VALORI EFFICACI DI GRANDEZZE ALTERNATE ARMONICHE

Una grandezza A(t) si definisce **periodica** di periodo T se, per ogni valore di t, si ha A(t+T) = A(t).

Una grandezza A(t) periodica si definisce **alternata** se

$$\frac{1}{T}\int_t^{t+T} A(t)\,dt = 0.$$

Nel caso di grandezze alternate **armoniche** si ha, ad esempio,

$A(t) = A_0 \sin(\omega t+\varphi)$

ECG: una grandezza quasi periodica

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Una resistenza R dissipa, istante per istante, la potenza $RI(t)^2$.

Spesso, p.es. effetti termici o misure con alcuni tipi di strumenti, è utile considerare la potenza mediata su un opportuno intervallo temporale. Nel caso di correnti periodiche di periodo T:

$$P_{med} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{T}\int_0^T RI^2(t)\,dt = R\left[\frac{1}{T}\int_0^T I^2(t)\,dt\right] = RI_{eff}^2$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I^2(t)\,dt}$$

I_{eff} rappresenta la corrente continua che produce gli stessi effetti dissipativi medi della I(t).

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$$\boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T I^2(t)\,dt}}$$

Qualora sia $I(t) = I_0 \sin(\omega t)$ si ha:

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T [I_0 \sin(\omega t)]^2\,dt} = I_0\sqrt{\frac{1}{\omega T}\int_0^{\omega T} \sin^2(x)\,dx}$$

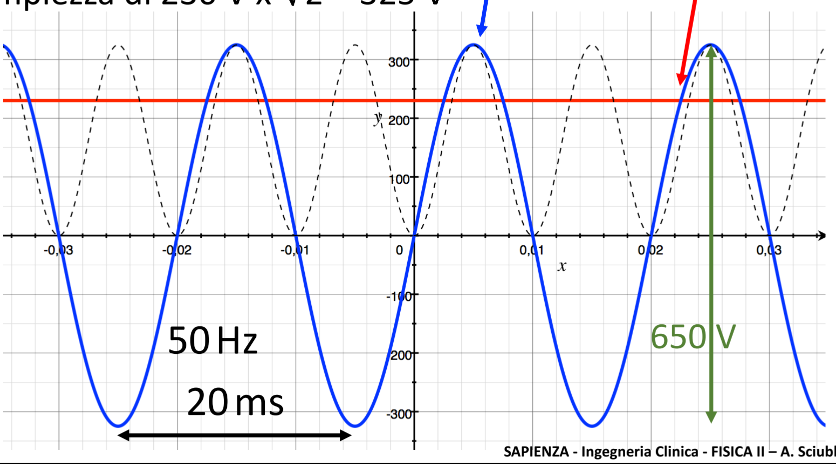
$$= I_0\sqrt{\frac{1}{2\pi}\left(\frac{x}{2} - \frac{\sin(2x)}{4}\right)\Bigg|_0^{2\pi}} = \frac{I_0}{\sqrt{2}}$$

Analogamente, se $A(t) = A_0 \sin(\omega t + \varphi) \rightarrow A_{eff} = \frac{A_0}{\sqrt{2}}$

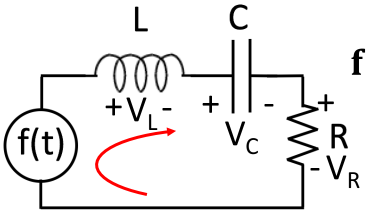
VALORI EFFICACI DI GRANDEZZE ALTERNATE ARMONICHE

$$f(t) = f_0 \sin(\omega t + \varphi) \rightarrow f_{eff} = \frac{f_0}{\sqrt{2}}$$

Per esempio la linea di alimentazione a 230 V efficaci corrisponde a un'ampiezza di 230 V x √2 = 325 V



CIRCUITI IN CORRENTE ALTERNATA (ARMONICA)



$$\mathbf{f(t) = f_0 \sin(\omega t)}$$

dopo il transitorio, a regime (soluzione particolare) si ha

$$\mathbf{I(t) = I_0 \sin(\omega t + \varphi)}$$

$$f_0 \sin(\omega t) - L\frac{dI}{dt} - \frac{1}{C}\int_0^t I dt - RI = 0$$

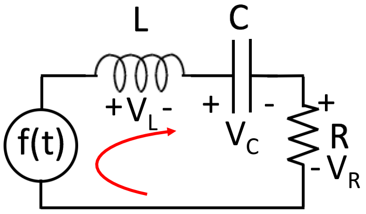
$$f(t) = V_L(t) + V_C(t) + V_R(t)$$

$$\mathbf{V_L(t)} = L\frac{dI}{dt} = I_0\, \omega L \cos(\omega t + \varphi) = \mathbf{I_0\, \omega L \sin\left(\omega t + \varphi + \frac{\pi}{2}\right)}$$

$$\mathbf{V_C(t)} = \frac{1}{C}\int_0^t I dt = -I_0 \frac{1}{\omega C}\cos(\omega t + \varphi) = \mathbf{I_0 \frac{1}{\omega C} \sin\left(\omega t + \varphi - \frac{\pi}{2}\right)}$$

$$\mathbf{V_R(t)} = RI = I_0 R \sin(\omega t + \varphi) = \mathbf{I_0 R \sin(\omega t + \varphi + 0)}$$

CIRCUITI IN CORRENTE ALTERNATA (ARMONICA)

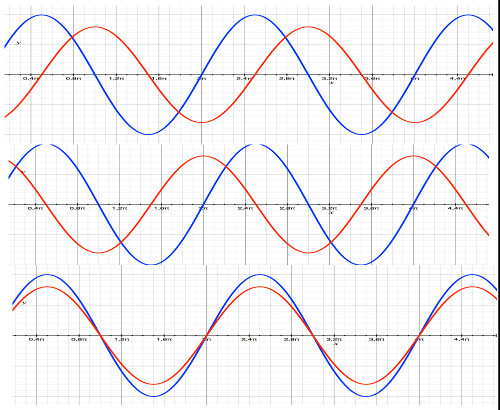


$$I = I_0 \sin(\omega t + \varphi)$$

$$V_L(t) = I_0\, \boldsymbol{\omega L} \sin\left(\omega t + \varphi + \frac{\pi}{2}\right)$$

$$V_C(t) = I_0 \frac{\mathbf{1}}{\boldsymbol{\omega C}} \sin\left(\omega t + \varphi - \frac{\pi}{2}\right)$$

$$V_R(t) = I_0\ \mathbf{R}\ \sin(\omega t + \varphi + 0)$$

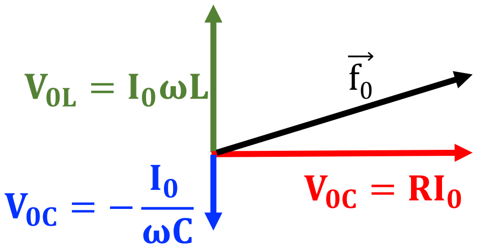


CIRCUITI IN CORRENTE ALTERNATA

$$\mathbf{V_L(t)} = I_0\,\boldsymbol{\omega}\mathbf{L}\sin\left(\omega t + \varphi + \frac{\pi}{2}\right)$$

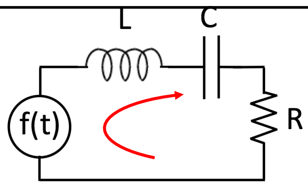
$$\mathbf{V_C(t)} = I_0\,\frac{\mathbf{1}}{\boldsymbol{\omega}\mathbf{C}}\sin\left(\omega t + \varphi - \frac{\pi}{2}\right)$$

$$\mathbf{V_R(t)} = I_0\ \mathbf{R}\ \ \sin(\omega t + \varphi + \mathbf{0})$$



$$f(t) = V_L(t) + V_C(t) + V_R(t)$$

RLC



$$f_0\sin(\omega t) - L\frac{dI}{dt} - \frac{1}{C}\int_0^t I dt - RI = 0$$

$$\omega f_0\cos(\omega t) - L\frac{d^2I}{dt^2} - \frac{I}{C} - R\frac{dI}{dt} = 0$$

$$\mathbf{I} = I_0\sin(\omega t + \varphi)\quad \dot{\mathbf{I}} = \omega I_0\cos(\omega t + \varphi)\quad \ddot{\mathbf{I}} = -\omega^2 I_0\sin(\omega t + \varphi)$$

$$\omega f_0\cos(\omega t) + L\,I_0\omega^2\,\mathbf{sin(\boldsymbol{\omega} t + \boldsymbol{\varphi})} + \\ -\frac{I_0}{C}\mathbf{sin(\boldsymbol{\omega} t + \boldsymbol{\varphi})} - R\,\omega\,I_0\cos(\omega t + \varphi) = 0$$

$$\omega f_0\cos(\omega t) + \left[I_0\,\omega^2 L - \tfrac{I_0}{C}\right][\sin(\omega t)\cos\varphi + \cos(\omega t)\sin\varphi] + \\ -I_0\omega R[\cos(\omega t)\cos\varphi - \sin(\omega t)\sin\varphi] = 0$$

$$\mathrm{COS}(\omega t)\left\{\omega f_0 + \left[I_0\,\omega^2 L - \tfrac{I_0}{C}\right]\sin\varphi - I_0\,\omega R\cos\varphi\right\} + \\ \mathrm{SIN}(\omega t)\left\{\left[I_0\,\omega^2 L - \tfrac{I_0}{C}\right]\cos\varphi + I_0\,\omega R\sin\varphi\right\} = 0$$

RLC

$$\mathrm{COS}(\omega t)\left\{\omega f_0 + \left[I_0\,\omega^2 L - \frac{I_0}{C}\right]\sin\varphi - I_0\,\omega R\cos\varphi\right\} +$$
$$\mathrm{SIN}(\omega t)\left\{\left[I_0\,\omega^2 L - \frac{I_0}{C}\right]\cos\varphi + I_0\,\omega R\sin\varphi\right\} = 0$$

$$1)\ \omega f_0 + \left[I_0\,\omega^2 L - \frac{I_0}{C}\right]\sin\varphi - I_0\,\omega R\cos\varphi = 0 \quad \times \sin\varphi$$
$$2)\ \left[I_0\,\omega^2 L - \frac{I_0}{C}\right]\cos\varphi + I_0\,\omega R\sin\varphi = 0 \quad \times \cos\varphi$$
$$+ \longrightarrow \boldsymbol{\omega f_0 \sin\varphi + \left[I_0\,\omega^2 L - \frac{I_0}{C}\right] = 0}$$

$$1)\ \omega f_0 + \left[I_0\,\omega^2 L - \frac{I_0}{C}\right]\sin\varphi - I_0\,\omega R\cos\varphi = 0 \quad \times \cos\varphi$$
$$2)\ \left[I_0\,\omega^2 L - \frac{I_0}{C}\right]\cos\varphi + I_0\,\omega R\sin\varphi = 0 \quad \times -\sin\varphi$$
$$+ \longrightarrow \boldsymbol{\omega f_0 \cos\varphi - I_0\,\omega R = 0}$$

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RLC

$$\rightarrow \omega f_0 \sin\varphi + \left[I_0\,\omega^2 L - \frac{I_0}{C}\right] = 0$$
$$\rightarrow \omega f_0 \cos\varphi - I_0\,\omega R = 0$$

$$\boxed{\begin{array}{l}\varphi_I - \varphi_V \\ \mathrm{tg}\,\varphi(\omega) = -\dfrac{\omega L - \dfrac{1}{\omega C}}{R}\end{array}}$$

$$(\omega f_0)^2\sin^2\varphi = \left[I_0\,\omega^2 L - \frac{I_0}{C}\right]^2 \quad \rightarrow f_0^{\,2}\sin^2\varphi = I_0^{\,2}\left[\omega L - \frac{1}{\omega C}\right]^2$$
$$(\omega f_0)^2\cos^2\varphi = [I_0\,\omega R]^2 \quad \rightarrow f_0^{\,2}\cos^2\varphi = I_0^{\,2}[R]^2$$

$$\mathbf{f(t) = f_0 \sin(\omega t)}$$
$$\mathbf{I(t) = I_0(\omega)\sin[\omega t + \varphi(\omega)]}$$

$$f_0^{\,2} = I_0^{\,2}\left[\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2\right]$$

$$\boxed{I_0(\omega) = \frac{f_0}{\sqrt{\left(\omega L - \dfrac{1}{\omega C}\right)^2 + R^2}}}$$

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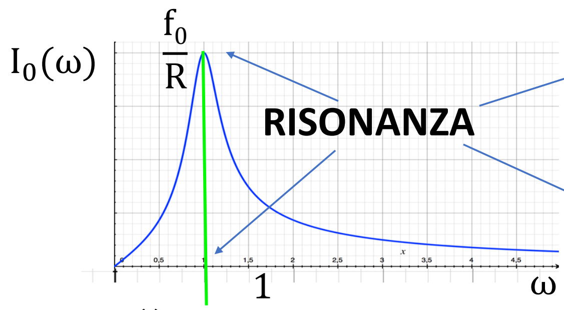
$$I_0(\omega) = \frac{f_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{\frac{f_0}{R}}{\sqrt{1 + \frac{1}{R^2}\left(\omega L - \frac{1}{\omega C}\right)^2}}$$



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = 0$$

LEZ 39

RLC

$$I_0(\omega) = \frac{f_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$|\overline{Z(\omega)}| = f_0/I_0 = f_{eff}/I_{eff} = \sqrt{R^2 + \left(\omega L - \underbrace{\frac{1}{\omega C}}\right)^2}$$

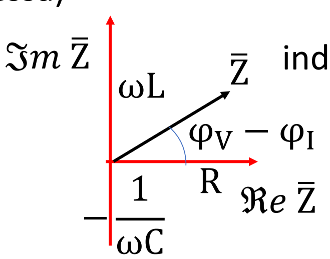
impedenza (complessa)

resistenza (reale)

reattanza (immaginaria)

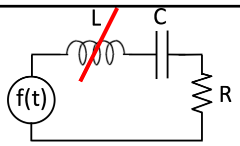
induttiva

capacitiva

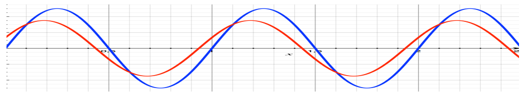


LEZ 39

RC



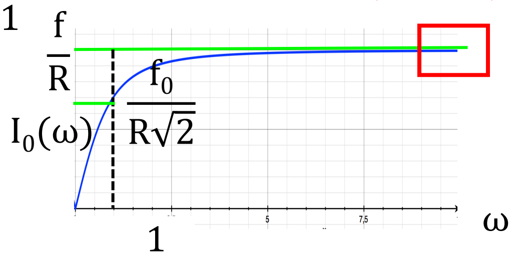
$$f_0 \sin(\omega t) - \cancel{L}\frac{dI}{dt} - \frac{1}{C}\int_0^t I dt - RI = 0$$



$f_0\sin(\omega t)$ $\quad I_0 \sin(\omega t + \varphi)$

$$I_0 = \frac{\frac{f_0}{R}}{\sqrt{\frac{1}{R^2}\left(\omega \cancel{L} - \frac{1}{\omega C}\right)^2 + 1}}$$

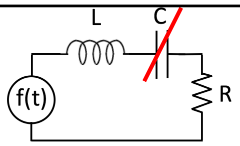
$$I_0 = \frac{\frac{f_0}{R}}{\sqrt{\left(\frac{1}{\omega RC}\right)^2 + 1}}$$



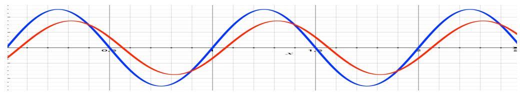
$$\omega = \frac{1}{RC}$$

$$|\overline{Z(\omega)}| = f_0/I_0 = f_{eff}/I_{eff} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

RL



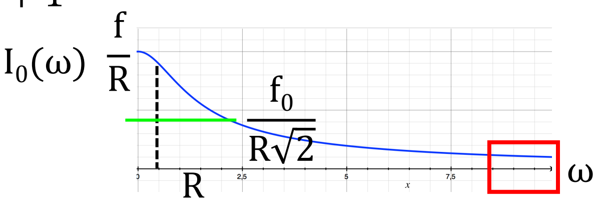
$$f_0 \sin(\omega t) - L\frac{dI}{dt} - \cancel{\frac{1}{C}}\int_0^t I dt - RI = 0$$



$f_0\sin(\omega t)$ $\quad I_0 \sin(\omega t + \varphi)$

$$I_0 = \frac{\frac{f_0}{R}}{\sqrt{\frac{1}{R^2}\left(\omega L - \cancel{\frac{1}{\omega C}}\right)^2 + 1}}$$

$$I_0 = \frac{\frac{f_0}{R}}{\sqrt{\left(\frac{\omega L}{R}\right)^2 + 1}}$$



$$\omega = \frac{R}{L}$$

$$|\overline{Z(\omega)}| = f_0/I_0 = f_{eff}/I_{eff} = \sqrt{R^2 + (\omega L)^2}$$

legge di Galileo Ferraris

$\mathbf{I(t) = I_0 sin(\omega t)}$

$$P(t) = V(t)I(t) = V_0 I_0 \sin(\omega t)\sin(\omega t + \varphi)$$

$\mathbf{V(t) = V_0 sin(\omega t + \varphi)}$

$$P_{med} = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{T}\int_0^T V_0 I_0 \sin(\omega t)\sin(\omega t + \varphi)\,dt =$$

$$= \frac{V_0 I_0}{T}\int_0^T \sin(\omega t)[\sin(\omega t)\cos\varphi + \cos(\omega t)\sin\varphi]\,dt =$$

$$= \frac{V_0 I_0}{T}\int_0^T [\sin^2(\omega t)\cos\varphi + \sin(\omega t)\cos(\omega t)\sin\varphi]\,dt =$$

$$= V_0 I_0 \left[\frac{\cos\varphi}{T}\int_0^T \sin^2(\omega t)\,dt + \frac{\sin\varphi}{T}\int_0^T \cancel{\sin(\omega t)\cos(\omega t)\,dt}\right] =$$

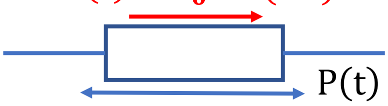
$$= V_0 I_0 \frac{\cos\varphi}{2} = \frac{V_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\varphi = \mathbf{V_{eff}\, I_{eff}\, \cos\varphi}$$

$\cos(\varphi_V - \varphi_I)$ ↑ fattore di potenza

LEZ 39

legge di Galileo Ferraris

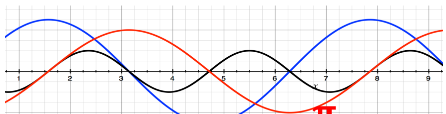
$\mathbf{I(t) = I_0 sin(\omega t)}$



$$P(t) = V(t)I(t) = V_0 I_0 \sin(\omega t)\sin(\omega t + \varphi)$$

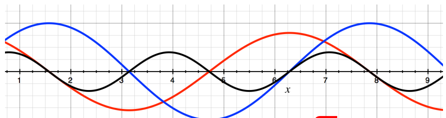
$\mathbf{V(t) = V_0 sin(\omega t + \varphi)}$

$$V_L(t) = \mathbf{I_0 \omega L} \sin\left(\omega t + \frac{\pi}{2}\right)$$



$$P_{med\_L} = V_{eff}\, I_{eff}\, \cos\left(+\frac{\pi}{2}\right) = 0$$

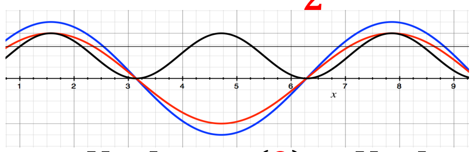
$$V_C(t) = \mathbf{I_0 \frac{1}{\omega C}} \sin\left(\omega t - \frac{\pi}{2}\right)$$



$$P_{med\_C} = V_{eff}\, I_{eff}\, \cos\left(-\frac{\pi}{2}\right) = 0$$

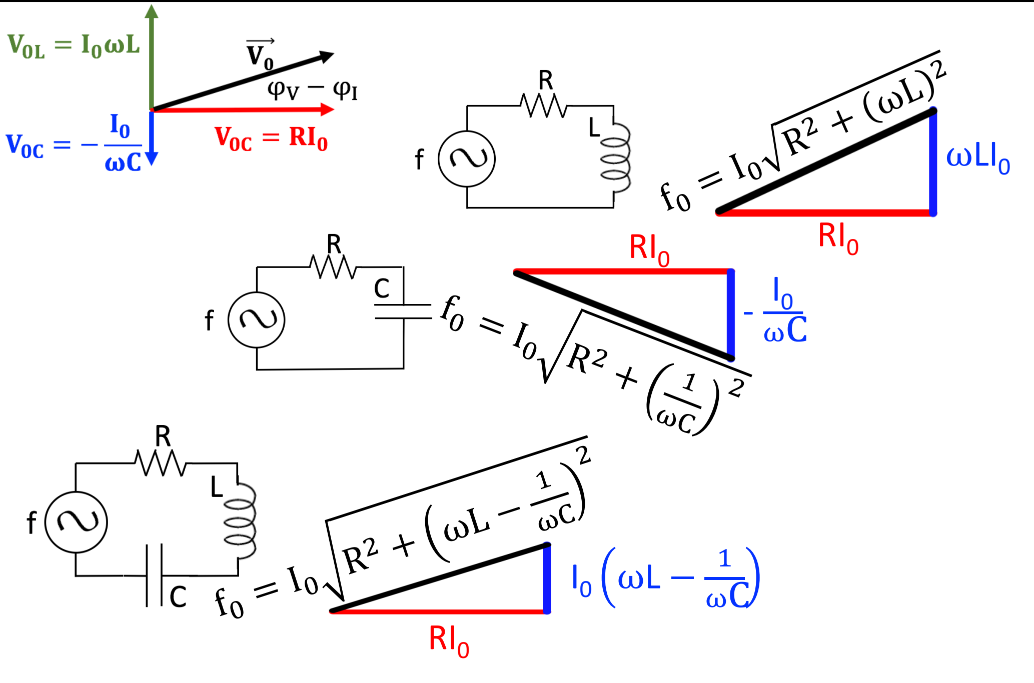
$$V_R(t) = \mathbf{I_0 R} \sin(\omega t + 0)$$

→



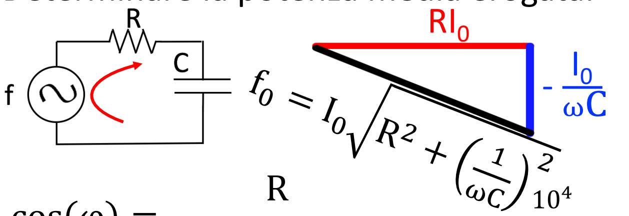
$$P_{med\_R} = V_{eff}\, I_{eff}\, \cos(0) = V_{eff}\, I_{eff}$$

LEZ 39



LEZ 39

Un generatore sinusoidale (f_{eff} = 230 V, ν = 50 Hz) alimenta, in serie, una resistenza R = 10 kΩ e una capacità C = 1 μF.
Determinare la potenza media erogata.



$$\mathbf{P_{med} = V_{eff}\, I_{eff}\, \cos\varphi}$$

$$f_{eff} = I_{eff}\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\cos(\varphi) = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{10^4}{\sqrt{10^8 + \left(\frac{1}{100\pi 10^{-6}}\right)^2}} = 0{,}95$$

$$I_{eff} = \frac{f_{eff}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{230}{\sqrt{10^8 + \left(\frac{1}{100\pi 10^{-6}}\right)^2}}\,\mathrm{A} = 22\ \mathrm{mA}$$

$$P_{med} = f_{eff}\,\frac{f_{eff}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}\,\frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = 230\ \mathrm{V} \times 22\ \mathrm{mA} \times 0{,}95 = 4{,}8\ \mathrm{W}$$

LEZ 39