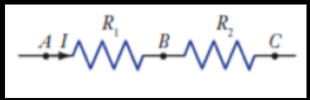
Complementi di fisica generale

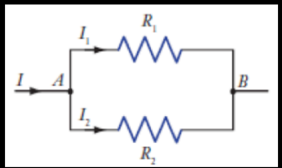
adalberto.sciubba@uniroma1.it

circuiti elettrici

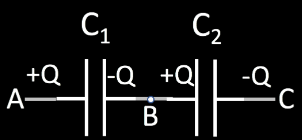
circuiti (R e C) in condizioni quasi stazionarie



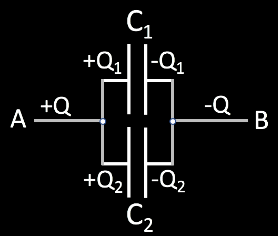
$$R_S = R_1 + R_2$$



$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \qquad R_p = \frac{R_1\,R_2}{R_1 + R_2}$$



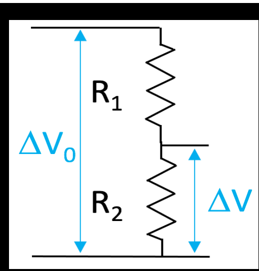
$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_S = \frac{C_1\,C_2}{C_1 + C_2}$$



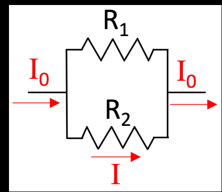
$$C_P = C_1 + C_2$$

$$U = \frac{1}{2} C\,\Delta V^2 \qquad P_G = f\ I$$

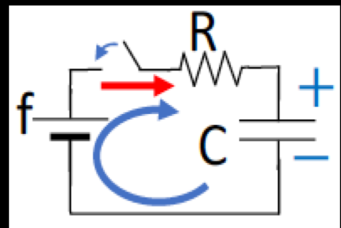
$$U = \frac{1}{2}\ L\,I^2 \qquad P_R = R\,I^2$$



$$\Delta V = \Delta V_0 \frac{R_2}{R_1 + R_2}$$



$$I = I_0 \frac{R_1}{R_1 + R_2}$$



se $\Delta V_C(0) = Q_0/C = 0$

$$Q(t) = f\,C\,(1 - e^{-t/\tau})$$

$$\Delta V_C(t) = f\,(1 - e^{-t/\tau})$$

$$I(t) = f/R\ e^{-t/\tau}$$

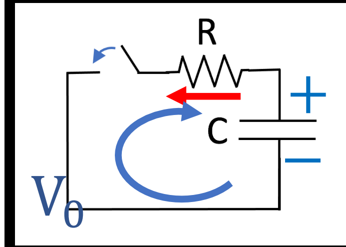
$dQ = I(t)\ dt \rightarrow$ la carica di un condensatore non cambia istantaneamente

SCARICA DI C

CONDIZIONI QUASI STAZIONARIE
CORRENTI LENTAMENTE VARIABILI

SCARICA DEL CONDENSATORE

(asintoticamente scarico)



$$\Delta V_C(0^+) = \Delta V_C(0^-) = \Delta V_0 = \frac{Q_0}{C} \neq 0$$

$$I(t) = \frac{dq}{dt} = -\frac{dQ(t)}{dt}$$

$t > 0$ $\quad V_0 + R\,I(t) - \frac{Q(t)}{C} = V_0 \quad \rightarrow R\,I(t) = \frac{Q(t)}{C} \quad \rightarrow -R\frac{dQ(t)}{dt} = \frac{Q(t)}{C}$

$$\rightarrow \frac{dQ(t)}{Q(t)} = -\frac{dt}{RC} \quad \rightarrow -\frac{dt}{RC} = \frac{dQ(t)}{Q(t)} \quad \rightarrow \int_0^t -\frac{dt}{RC} = \int_{Q_0}^{Q(t)} \frac{dQ(t)}{Q(t)} = \int_{Q_0}^{Q(t)} d\ln Q(t)$$

$$\rightarrow -\frac{t}{RC} = \ln\left(\frac{Q(t)}{Q_0}\right) \quad \rightarrow e^{-\frac{t}{RC}} = \frac{Q(t)}{Q_0}$$

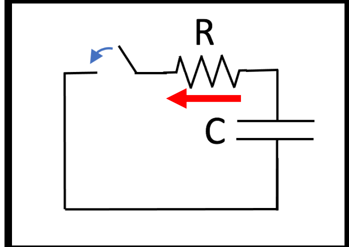
$$\boxed{Q(t) = Q_0\, e^{-\frac{t}{RC}}}$$

$$\boxed{\Delta V_C(t) = \frac{Q(t)}{C} = \frac{Q_0}{C}\, e^{-\frac{t}{RC}} = \Delta V_0\, e^{-\frac{t}{RC}}}$$

$$\boxed{I(t) = -\frac{dQ(t)}{dt} = \frac{Q_0}{RC}\, e^{-\frac{t}{RC}} = \frac{\Delta V_0}{R}\, e^{-\frac{t}{RC}}}$$

CONDIZIONI QUASI STAZIONARIE
CORRENTI LENTAMENTE VARIABILI

SCARICA DEL CONDENSATORE
(asintoticamente scarico)

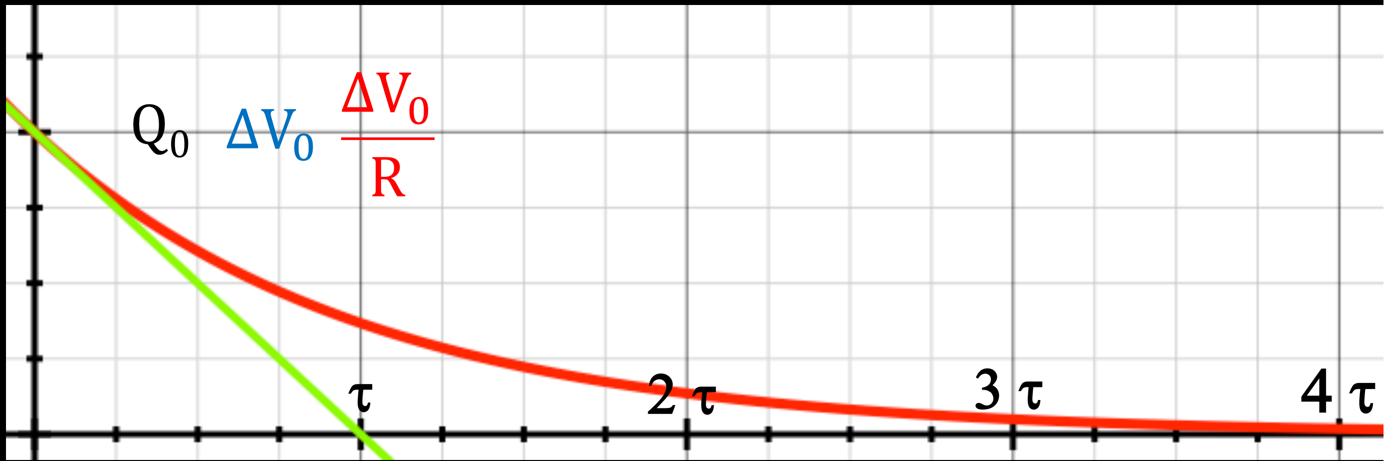


$$Q(t) = Q_0\, e^{-\frac{t}{RC}}$$

$$\Delta V_C(t) = \Delta V_0\, e^{-\frac{t}{RC}}$$

$$I(t) = \frac{\Delta V_0}{R}\, e^{-\frac{t}{RC}}$$

$I(t)$ $\Delta V_C(t)$ $Q(t)$

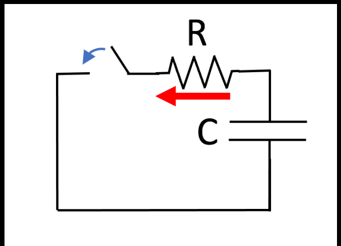


RC = τ

costante di tempo del circuito

CORRENTI LENTAMENTE VARIABILI: POTENZA

SCARICA DEL CONDENSATORE
(asintoticamente scarico)



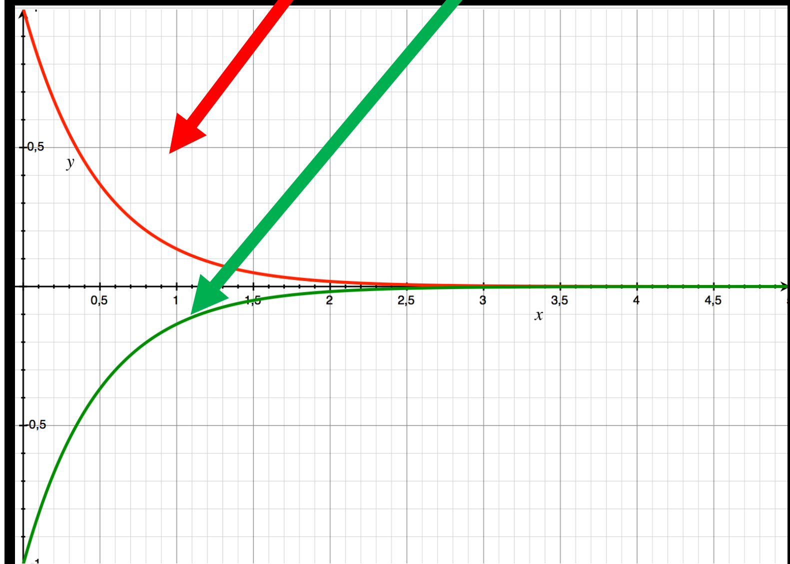
$$\Delta V_C(t) = \Delta V_0 \; e^{-\frac{t}{RC}}$$

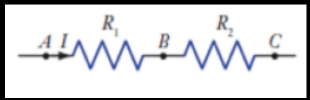
$$I(t) = \frac{\Delta V_0}{R} e^{-\frac{t}{RC}}$$

$$P_R = R\, I^2(t) = R\, \frac{{\Delta V_0}^2}{R^2}\, e^{-\frac{2t}{RC}} = \frac{{\Delta V_0}^2}{R}\, e^{-\frac{2t}{RC}}$$

$$P_C = \frac{dU_C(t)}{dt} = \frac{d\left(\frac{1}{2}\, C\, \Delta V_C(t)^2\right)}{dt} = \frac{1}{2}\, C\, 2\, \Delta V_C(t) \frac{d\, \Delta V_C(t)}{dt} < 0$$

$$= C\; \Delta V_0\, e^{-\frac{t}{RC}} \left[-\frac{\Delta V_0}{RC} e^{-\frac{t}{RC}}\right] = -\frac{{\Delta V_0}^2}{R}\, e^{-\frac{2t}{RC}}$$





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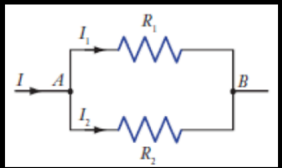

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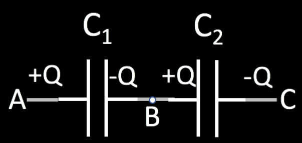

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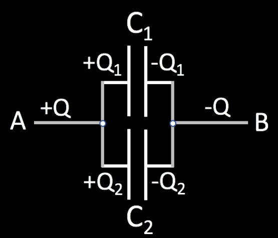

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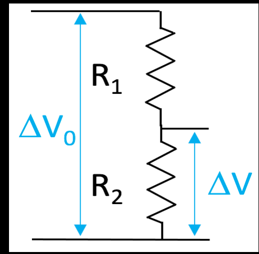
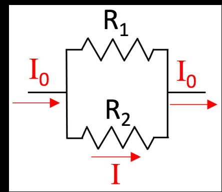
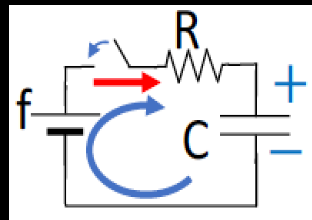


$$\Delta V = \Delta V_0\,\frac{R_2}{R_1 + R_2}$$



$$I = I_0\,\frac{R_1}{R_1 + R_2}$$

se $\Delta V_C(0) = Q_0/C = 0$

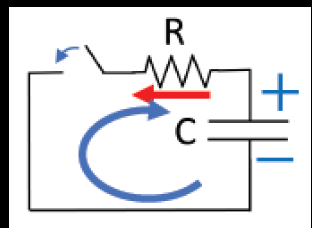


$Q(t) = f\,C\,(1 - e^{-t/\tau})$

$\Delta V_C(t) = f\,(1 - e^{-t/\tau})$

$I(t) = f/R\ e^{-t/\tau}$

se $\Delta V_C(\infty) = Q(\infty)/C = 0$



$Q(t) = Q_0\ e^{-t/\tau}$

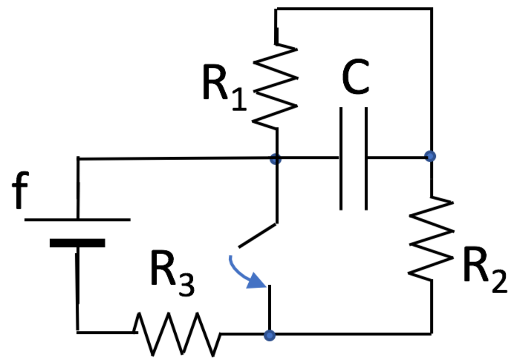
$\Delta V_C(t) = \Delta V_C(0)\ e^{-t/\tau}$

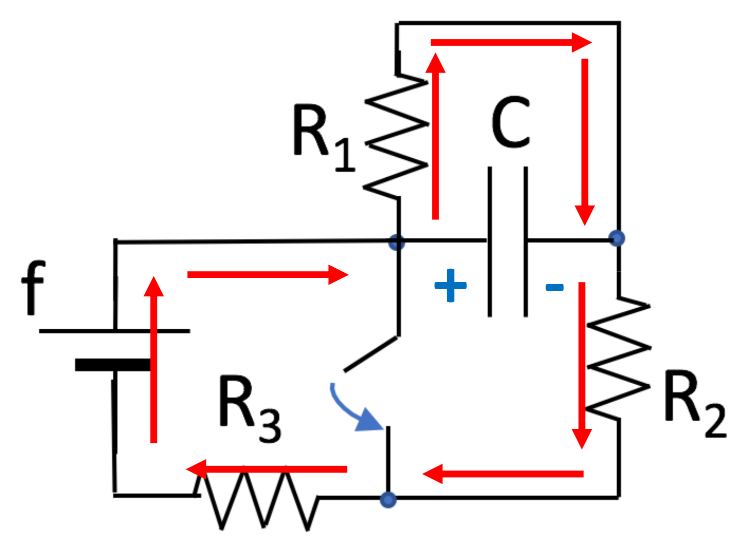
$I(t) = \Delta V_C(0)/R\ e^{-t/\tau}$

Il circuito in figura è in condizioni stazionarie quando, all'istante $t = 0$, viene chiuso l'interruttore. Determinare l'espressione della differenza di potenziale $\Delta V_C(t)$ ai capi del condensatore.

Dati: $R_1 = R_2 = R_3 = R$ con $R = 200\ \Omega$; $f = 15$ V; $C = 20$ nF.

>>> soluzione: $V_c(t) = (5\ \mathrm{V})\ e^{-t/2\mu s}$

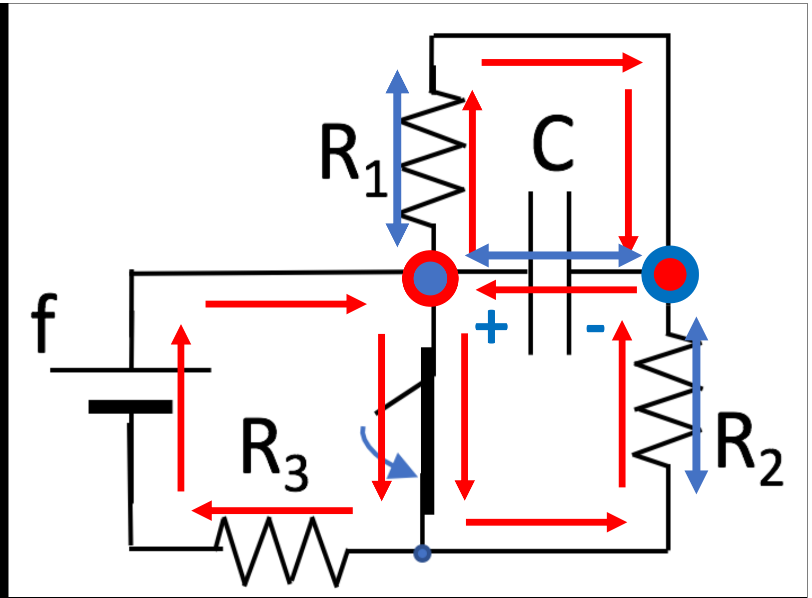


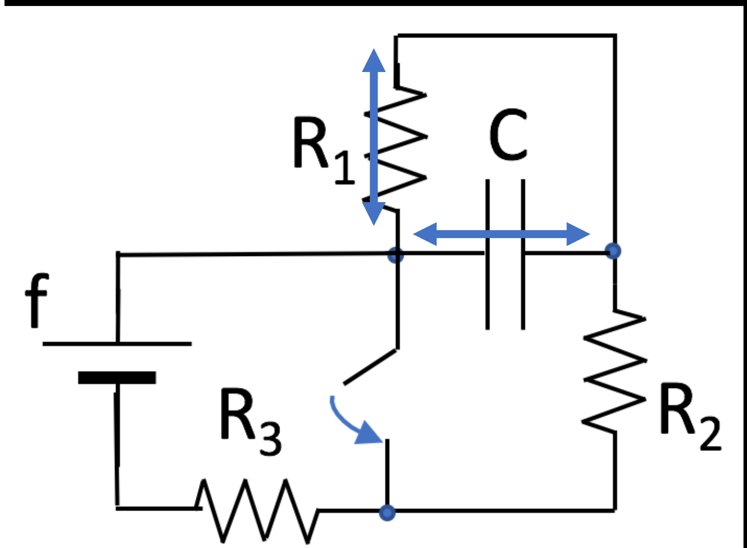


$I = f/(R_1 + R_2 + R_3)$

$I_G = f/R_3$

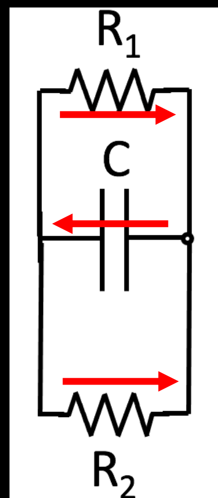
COMMUTAZIONE

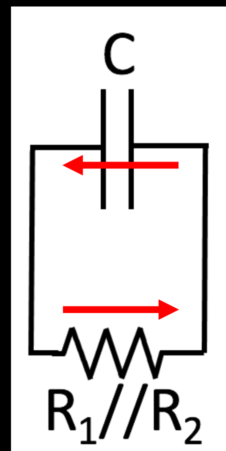




$\Delta V_C(0) = R_1\, I$
$= R_1\, f/(R_1 + R_2 + R_3)$

$\Delta V_C(t) = \Delta V_{R1}(t) = \Delta V_{R2}(t)$





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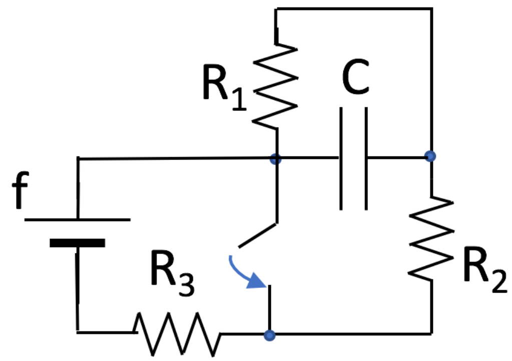
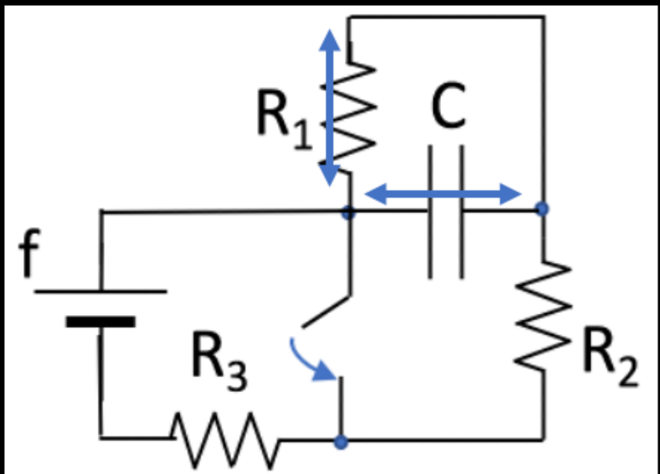




$V_c(2\ \mu s) = 1{,}8$ V

$V_c(4\ \mu s) = 0{,}7$ V

$V_c(6\ \mu s) = 0{,}2$ V

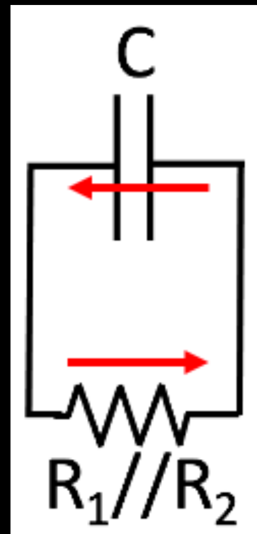
$\Delta V_C(0) = R_1\ I$

$= R_1\ f/(R_1 + R_2 + R_3) = f/3 = 5$ V

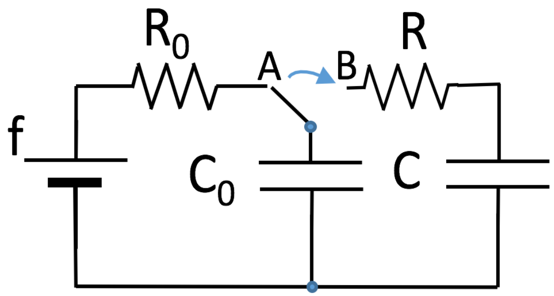
$\tau = [(R_1\ R_2)/(R_1+R_2)]\ C$

$R_1//R_2 = R^2/2R = R/2 = 100\ \Omega$

$\tau = R/2\ C = 100 \times 20 \times 10^{-9}$ s $= 2\ \mu s$

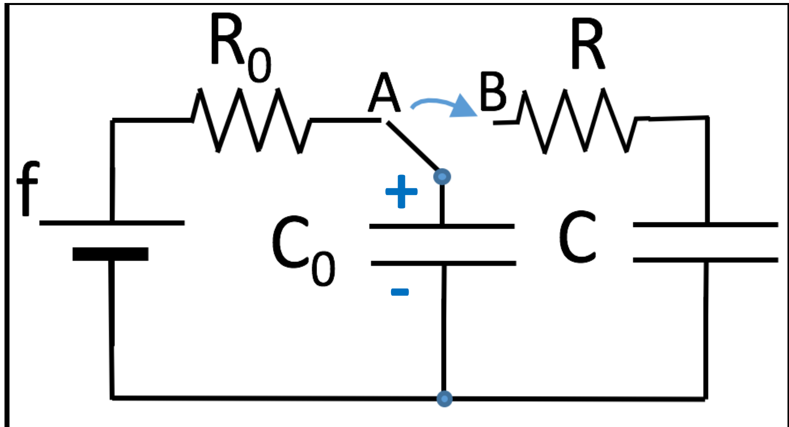


Nel circuito in figura $f = 10$ V, $C_0 = 20\ \mu F$, $C = 30\ \mu F$, i valori di R_0 e R sono ininfluenti. In condizioni stazionarie il condensatore C è scarico. All'istante $t = 0$ il deviatore viene spostato dalla posizione A a B.

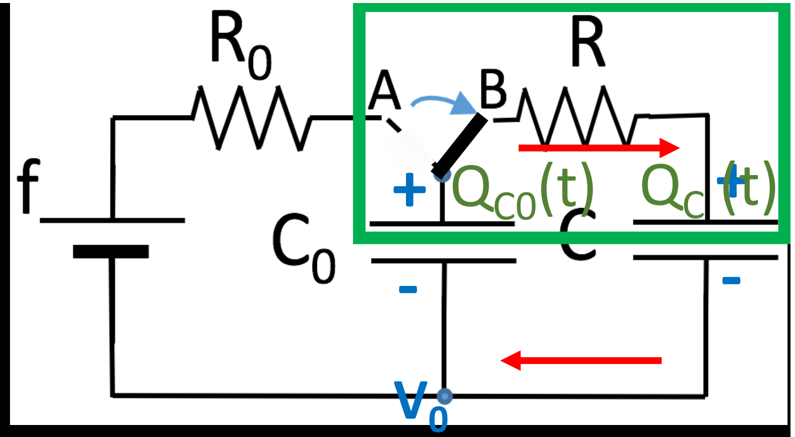


Determinare l'energia che viene dissipata in R dall'istante della commutazione fino a quando non viene ristabilito l'equilibrio.

[redacted]



COMMUTAZIONE



$Q_C(0^-) = 0$

$I_G = 0$

$\Delta V_{C0}(0^-) = f$

$Q_{TOT}(0^-) = Q_{C0}(0^-) + Q_C(0^-) = f\, C_0$

$V_0 + \Delta V_{C0}(t) - R\, I(t) - \Delta V_C(t) = V_0$

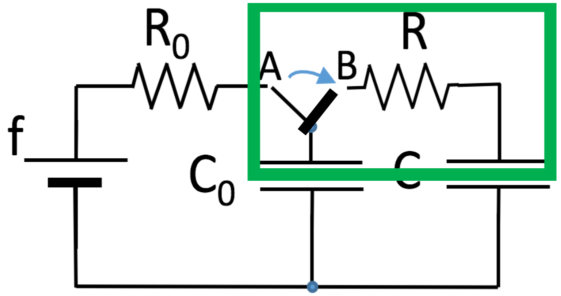
$I(t) = [\Delta V_{C0}(t) - \Delta V_C(t)]/R$

$I(t) = [Q_{C0}(t)/C_0 - Q_C(t)/C]/R$

$Q_{C0}(t) + Q_C(t) = Q_{TOT}(0)$

Nel circuito in figura f = 10 V, C_0 = 20 μF, C = 30 μF, i valori di R_0 e R sono ininfluenti. In condizioni stazionarie il condensatore C è scarico. All'istante t = 0 il deviatore viene spostato dalla posizione A a B.

Determinare l'energia che viene dissipata in R dall'istante della commutazione fino a quando non viene ristabilito l'equilibrio.



$I_R(t) = [\Delta V_{C0}(t) - \Delta V_C(t)]/R$

$Q_{C0}(t) + Q_C(t) = Q_{TOT}(0) = f\, C_0$

ASINTOTICAMENTE

$I(\infty) = [\Delta V_{C0}(\infty) - \Delta V_C(\infty)]/R = 0$

$\Delta V_{C0}(\infty) = \Delta V_C(\infty) \rightarrow C_0 // C$

$C_P = C_0 + C$

energia dissipata in R: $E_R = U_i - U_f$

$U_i = ½\, Q_{TOT}(0)^2/C_0 = ½\, f^2 C_0$ $\qquad$ $U_f = ½\, Q_{TOT}(0)^2/C_P = ½\, f^2 C_0^2/C_P$

$E_R = U_i - U_f = ½\, f^2 C_0 (1 - C_0/C_P) = 600\ \mu J$

ESONERO #$@% !!!

"CARICA" L

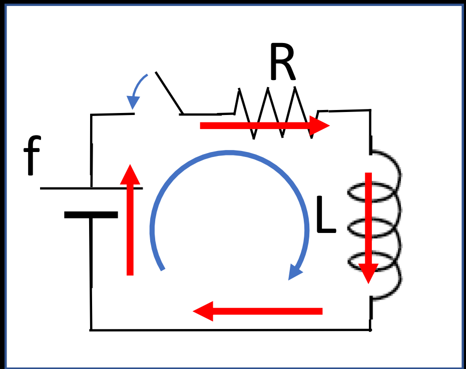
INDUTTANZE

$$\text{f.e.m.} = -\frac{d\Phi_S(\vec{B})}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

$$\text{f.e.m.} = \oint_\gamma \vec{E}\,\vec{dl}$$

IN CONDIZIONI STAZIONARIE NON C'È d.d.p. AI CAPI DELLE INDUTTANZE

$V_0 + f$ $\quad$ $V_0 + f - R\,I$



V_0 $\quad$ $V_0 + f - R\,I + f_{IND}$

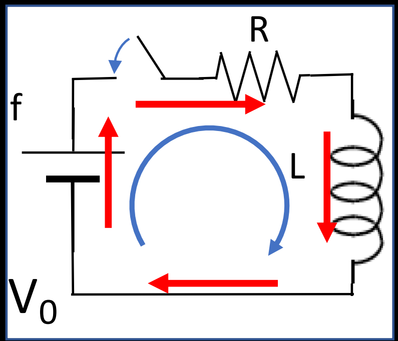
f_{IND} SI OPPONE A BRUSCHE VARIAZIONI DI FLUSSO → $I_L(0^+) = I_L(0^-)$

$$\rightarrow f - R\,I + f_{IND} = 0$$

$$\boxed{\rightarrow f - R\,I - L\frac{dI}{dt} = 0}$$

CORRENTI LENTAMENTE VARIABILI

"CARICA" DELL'INDUTTANZA (inizialmente "scarica")



$$I_L(0^+) = I_L(0^-) = 0$$

$t > 0$
$$f - R\,I - L\frac{dI}{dt} = 0$$

$$f - R\,I = L\frac{dI}{dt}$$

$$f/R - I = L/R\,\frac{dI}{dt}$$

$L/R = \tau$

costante di tempo del circuito

$$\frac{dt}{L/R} = \frac{dI}{f/R - I} = \frac{-d(-I)}{f/R - I} = -\frac{d(f/R - I)}{f/R - I} = -d\ln(f/R - I)$$

$$\int_0^t \frac{dt}{L/R} = \int_0^{I(t)} -d\ln(f/R - I)$$

$$\frac{t-0}{L/R} = -\ln\left(\frac{f/R - I(t)}{f/R - 0}\right)$$

$$-\frac{t}{L/R} = \ln\left(\frac{f/R - I(t)}{f/R}\right)$$

$$e^{-\frac{t}{L/R}} = \frac{f/R - I(t)}{f/R}$$

$$\boxed{I(t) = f/R\,(1 - e^{-\frac{t}{L/R}})}$$

Complementi di fisica generale

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LUNEDÌ 2 MAGGIO ORE 10 - 11

esercizi su

correnti lentamente variabili R, C, L