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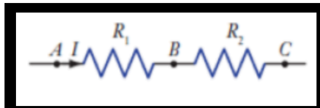
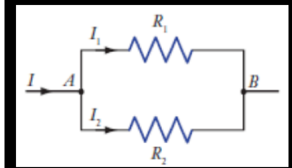
circuiti elettrici

esercitazione su:

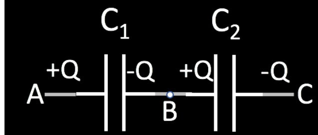
circuiti in condizioni quasi stazionarie



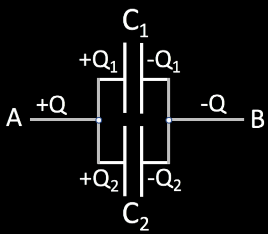
$$R_S = R_1 + R_2$$



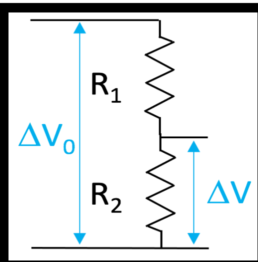
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \qquad R_p = \frac{R_1\, R_2}{R_1 + R_2}$$



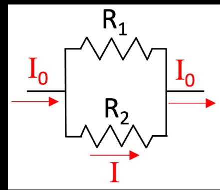
$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_S = \frac{C_1\, C_2}{C_1 + C_2}$$



$$C_P = C_1 + C_2$$



$$\Delta V = \Delta V_0 \frac{R_2}{R_1 + R_2}$$



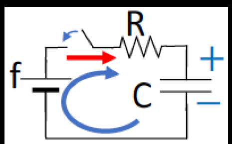
$$I = I_0 \frac{R_1}{R_1 + R_2}$$

$$U = \frac{1}{2} C\, \Delta V^2$$

$$U = \frac{1}{2}\, L\, I^2$$

$$P_G = f\ I$$

$$P_R = R\, I^2$$

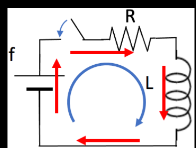


se $\Delta V_C(0) = Q(0)/C = 0$

$Q(t) = f\,C\,(1 - e^{-t/\tau})$

$I(t) = f/R\ e^{-t/\tau}$

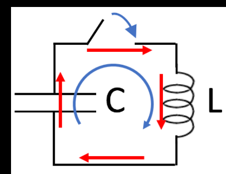
$\Delta V_C(t) = f\,(1 - e^{-t/\tau})$



se $I(0) = 0$

$I(t) = I(\infty)\,(1 - e^{-t/\tau}) = f/R\,(1 - e^{-t/\tau})$

$\Delta V_L(t) = L\,dI/dt = L\,I(\infty)/\tau\ e^{-t/\tau} = f\,e^{-t/\tau}$

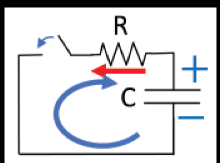


se $Q(0) = Q_0$ e $I(0) = 0$

$Q(t) = Q_0 \cos(\omega t)$

$I(t) = Q_0 \omega \sin(\omega t)$

$$\omega = \frac{1}{\sqrt{LC}}$$

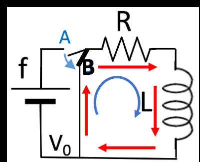


se $\Delta V_C(\infty) = Q(\infty)/C = 0$

$Q(t) = Q_0\, e^{-t/\tau}$

$I(t) = \Delta V_C(0)/R\ e^{-t/\tau}$

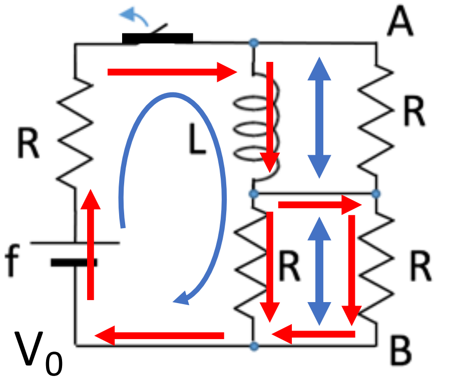
$\Delta V_C(t) = \Delta V_C(0)\, e^{-t/\tau}$



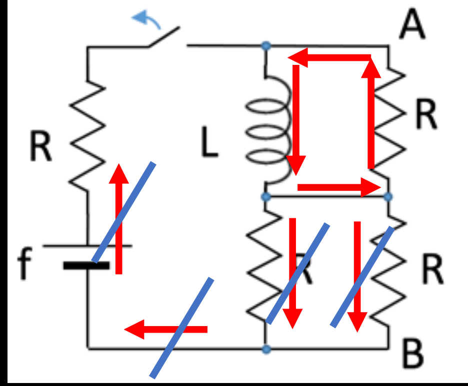
se $I(0) = I_0$ e $I_L(\infty) = 0$

$I(t) = I_0\, e^{-t/\tau}$

$\Delta V_L(t) = L\,dI/dt = L\,I_0/\tau\ e^{-t/\tau} = R\,I_0\, e^{-t/\tau}$



$$I(0^{+}) = I(0^{-}) = \frac{2}{3}\, f/R$$



$$V_0 + f - R\,I - R_P\,I = V_0$$

$$f - R\,I - R/2\;\;I = 0$$

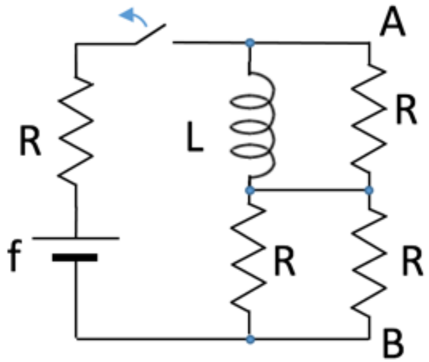
$$f = 3/2\;R\,I$$

$$I(0^{-}) = \frac{2}{3}\, f/R$$

COMMUTAZIONE

$$\tau = \frac{L}{R}$$

$$I(t) = I(0)\; e^{-\frac{t}{\tau}}$$

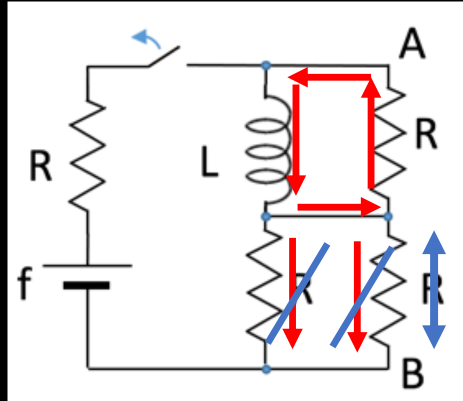


3) Determinare l'espressione della tensione $\Delta V_{AB}(t)$ presente tra i punti A e B dopo l'apertura dell'interruttore.

>>> soluzione: $\Delta V_{AB}(t) = \Delta V_R(t) = R\, f/(R+R/2)\, \exp[-t/(L/R)]$

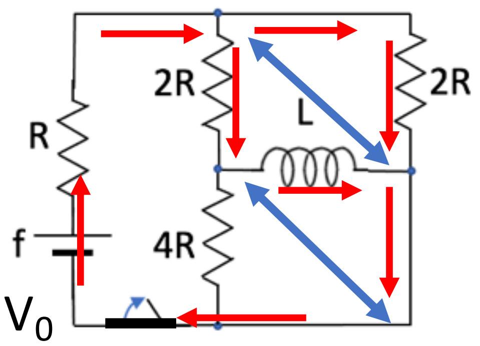
$$I(0^+) = I(0^-) = \frac{2}{3}\, f/R$$

$$\tau = \frac{L}{R} \qquad I(t) = I(0)\, e^{-\frac{t}{\tau}}$$



$$\Delta V_{AB}(t) = R\, I(t)$$

$$\Delta V_{AB}(t) = R\, I(0)\, e^{-\frac{t}{\tau}} = R \frac{2}{3} f/R\, e^{-\frac{t}{L/R}} = \frac{2}{3} f\, e^{-\frac{t}{L/R}}$$



4) Il circuito in figura è a regime quando, all'istante t = 0, l'interruttore viene aperto. Ricavare l'andamento I(t) della corrente che scorre in 4R per t > 0
>>> soluzione: f/4R exp[-t/(L/2R)]

$$V_0 + f - R\,I - R_P\,I = V_0$$

$$f - R\,I - (2R//2R)\;I = 0$$

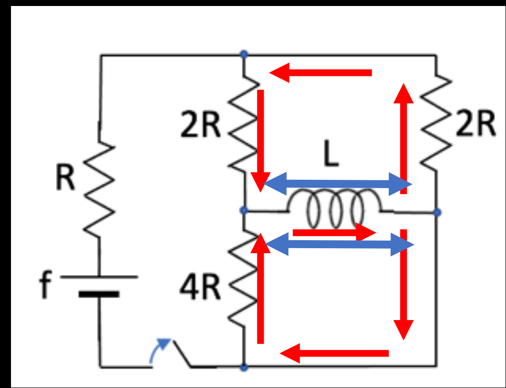
$$f = 2\,R\,I$$

$$I_G(0^-) = \frac{1}{2}\,f/R$$

$$I_L(0^-) = \frac{1}{2}I_G(0^-) = \frac{1}{4}\,f/R$$

$$I_L(0^+) = I_L(0^-) = \frac{1}{4}\,f/R$$

COMMUTAZIONE



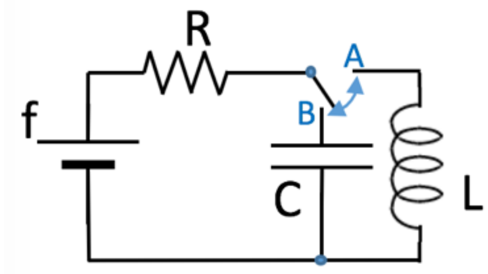
$$\tau = \frac{L}{(2R+2R)//4R} = \frac{L}{2R}$$

$$I_L(t) = I(0)e^{-\frac{t}{\tau}} = \frac{1}{4}\frac{f}{R}e^{-\frac{t}{L/2R}}$$

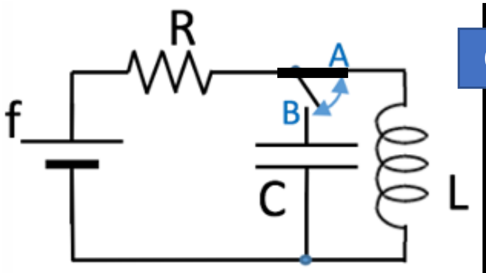
$$I_{4R}(t) = \frac{1}{2}I_L(t) = \frac{1}{8}\frac{f}{R}e^{-\frac{t}{L/2R}}$$

6) Il deviatore del circuito in figura è inizialmente in posizione A e la capacità è scarica. Il deviatore viene messo in posizione B, si aspetta che il sistema raggiunga l'equilibrio e poi viene rimesso in posizione A. In seguito alle due commutazioni in uno dei due casi la potenza erogata dal generatore (f = 2 V) scende esponenzialmente da 20 mW a 0 mW con una costante di tempo di 2 μs; nell'altro caso la potenza erogata sale esponenzialmente da 0 mW a 20 mW con la stessa costante di tempo. Determinare il valore dei tre componenti R, L, C.

>>> soluzione: 200 Ω; 10 nF; 0,4 mH



$$P(t) = f\,I(t)$$



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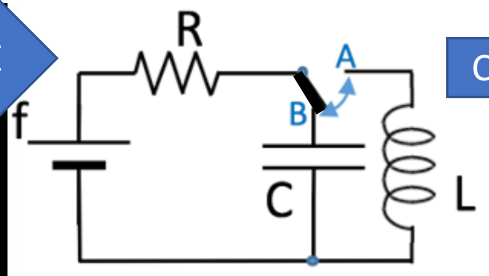

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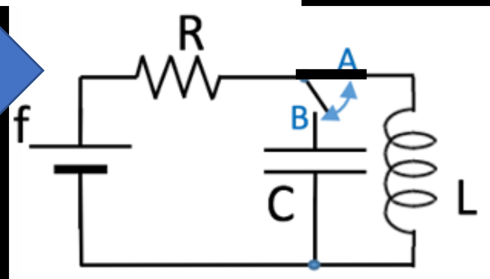


$$I(t) = \frac{f}{R}$$

$$P(t) = P_0 = \frac{f^2}{R}$$

$$R = \frac{f^2}{P_0} = \frac{4\ V^2}{20 \cdot 10^{-3} W} = 200\ \Omega$$

$$I(t) = \frac{f}{R} e^{-\frac{t}{RC}}$$

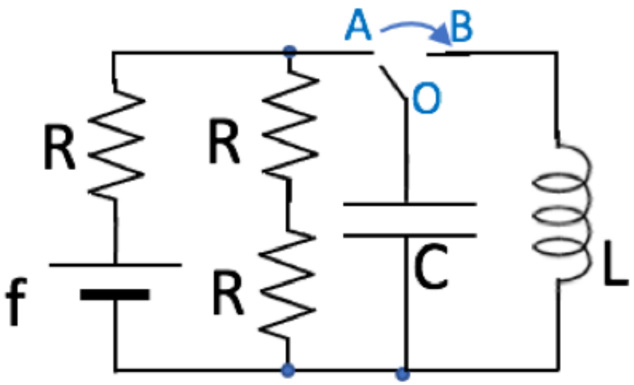
$$P(t) = \frac{f^2}{R} e^{-\frac{t}{RC}}$$

$$C = \frac{\tau}{R} = \frac{2 \cdot 10^{-6}\ s}{200\ \Omega} = 10\ nF$$

$$I(t) = \frac{f}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$$

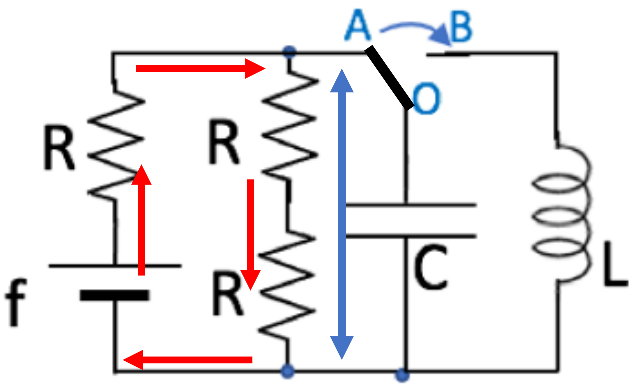
$$P(t) = \frac{f^2}{R}\left(1 - e^{-\frac{t}{L/R}}\right)$$

$$L = R\tau = 200\,\Omega \times 2\,10^{-6}\ s = 0{,}4\ mH$$



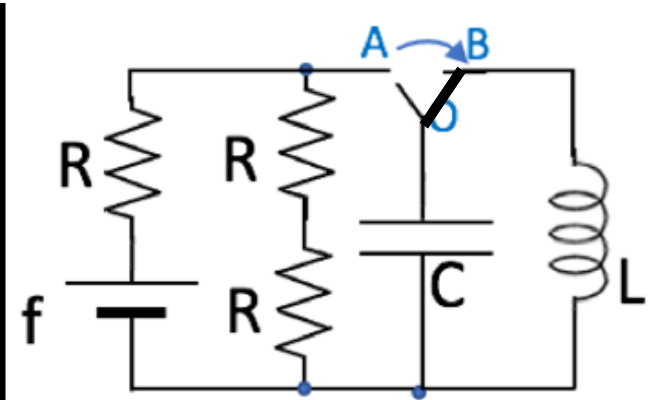
7) All'istante t = 0 il deviatore commuta dalla posizione A alla posizione B e la carica inizia ad oscillare. Determinare la massima intensità di corrente che successivamente scorre nel condensatore

>>> soluzione: $I_{max} = 2/3\ f\ (C/L)^{1/2}$



$$Q(0^+) = Q(0^-) = 2/3\ fC$$
$$I_L(0^+) = I_L(0^-) = 0$$

COMMUTAZIONE



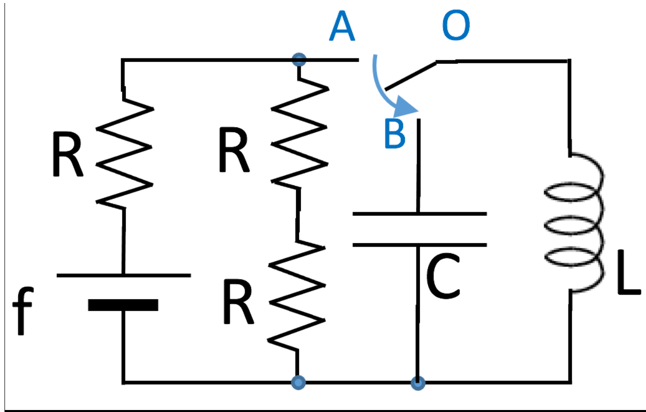
$$\Delta V_C(0^-) = (R+R)\, I(0^-) = 2R\,\frac{f}{3R} = \frac{2}{3} f$$

$$Q(t) = Q_0 \cos(\omega t)$$
$$I(t) = Q_0\, \omega \sin(\omega t) \rightarrow I_{MAX} = Q_0\, \omega$$

$$\omega = \frac{1}{\sqrt{LC}}$$

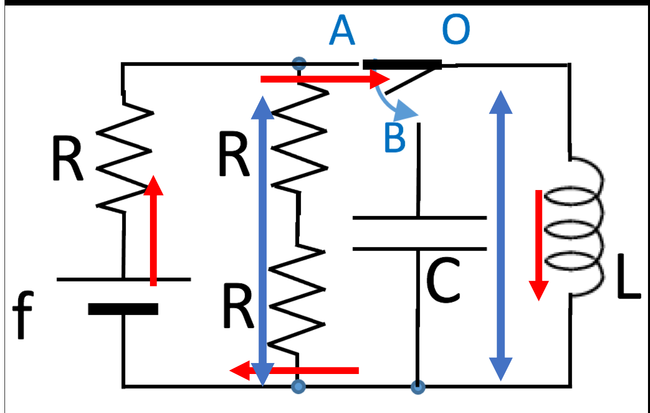
$$U_{CMAX} = U_{LMAX} \qquad \frac{1}{2}\frac{Q_0^2}{C} = \frac{1}{2} L I_{MAX}^2 \rightarrow I_{MAX}^2 = \frac{Q_0^2}{LC}$$

$$= \frac{2}{3} f \sqrt{\frac{C}{L}}$$

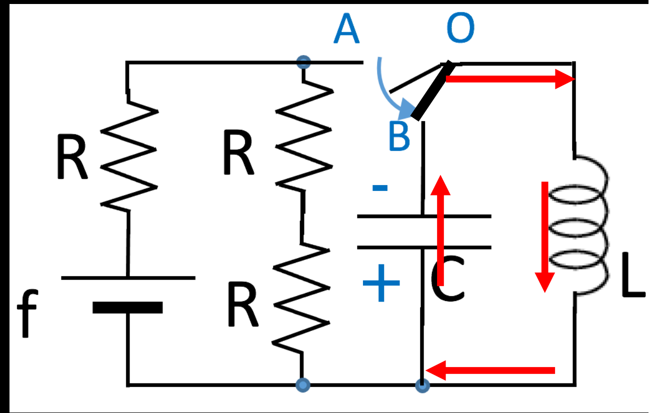


Il condensatore è scarico quando, all'istante t = 0, il deviatore commuta dalla posizione A alla posizione B e la corrente inizia ad oscillare. Determinare la <u>massima</u> carica del condensatore

$$Q(0^+) = Q(0^-) = 0$$
$$I_L(0^+) = I_L(0^-) = f/R$$



COMMUTAZIONE



$$\omega = \frac{1}{\sqrt{LC}}$$

$$U_{CMAX} = U_{LMAX} \qquad \frac{1}{2}\frac{Q^2_{MAX}}{C} = \frac{1}{2}LI_0^2 \rightarrow Q^2_{MAX} = LC\, I_0^2 = \frac{I_0^2}{\omega^2}$$

$$Q_{MAX} = \frac{I_0}{\omega} = \frac{f}{R}\sqrt{LC}$$

$$dQ > 0 \rightarrow dQ = I(t)\, dt \rightarrow I = dQ/dt$$

$$Q(t) = A\cos(\omega t + \varphi)$$
$$I(t) = -A\,\omega \sin(\omega t + \varphi)$$

$$Q(0) = A\cos(\varphi) = 0 \rightarrow \varphi = \pi/2$$
$$I(0) = -A\,\omega\sin(\pi/2) = I_0 \rightarrow A = -I_0/\omega$$

$$Q(t) = I_0/\omega\, \sin(\omega t)$$
$$I(t) = I_0 \cos(\omega t)$$

Complementi di fisica generale

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VENERDÌ 6 MAGGIO ORE 8:30-10:00

correnti lentamente variabili