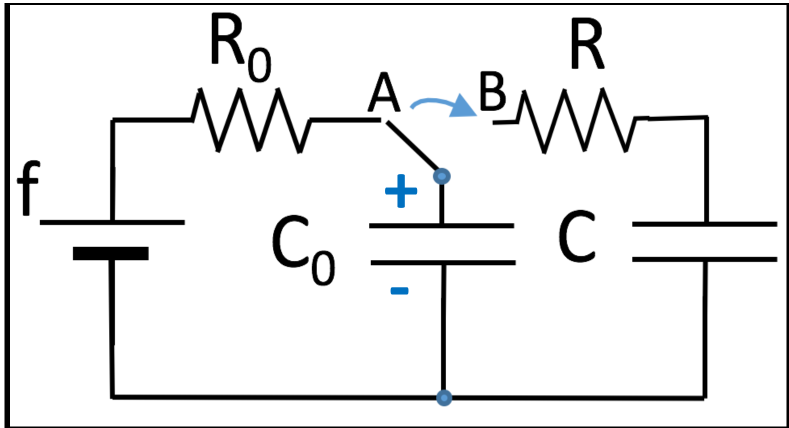
$R\, f/(R_0+R)\, e^{-t^*/\tau} = f\,(1 - e^{-t^*/\tau})$

$R/(R_0+R)\, e^{-t^*/\tau} = (1 - e^{-t^*/\tau})$

$R\, e^{-t^*/\tau} = (R_0+R)\,(1 - e^{-t^*/\tau}) = (R_0+R) - (R_0+R)\, e^{-t^*/\tau}$

$[R+(R_0+R)]\, e^{-t^*/\tau} = (R_0+R) \rightarrow e^{-t^*/\tau} = (R_0+R)/(R_0+2R)$

$-t^*/\tau = \ln[(R_0+R)/(R_0+2R)]$ $\quad t^* = \tau \ln[(R_0+2R)/(R_0+R)] = \tau \ln(3/2)$



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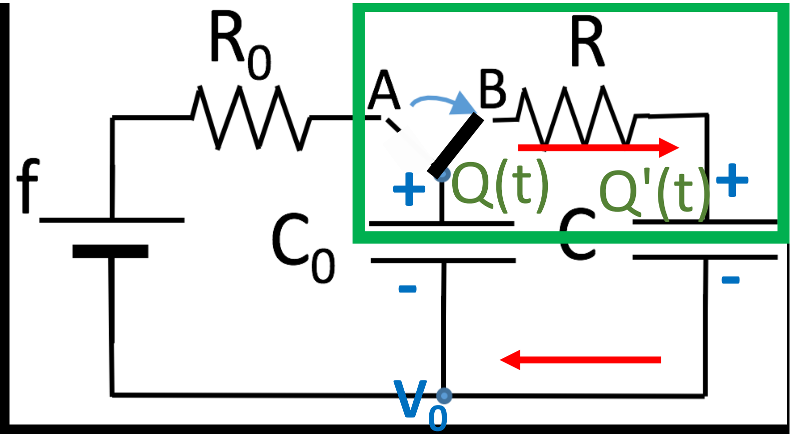


$Q_C(0^-) = 0$

$I_G = 0$

$\Delta V_{C0}(0^-) = f$

$Q(0^-) = f\, C_0$

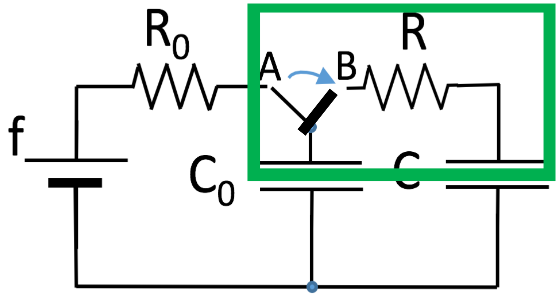
$V_0 + \Delta V_{C0}(t) - R\, I(t) - \Delta V_C(t) = V_0$

$I(t) = [\Delta V_{C0}(t) - \Delta V_C(t)]/R$

$I(t) = [Q(t)/C_0 - Q'(t)/C]/R$

$Q(t) + Q'(t) = Q(0)$

7) Nel circuito in figura $f = 10$ V, $C_0 = 20\ \mu F$, $C = 30\ \mu F$, i valori di R_0 e R sono ininfluenti. In condizioni stazionarie il condensatore C è scarico. All'istante t = 0 il deviatore viene spostato dalla posizione A a B.



Determinare l'energia che viene dissipata in R dall'istante della commutazione fino a quando non viene ristabilito l'equilibrio.

$I(t) = [\Delta V_{C0}(t) - \Delta V_C(t)]/R$

$Q(t) + Q'(t) = Q(0) = f\, C_0$

ASINTOTICAMENTE

$I(\infty) = [\Delta V_{C0}(\infty) - \Delta V_C(\infty)]/R = 0$

$\Delta V_{C0}(\infty) = \Delta V_C(\infty) \rightarrow C_0 \,//\, C$

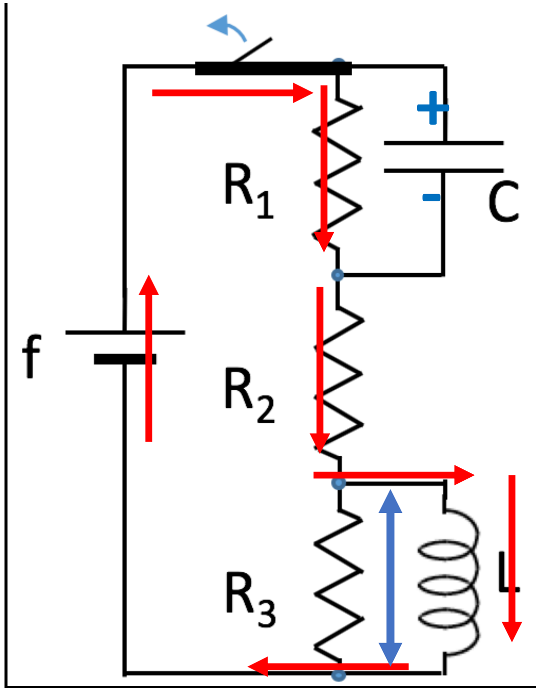
$C_P = C_0 + C$

energia dissipata in R: $E_R = U_i - U_f$

$U_i = \frac{1}{2} Q(0)^2/C_0 = \frac{1}{2} f^2 C_0$

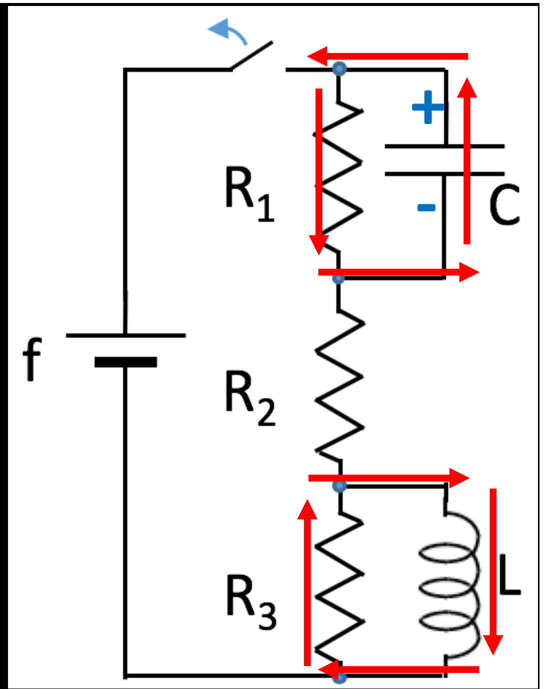
$U_f = \frac{1}{2} Q(0)^2/C_P = \frac{1}{2} f^2 C_0^2/C_P$

$E_R = U_i - U_f = \frac{1}{2} f^2 C_0 (1 - C_0/C_P) = \frac{1}{2} f^2 C_0 (C_P - C_0)/C_P = \frac{1}{2} f^2 C_0 C/(C_0 + C)$



$\Delta V_C(0) = R_1\, I = R_1\, f/(R_1 + R_2\,)$

COMMUTAZIONE



$I = f/(R_1 + R_2)$

$I_{R2} = 0$

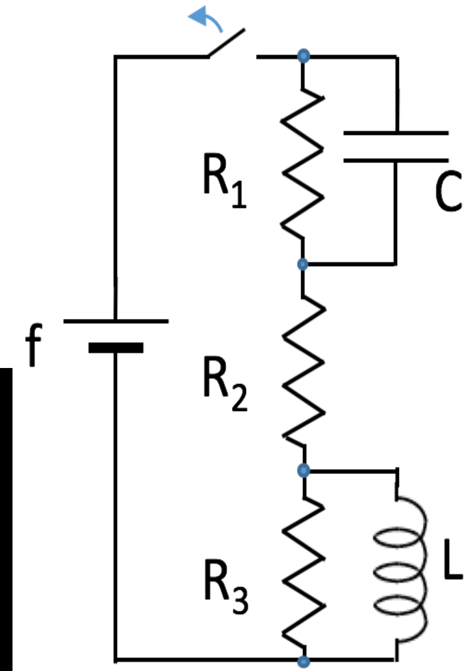
$\Delta V_C(t) = \Delta V_C(0)\, e^{-t/R1C}$

8) Il circuito in figura è inizialmente in condizioni stazionarie. Determinare dopo quanto tempo dall'apertura dell'interruttore la differenza di potenziale ai capi della capacità arriva al valore $V^* = 1$ V.

Dati: $f = 6$ V; $R_1 = R_2 = R_3 = R = 100\ \Omega$; $C = 10$ nF; $L = 0{,}1$ mH

>>> soluzione: $t^* = 1{,}1\ \mu s$

8) $t^* = \ln 3\ \mu s$



$$\Delta V_C(0) = R_1 I = R_1 f/(R_1 + R_2) = 100 \times 6/(100+100) = 3 \text{ V}$$

$$\Delta V_C(t) = \Delta V_C(0)\, e^{-t/R1C}$$

$$V^* = \Delta V_C(t^*) = \Delta V_C(0)\, e^{-t^*/R1C}$$

$$V^*/\Delta V_C(0) = e^{-t^*/R1C}$$

$$t^* = R_1 C \ln[\Delta V_C(0)/V^*] = 100 \times 10^{-8} \ln 3 \text{ s} = \ln 3\ \mu s$$

$$\ln[\Delta V_C(0)/V^*] = t^*/R_1 C$$

ESONERO...

Complementi di fisica generale

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VENERDÌ 29 APRILE ORE 8:30-10:00

correnti lentamente variabili R, C, L