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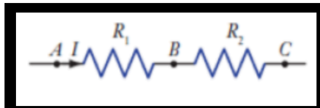
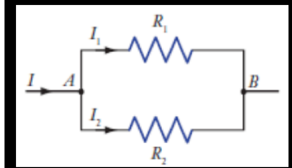
circuiti elettrici

esercitazione su:

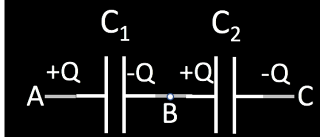
circuiti in condizioni quasi stazionarie



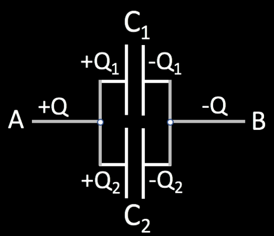
$$R_S = R_1 + R_2$$



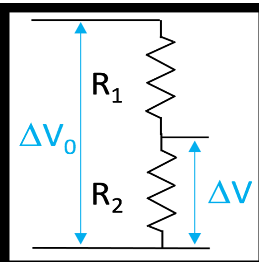
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \qquad R_p = \frac{R_1\, R_2}{R_1 + R_2}$$



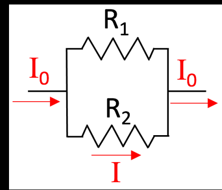
$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_S = \frac{C_1\, C_2}{C_1 + C_2}$$



$$C_P = C_1 + C_2$$



$$\Delta V = \Delta V_0 \frac{R_2}{R_1 + R_2}$$



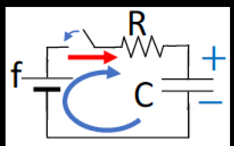
$$I = I_0 \frac{R_1}{R_1 + R_2}$$

$$U = \frac{1}{2} C\, \Delta V^2$$

$$U = \frac{1}{2}\, L\, I^2$$

$$P_G = f\ I$$

$$P_R = R\, I^2$$

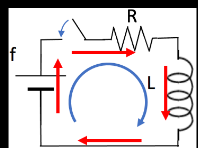


se $\Delta V_C(0) = Q(0)/C = 0$

$Q(t) = f\,C\,(1 - e^{-t/\tau})$

$I(t) = f/R\ e^{-t/\tau}$

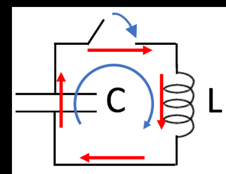
$\Delta V_C(t) = f\,(1 - e^{-t/\tau})$

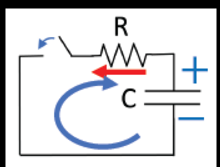


se $I(0) = 0$

$I(t) = I(\infty)\,(1\text{-}e^{-t/\tau}) = f/R\,(1\text{-}e^{-t/\tau})$

$\Delta V_L(t) = L\,dI/dt = L\,I(\infty)/\tau\ e^{-t/\tau} = f\,e^{-t/\tau}$



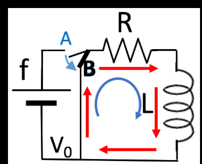


se $\Delta V_C(\infty) = Q(\infty)/C = 0$

$Q(t) = Q_0\,e^{-t/\tau}$

$I(t) = \Delta V_C(0)/R\ e^{-t/\tau}$

$\Delta V_C(t) = \Delta V_C(0)\ e^{-t/\tau}$



se $I(0) = I_0$ e $I_L(\infty) = 0$

$I(t) = I_0\,e^{-t/\tau}$

$\Delta V_L(t) = L\,dI/dt = L\,I_0/\tau\ e^{-t/\tau} = R\,I_0\,e^{-t/\tau}$

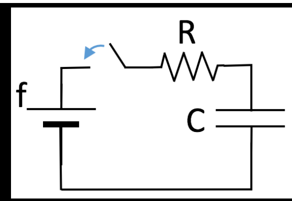
se $Q(0) = Q_0$ e $I(0) = 0$

$Q(t) = Q_0 \cos(\omega t)$

$I(t) = Q_0\omega \sin(\omega t)$

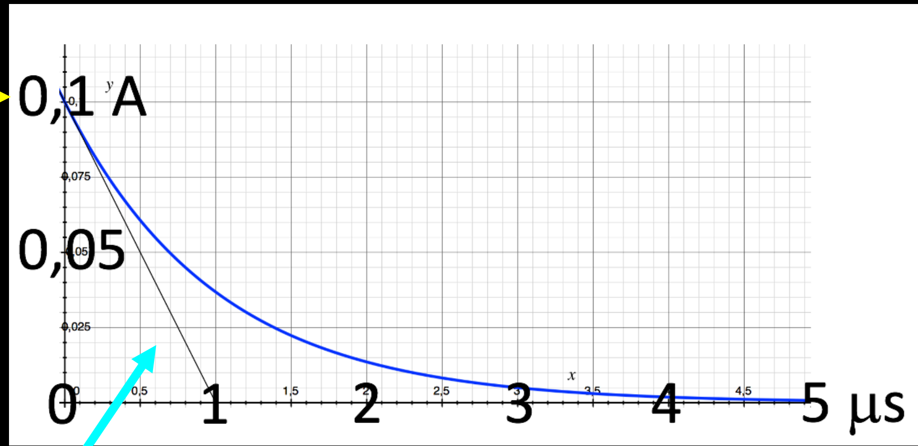
$$\omega = \frac{1}{\sqrt{LC}}$$

1) In figura è riportata la corrente che scorre nel condensatore, inizialmente scarico, dall'istante di chiusura dell'interruttore. Sapendo che C = 20 nF determinare i valori di R e f.



>>>> R = 50 Ω, f = 5 V

$I_0 = f/R$ →



$\tau = RC = 1\ \mu s \rightarrow R = 1\ \mu s / 20\ nF = 50\ \Omega$

$I_0 = f/R = 0{,}1\ A \rightarrow f = 0{,}1\ A \ \ 50\ \Omega = 5\ V$

$$I(t) = I_0 - \frac{I_0}{\tau} t$$

τ

$$\frac{dI}{dt} = \frac{d\left(I_0 e^{-\frac{t}{\tau}}\right)}{dt} = -\frac{I_0 e^{-\frac{t}{\tau}}}{\tau} \quad \rightarrow \left.\frac{dI}{dt}\right|_0 = -\frac{I_0}{\tau}$$

$$I(t^*) = I_0 - \frac{I_0}{\tau} t^* = 0 \rightarrow t^* = \tau$$

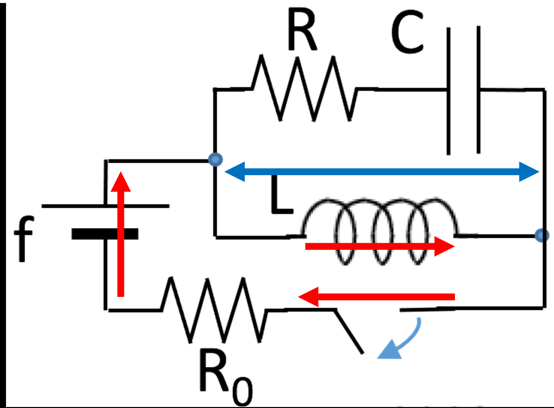
se $\Delta V_C(0) = Q(0)/C = 0$

$Q(t) = f\, C\, (1 - e^{-t/\tau})$

$I(t) = f/R\ e^{-t/\tau}$

$\Delta V_C(t) = f\, (1 - e^{-t/\tau})$

2) Il generatore eroga una potenza di 1 W quando, a t = 0, viene aperto l'interruttore.
Dopo un tempo t* l'energia immagazzinata nel circuito si è dimezzata ed è ugualmente ripartita fra la capacità e l'induttanza.
Determinare il valore della corrente nell'induttanza all'istante t*.
Dati: R_0 = 25 Ω, R = 200 Ω, L = 1 mH, C = 100 nF.



>>>> $I_L(t^*)$ = 0,1 A

$$f - R_0 I_0 = 0 \rightarrow I_0 = \frac{f}{R_0}$$

$$P_G = f\ I_0 = f \frac{f}{R_0} = \frac{f^2}{R_0}$$

$$f^2 = P_G R_0$$

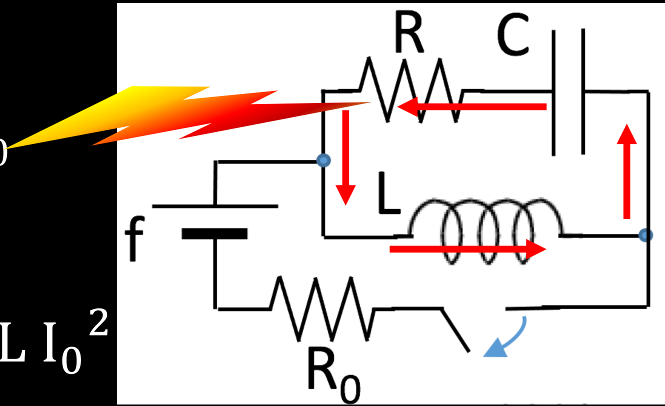
$$\rightarrow f = \sqrt{1\ W\ \ 25\ \Omega} = 5\ V$$

$$I_0 = \frac{f}{R_0} = \frac{5\ V}{25\ \Omega} = 0{,}2\ A$$

$$P_G = f\ I$$

$$U_0 = \frac{1}{2}\ L\, I_0{}^2$$

$$U_{TOT}(t^*) = \frac{1}{2}\ U_0$$

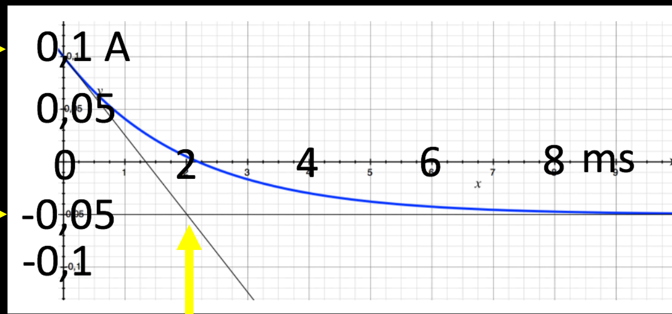


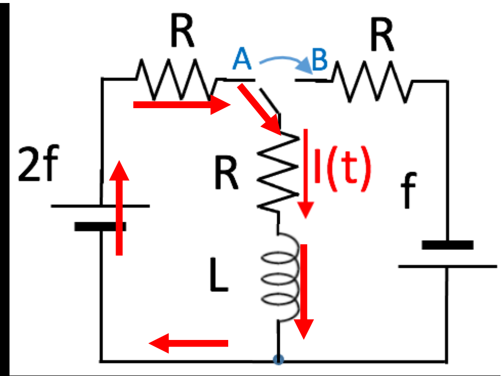
$$U_L(t^*) = \frac{1}{2} U_{TOT}(t^*) = \frac{1}{4}\ U_0 = \frac{1}{8}\ L\, I_0{}^2$$

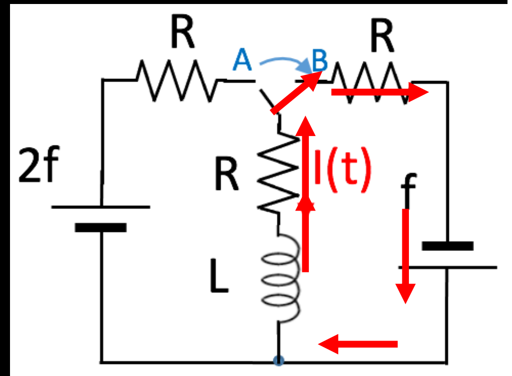
$$U_L(t^*) = \frac{1}{2}\ L\, I_L^2(t^*) = \frac{1}{8} L\, I_0{}^2 \rightarrow \frac{1}{2}\ I_L^2(t^*) = \frac{1}{8}\ I_0{}^2$$

$$\rightarrow I_L(t^*) = \frac{1}{2}\ I_0 = 0{,}1\ A$$

3) In figura è riportato l'andamento della corrente I(t).
Sapendo che L = 0,2 H determinare il valore di f.
>>>> f = 5 V







prima della commutazione

la corrente vale $I(0^-) = 2f/2R = f/R = 0{,}1$ A

$\tau = 2$ ms

dopo molto tempo dalla commutazione

la corrente vale $I(\infty) = -f/2R = -0{,}05$ A

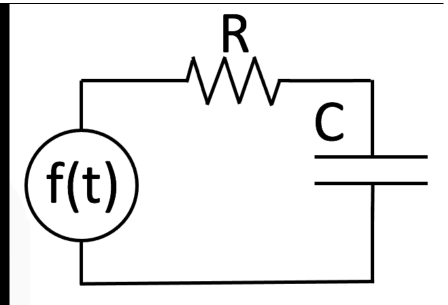
durante il transitorio la corrente evolve

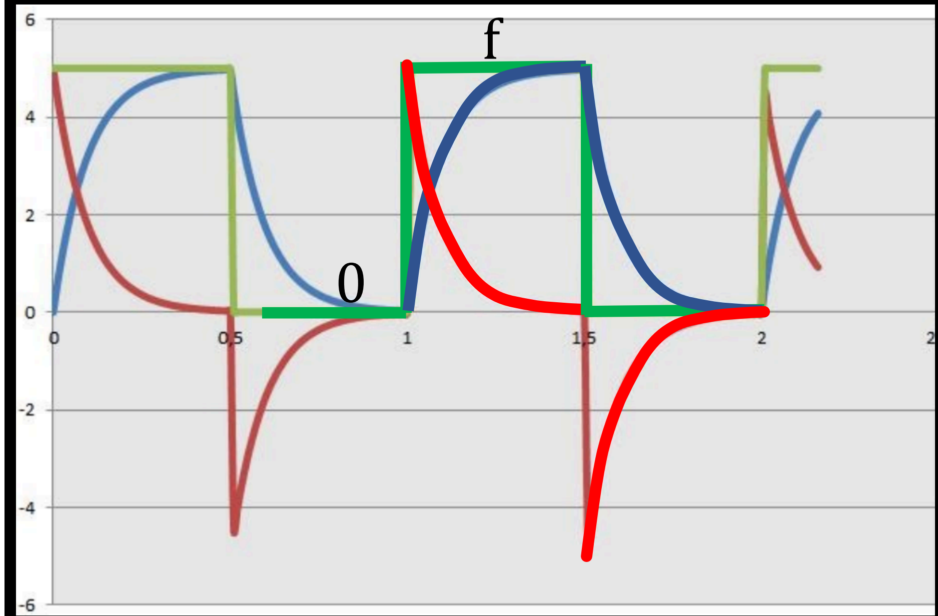
con la costante di tempo $\tau = L/2R = 2$ ms

→ $R = L/2\tau = 0{,}2\ \text{H}/(2 \times 2\ \text{ms}) = 50\ \Omega$

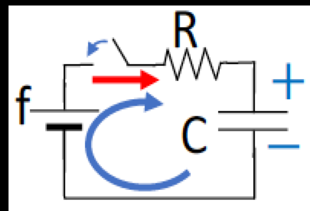
$I(0) = f/R$ → $f = 0{,}1\ \text{A} \times 50\ \Omega = 5$ V

4) In figura sono riportati gli andamenti temporali di tre tensioni comprese fra –5 V e +5V.
Stabilire quale andamento è quello del generatore, quale della resistenza e quale della capacità.





se $\Delta V_C(0) = Q(0)/C = 0$



$Q(t) = f\,C\,(1 - e^{-t/\tau})$

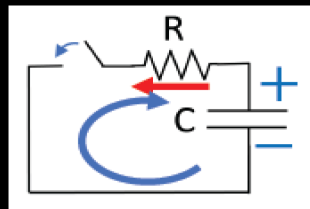
$I(t) = f/R\; e^{-t/\tau}$

$\Delta V_C(t) = f\,(1 - e^{-t/\tau})$

$\Delta V_R(t) = R\,I(t) = f\,e^{-t/\tau}$

$\Delta V_C(t)+\Delta V_R(t) = f$

se $\Delta V_C(\infty) = Q(\infty)/C = 0$



$Q(t) = Q_0\,e^{-t/\tau}$

$I(t) = \Delta V_C(0)/R\; e^{-t/\tau}$

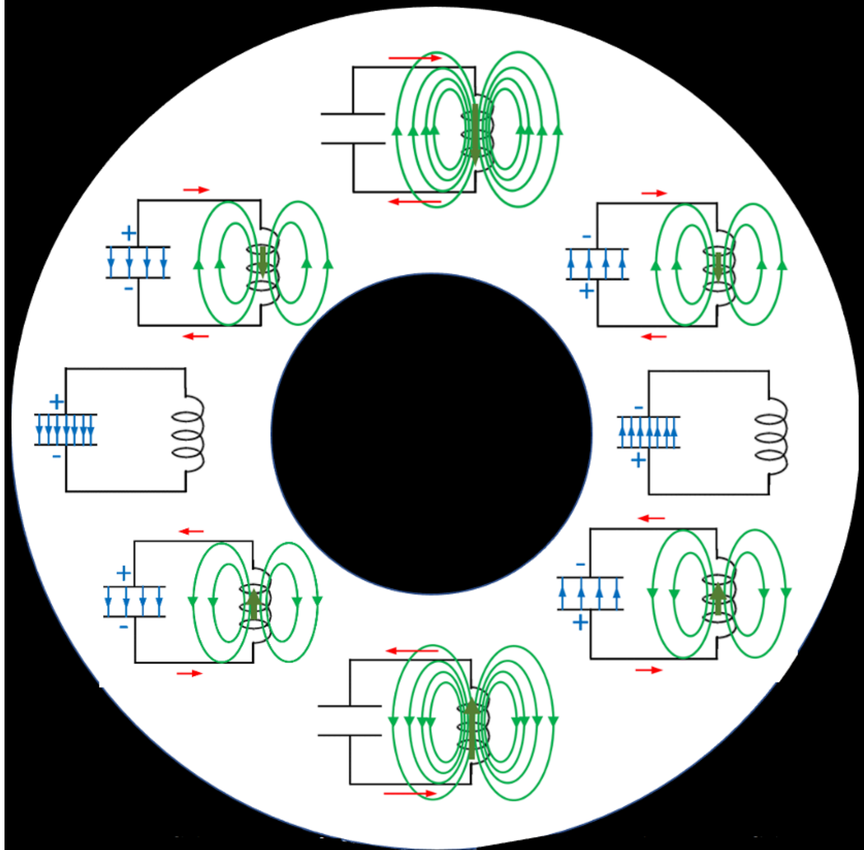
$\Delta V_C(t) = \Delta V_C(0)\,e^{-t/\tau}$

$\Delta V_R(t) = -R\,I(t) = -\Delta V_C(0)\,e^{-t/\tau}$

$\Delta V_C(t)+\Delta V_R(t) = 0$

5) Prima della chiusura dell'interruttore la capacità da 400 nF ha un'energia di 1 μJ. Dopo quanto tempo dalla chiusura dell'interruttore l'induttanza da 0,1 H ha per la prima volta la stessa energia?

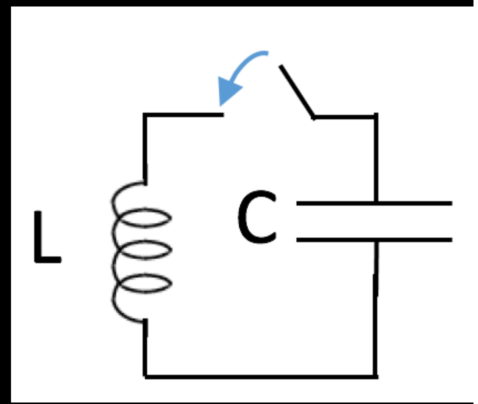
>>>> 0,314 ms



$$t = \frac{1}{4}\,T = \frac{1}{4}\,\frac{2\pi}{\omega} = \frac{\pi}{2}\sqrt{LC}$$

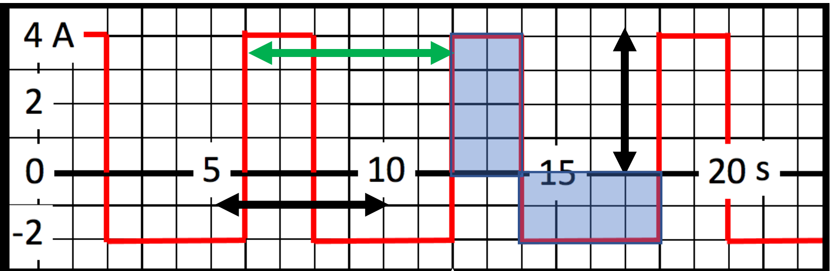
$$= \frac{\pi}{2}\sqrt{0{,}1\ \mathrm{H}\ 400 \cdot 10^{-9}\ \mathrm{F}}$$

$$= \frac{\pi}{2}\sqrt{4 \cdot 10^{-8}}\ \mathrm{s} = \pi\ 10^{-4}\mathrm{s}$$



$$\omega = \frac{1}{\sqrt{LC}}$$

6) In figura è riportato l'andamento della corrente I(t). Determinarne il periodo, la pulsazione, stabilire se si tratta di una corrente alternata e calcolarne il valore efficace



$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I(t)^2 dt}$$

>>>>> 6 s; 1,05 rad/s; 2,83 A

$$T = 6\ s \qquad \omega = \frac{2\pi}{T} = \frac{\pi}{3} = 1{,}047\ rad/s$$

$$I_{medio} = \frac{1}{T}\int_0^T I(t)dt = \frac{\int_0^{2s} I(t)dt + \int_{2s}^{6s} I(t)dt}{T} = \frac{4\ A\ 2\ s - 2\ A\ 4\ s}{6\ s} = 0$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I(t)^2 dt} = \sqrt{\frac{1}{6\ s}\left[\int_0^{2s}(4\ A)^2 dt + \int_{2s}^{6s}(-2\ A)^2 dt\right]} = \sqrt{\frac{16\ A^2 \times 2\ s + 4\ A^2 \times 4\ s}{6\ s}}$$

$$= \sqrt{\frac{(32+16)\ A^2\ s}{6\ s}} = \sqrt{8\ A^2} = 2\sqrt{2}\ A$$

Complementi di fisica generale

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VENERDÌ 14 MAGGIO ORE 8:30-10:00

dalle 8:30 alle 8:40: OPIS 8URYRLQU

sviluppi in serie di potenze
sviluppi in armoniche di Fourier

cenni onde elettromagnetiche -> ottica