Complementi di fisica generale

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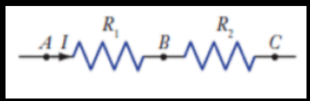
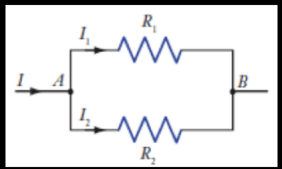
circuiti elettrici

circuiti in condizioni quasi stazionarie

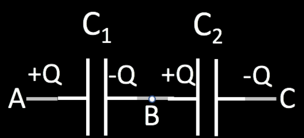
induttanza



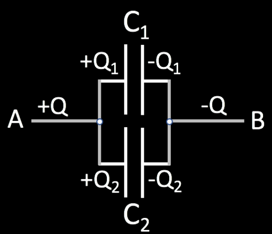
$$R_S = R_1 + R_2$$



$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \qquad R_p = \frac{R_1\, R_2}{R_1 + R_2}$$



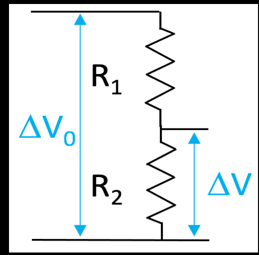
$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_S = \frac{C_1\, C_2}{C_1 + C_2}$$



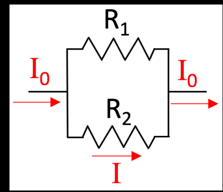
$$C_P = C_1 + C_2$$

$$U = \frac{1}{2} C\, \Delta V^2 \qquad P_G = f\ I$$

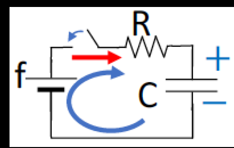
$$U = \frac{1}{2}\, L\, I^2 \qquad P_R = R\, I^2$$



$$\Delta V = \Delta V_0 \frac{R_2}{R_1 + R_2}$$



$$I = I_0 \frac{R_1}{R_1 + R_2}$$

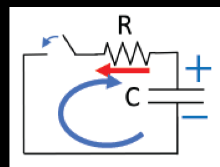


se $\Delta V_C(0) = Q(0)/C = 0$

$Q(t) = f\, C\, (1 - e^{-t/\tau})$

$I(t) = f/R\ e^{-t/\tau}$

$\Delta V_C(t) = f\, (1 - e^{-t/\tau})$



se $\Delta V_C(\infty) = Q(\infty)/C = 0$

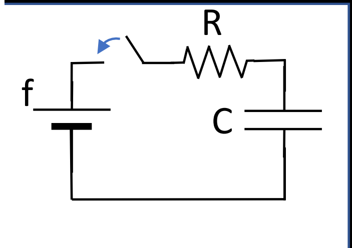
$Q(t) = Q_0\, e^{-t/\tau}$

$I(t) = \Delta V_C(0)/R\ e^{-t/\tau}$

$\Delta V_C(t) = \Delta V_C(0)\, e^{-t/\tau}$

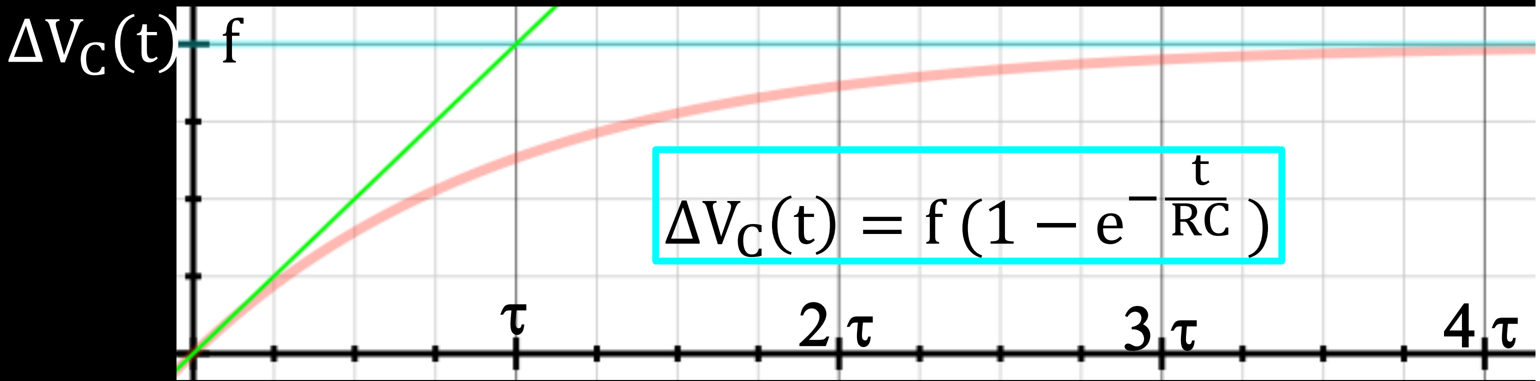
CORRENTI LENTAMENTE VARIABILI

CARICA DEL CONDENSATORE (inizialmente scarico)

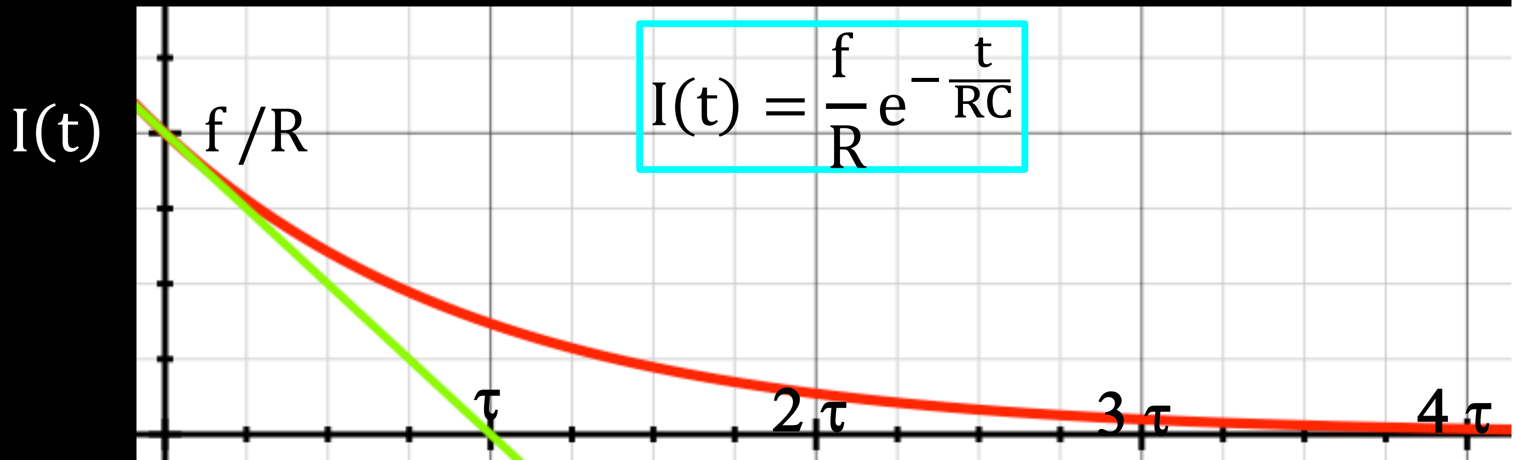


RC = τ

costante di tempo del circuito



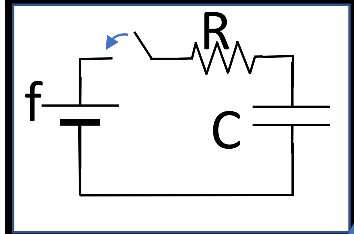
$$\Delta V_C(t) = f\left(1 - e^{-\frac{t}{RC}}\right)$$



$$I(t) = \frac{f}{R} e^{-\frac{t}{RC}}$$

CORRENTI LENTAMENTE VARIABILI: ENERGIA

CARICA DEL CONDENSATORE (inizialmente scarico)



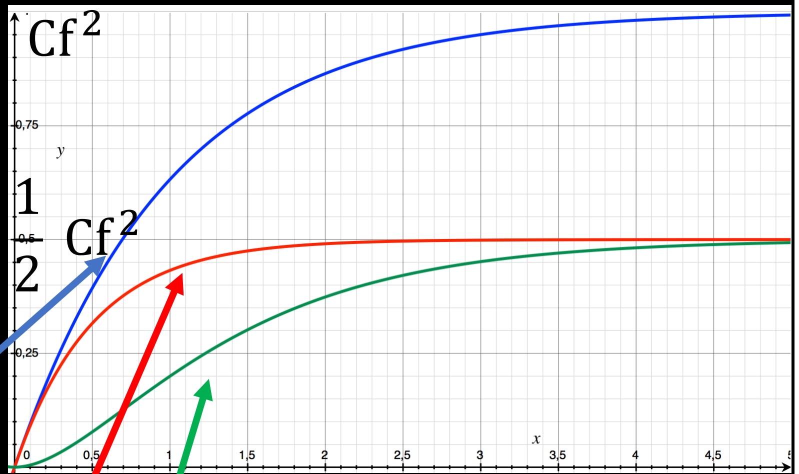
$$P_G = f\,I(t) = \frac{f^2}{R}\,e^{-\frac{t}{RC}}$$

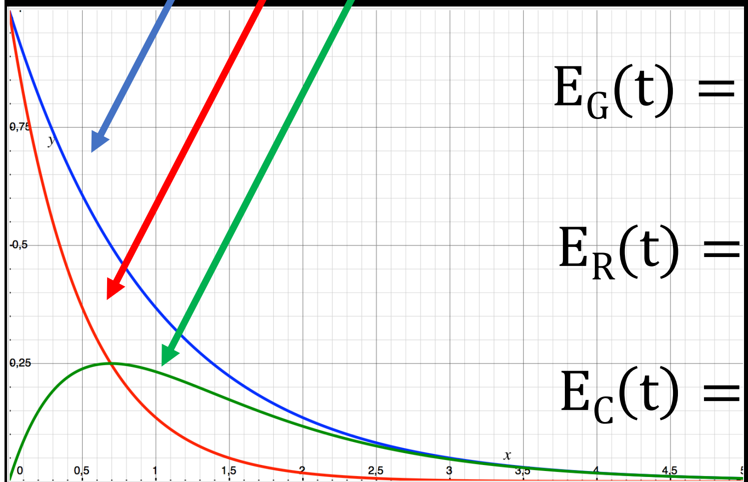
$$P_R = R\,I^2(t) = \frac{f^2}{R}\,e^{-\frac{2t}{RC}}$$

$$P_C = \frac{dU_C(t)}{dt} = \frac{f^2}{R}\left(e^{-\frac{t}{RC}} - e^{-\frac{2t}{RC}}\right)$$

$$\Delta V_C(t) = f\left(1 - e^{-\frac{t}{RC}}\right)$$

$$I(t) = \frac{f}{R}e^{-\frac{t}{RC}}$$





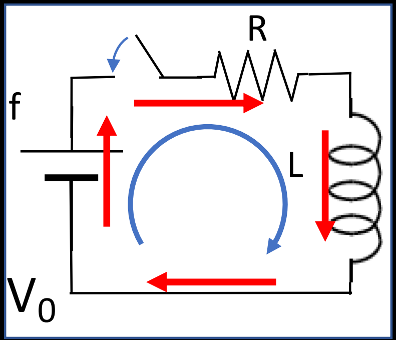
$$E_G(t) = \int_0^t f\,I(t)\,dt = \int_0^t \frac{f^2}{R}\,e^{-\frac{t}{RC}}\,dt = Cf^2\left(1 - e^{-\frac{t}{RC}}\right)$$

$$E_R(t) = \int_0^t R\,I^2(t)\,dt = \int_0^t \frac{f^2}{R}\,e^{-\frac{2t}{RC}}\,dt = \frac{1}{2}Cf^2\left(1 - e^{-\frac{2t}{RC}}\right)$$

$$E_C(t) = \frac{1}{2}C\,\Delta V_C(t)^2 = \frac{1}{2}Cf^2\left(1 - e^{-\frac{t}{RC}}\right)^2$$

CORRENTI LENTAMENTE VARIABILI

"CARICA" DELL'INDUTTANZA (inizialmente "scarica")



$I(0^+) = I(0^-) = 0$

$t > 0 \quad V_0 + f - R\,I + f_{IND} = V_0$

$$f - R\,I - L\frac{dI}{dt} = 0$$

$$f - R\,I = L\frac{dI}{dt}$$

$$f/R - I = L/R\,\frac{dI}{dt}$$

$$\frac{dt}{L/R} = \frac{dI}{f/R - I} = \frac{-d(-I)}{f/R - I} = -\frac{d(f/R - I)}{f/R - I}$$

$L/R = \tau$

costante di tempo del circuito

$$\int_0^t \frac{dt}{L/R} = \int_0^{I(t)} -d\ln(f/R - I)$$

$$\frac{t - 0}{L/R} = -\ln\left(\frac{f/R - I(t)}{f/R - 0}\right)$$

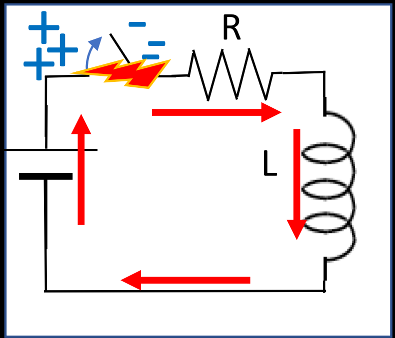
$$-\frac{t}{L/R} = \ln\left(\frac{f/R - I(t)}{f/R}\right)$$

$$e^{-\frac{t}{L/R}} = \frac{f/R - I(t)}{f/R}$$

$$I(t) = f/R\,(1 - e^{-\frac{t}{L/R}})$$

CORRENTI LENTAMENTE VARIABILI

"SCARICA" DELL'INDUTTANZA (inizialmente "carica")



$t > 0$???

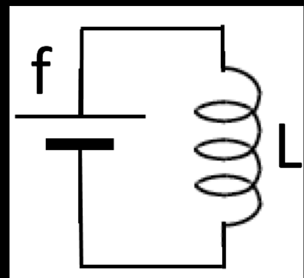
$I(0^-) = I_0$

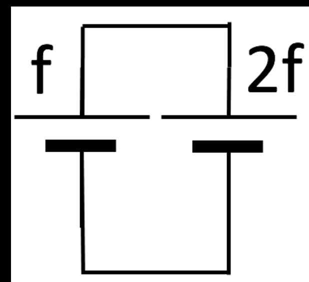
$I(0^+) = I(0^-)$

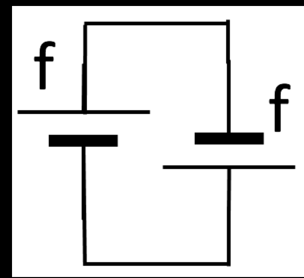
$I(0^+) = 0$

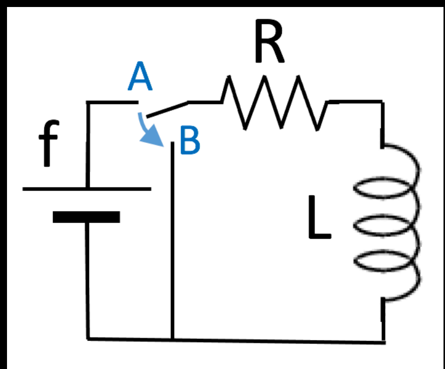
il modello non è realistico!!!

altri modelli non realistici
(per la mancanza di resistenze)





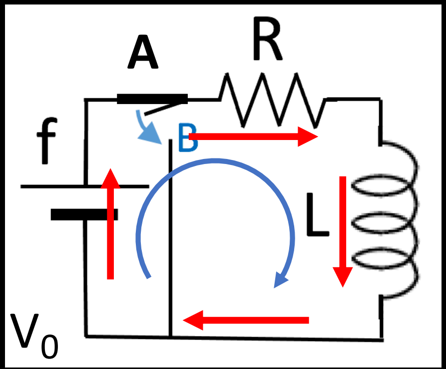




così, invece dopo la commutazione
la corrente continua a scorrere

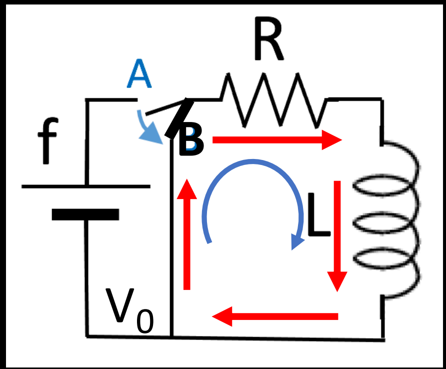
CORRENTI LENTAMENTE VARIABILI

"SCARICA" DELL'INDUTTANZA (inizialmente "carica")



$I(0^+) = I(0^-) = f/R = I_0$

COMMUTAZIONE



$t < 0$ (A) $\quad V_0 + f - R\,I + f_{IND} = V_0$

$$f - R\,I - L\frac{dI}{dt} = 0$$

$$f - R\,I = 0$$

$$I(0^-) = f/R = I_0$$

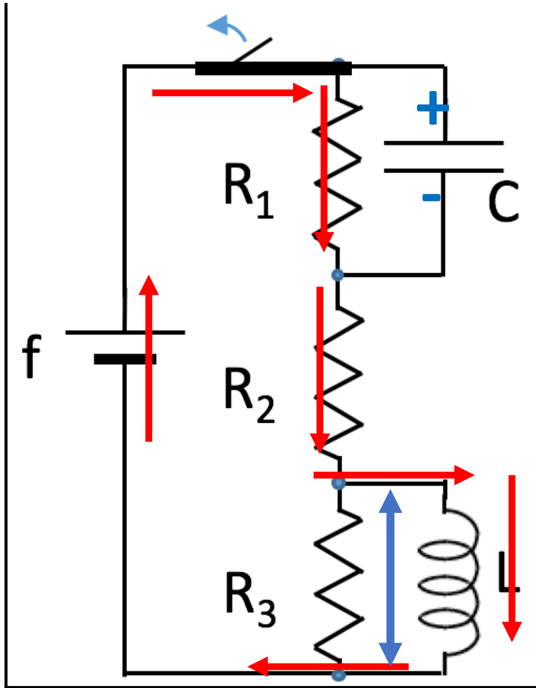
$t > 0$ (B) $\quad V_0 - R\,I + f_{IND} = V_0$

$$-R\,I - L\frac{dI}{dt} = 0 \rightarrow -R\,I = L\frac{dI}{dt} \rightarrow -\frac{dt}{L/R} = \frac{dI}{I}$$

$$\int_0^t -\frac{dt}{L/R} = \int_{I_0}^{I(t)} d\ln(I) \rightarrow -\frac{t-0}{L/R} = \ln\left(\frac{I(t)}{I_0}\right)$$

$$e^{-\frac{t}{L/R}} = \frac{I(t)}{I_0}$$

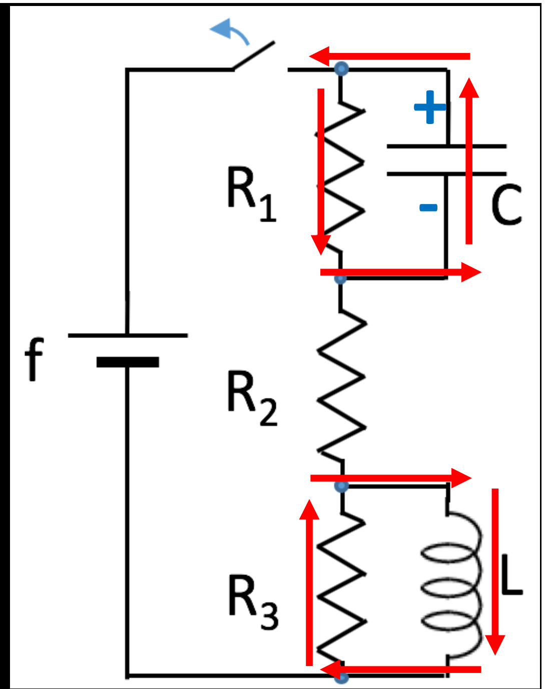
$$\boxed{I(t) = I_0\, e^{-\frac{t}{L/R}}}$$



$R_1 = R_2 = R_3 = R$

$\Delta V_C(0) = R_1 I = R_1 f/(R_1 + R_2) = f/2$

COMMUTAZIONE



$I_{R2} = 0$

$I = f/(R_1 + R_2) = f/2R$

$\tau_C = R_1 C$ $\quad \Delta V_C(t) = \Delta V_C(0)\, e^{-t/\tau C} = f/2\, e^{-t/\tau C}$

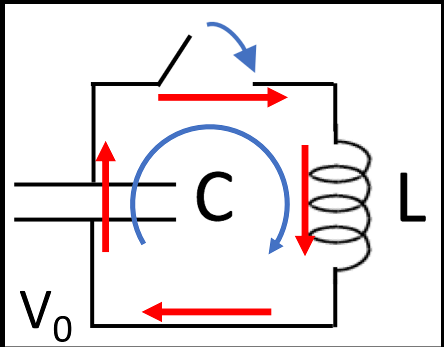
$\tau_L = L/R_3$ $\quad I_L(t) = I_L(0)\, e^{-t/\tau L} = f/2R\, e^{-t/\tau L}$

$U_C(t) = ½\, C\, (f/2\, e^{-t/\tau C})^2$

$U_L(t) = ½\, L\, (f/2R\, e^{-t/\tau L})^2$

CORRENTI LENTAMENTE VARIABILI

CIRCUITO OSCILLANTE



$Q(0^-) = Q_0$

$I(0^-) = 0$

C: $Q(0^+) = Q(0^-) = Q_0$

L: $I(0^+) = I(0^-) = 0$

$t > 0 \quad V_0 + \Delta V_C(t) + f_{IND} = V_0$

$$\frac{Q(t)}{C} - L\frac{dI(t)}{dt} = 0$$

$dQ < 0 \rightarrow dQ = -I(t)\,dt \rightarrow I(t) = -dQ/dt$

$$\frac{Q(t)}{C} - L\frac{d\left(-\frac{dQ}{dt}\right)}{dt} = 0$$

$$\frac{Q(t)}{C} + L\frac{d^2Q}{dt^2} = 0$$

$$\frac{d^2Q(t)}{dt^2} + \frac{Q(t)}{LC} = 0$$

$$\frac{d^2u(t)}{dt^2} + \omega^2 u(t) = 0$$

$$\omega^2 = \frac{1}{LC} \rightarrow \omega = \frac{1}{\sqrt{LC}}$$

MOTO ARMONICO

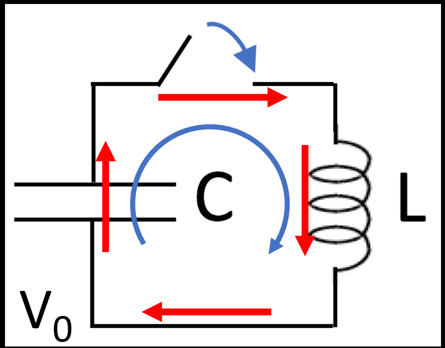
- $x(t) = A\cos(\omega t + \varphi)$
- $v_x(t) = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi)$
- $a_x(t) = \frac{dv_x}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \varphi) = -\omega^2 x(t)$
- $\frac{d^2x}{dt^2} + \omega^2 x = 0$

$$\frac{d^2u(t)}{dt^2} + \omega^2 u(t) = 0 \leftrightarrow u(t) \text{ armonica di periodo } T = \frac{2\pi}{\omega}$$

- $y(t) = A\sin(\omega t + \varphi)$
- $v_y(t) = \frac{dy}{dt} = A\omega\cos(\omega t + \varphi)$
- $a_y(t) = \frac{dv_y}{dt} = \frac{d}{dt}\frac{dy}{dt} = \frac{d^2y}{dt^2} = -A\omega^2\sin(\omega t + \varphi) = -\omega^2 y(t)$
- $\frac{d^2y}{dt^2} + \omega^2 y = 0$

CORRENTI LENTAMENTE VARIABILI

CIRCUITO OSCILLANTE



$Q(0^-) = Q_0$

$I(0^-) = 0$

C: $Q(0^+) = Q(0^-) = Q_0$

L: $I(0^+) = I(0^-) = 0$

$$\frac{d^2Q(t)}{dt^2} + \frac{Q(t)}{LC} = 0$$

$$\frac{d^2u(t)}{dt^2} + \omega^2 u(t) = 0$$

$$\omega^2 = \frac{1}{LC} \rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$Q(t) = A\cos(\omega t + \varphi)$$

$$I(t) = -\frac{dQ(t)}{dt} = A\omega \sin(\omega t + \varphi)$$

$$I(0) = A\omega \sin(0 + \varphi) = 0$$

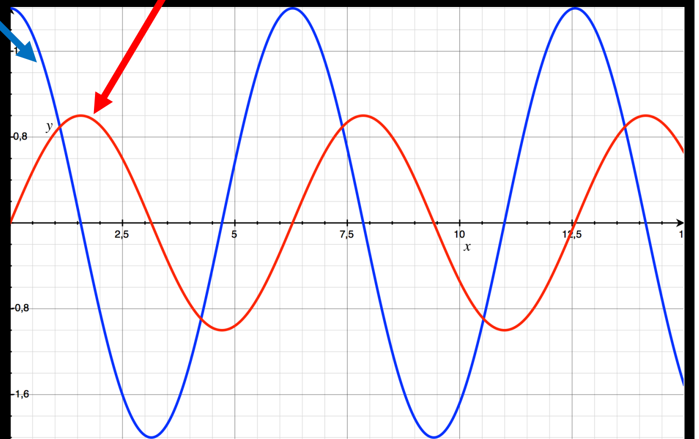
$$\rightarrow A\omega \sin(\varphi) = 0 \rightarrow \varphi = 0$$

$$Q(0) = A\cos(0 + \varphi) = Q_0$$

$$\rightarrow A\cos(0) = Q_0 \rightarrow A = Q_0$$

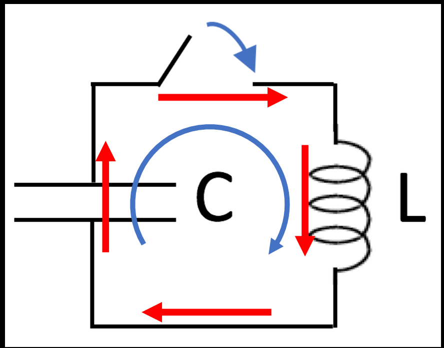
$$Q(t) = Q_0 \cos(\omega t)$$

$$I(t) = Q_0\, \omega \sin(\omega t)$$



CORRENTI LENTAMENTE VARIABILI

CIRCUITO OSCILLANTE



C: $Q(0^+) = Q(0^-) = Q_0$

L: $I(0^+) = I(0^-) = 0$

$$\omega^2 = \frac{1}{LC}$$

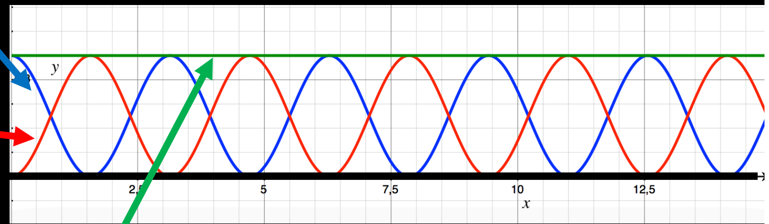
$$U_C(t) = \frac{1}{2}\frac{Q(t)^2}{C} = \frac{1}{2}\frac{{Q_0}^2}{C}\cos^2(\omega t)$$

$$U_C(0) = \frac{1}{2}\frac{{Q_0}^2}{C}$$

$$U_L(0) = \frac{1}{2} L\, I(0)^2 = 0$$

$$U_L(t) = \frac{1}{2} L\, I(t)^2 = \frac{1}{2} L\, {Q_0}^2\omega^2 \sin^2(\omega t)$$

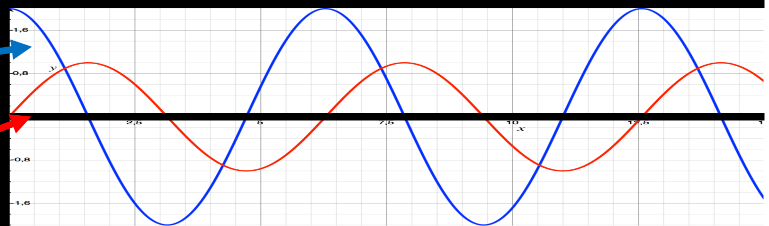
$$= \frac{1}{2} L\, {Q_0}^2 \frac{1}{LC}\sin^2(\omega t) = \frac{1}{2}\frac{{Q_0}^2}{C}\sin^2(\omega t)$$



$$U_C(t) + U_L(t) = \frac{1}{2}\frac{{Q_0}^2}{C}\cos^2(\omega t) + \frac{1}{2}\frac{{Q_0}^2}{C}\sin^2(\omega t) = \frac{1}{2}\frac{{Q_0}^2}{C}$$

$$Q(t) = Q_0\cos(\omega t)$$

$$I(t) = Q_0\,\omega\sin(\omega t)$$



Complementi di fisica generale

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ESONERO

15/5 o ... 8/5 ?

LUNEDÌ 3 MAGGIO ORE 10 - 11

esercizi su

correnti lentamente variabili R, C, L