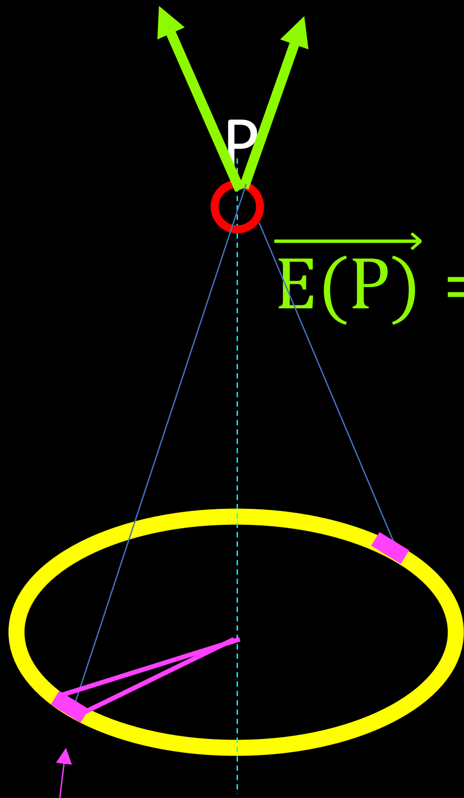
1) ELETTROSTATICA NEL VUOTO

esempio: anello carico (su asse)



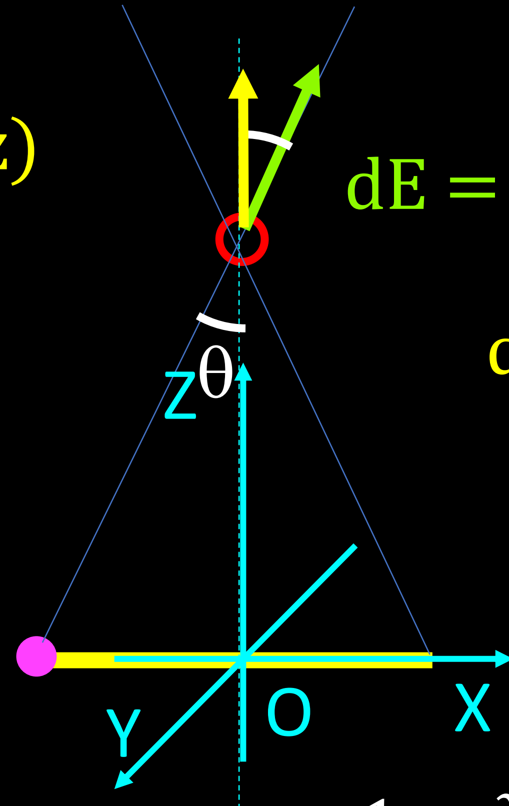
$$\overrightarrow{E(P)} = \hat{k}\, E_z(z)$$

$$\overrightarrow{E(P)} = ???$$

SIMMETRIA

$$dq = \lambda R d\varphi$$

$$\lambda = \frac{q}{2\pi R}$$



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\varphi}{z^2+R^2}$$

$$dE_z = dE\cos\theta$$

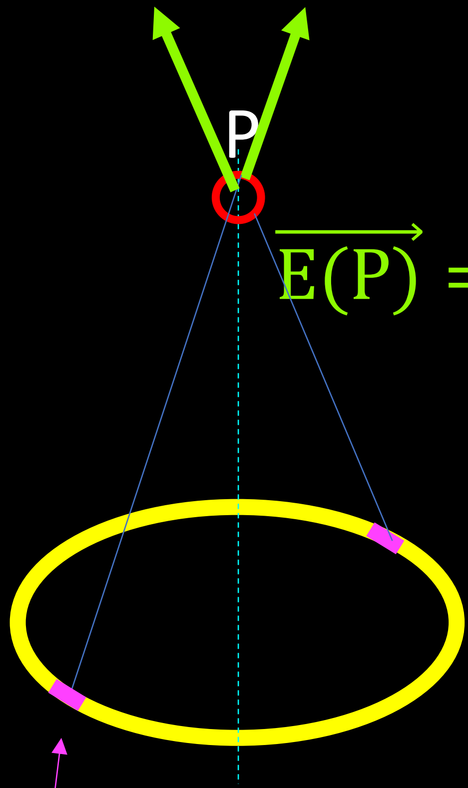
$$dE_z = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R d\varphi}{z^2+R^2}\frac{z}{\sqrt{z^2+R^2}}$$

$$E_z = \int_0^{2\pi}\frac{1}{4\pi\varepsilon_0}\frac{\lambda R z d\varphi}{(z^2+R^2)^{3/2}} = \frac{1}{2\varepsilon_0}\frac{\lambda R z}{(z^2+R^2)^{3/2}}$$

>>>

e... se fosse z >> R?

esempio: anello carico (su asse)



$\overrightarrow{E(P)} = ???$

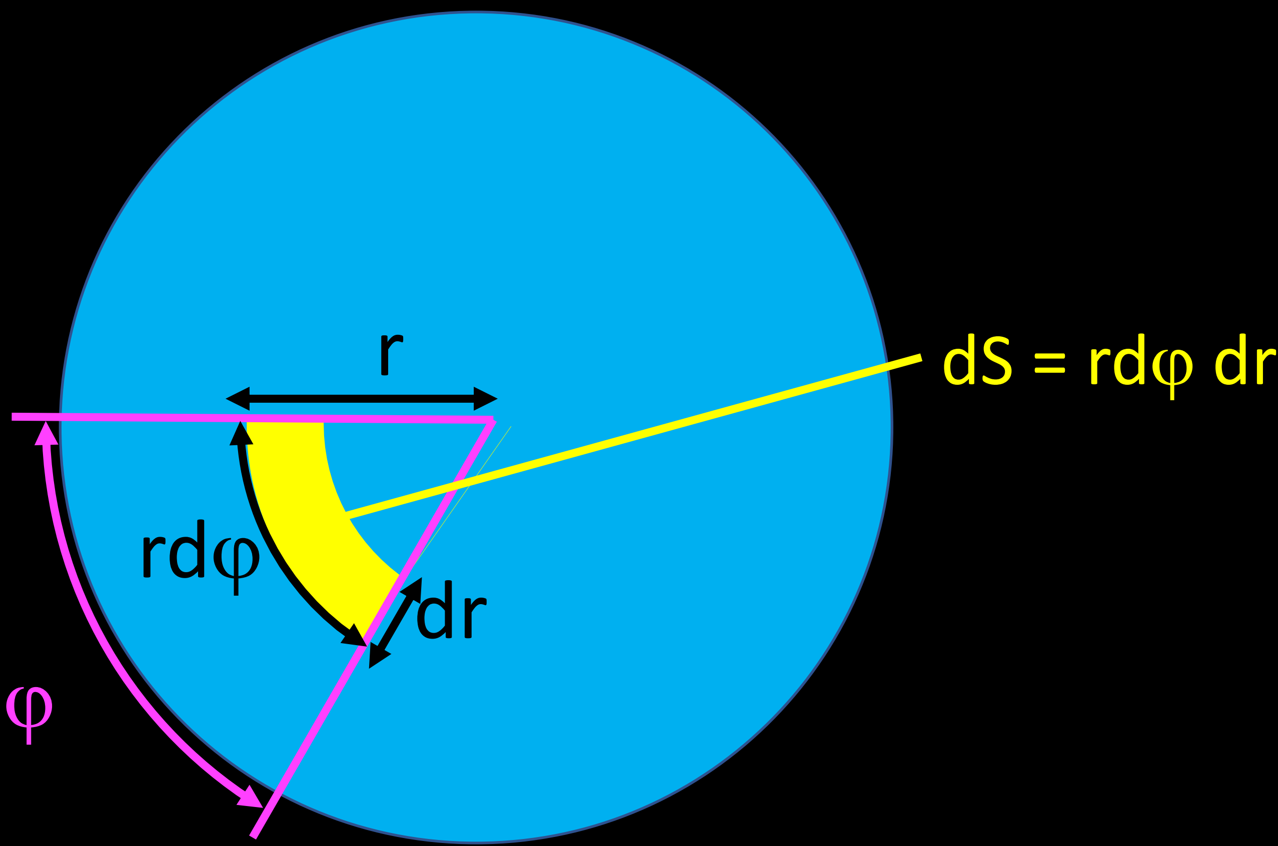
$\lambda = \dfrac{q}{2\pi R}$

$dq = \lambda R d\varphi$

se fosse z >> R (grande distanza)

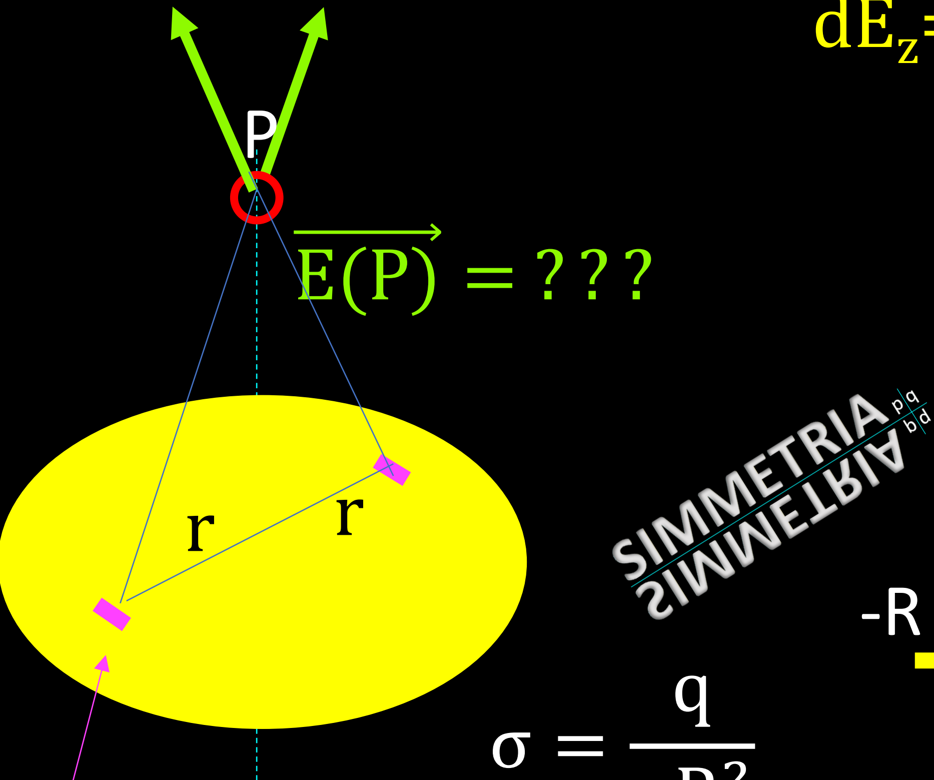
$$>>> \quad E_z = \frac{1}{2\varepsilon_0}\frac{\lambda R z}{(z^2+R^2)^{3/2}} \rightarrow \frac{1}{2\varepsilon_0}\frac{\lambda R z}{(z^2)^{3/2}} = \frac{1}{2\varepsilon_0}\frac{q/2\pi}{z^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}$$

porzione puntiforme di superficie con simmetria circolare



1) ELETTROSTATICA NEL VUOTO

esempio: disco carico (su asse)

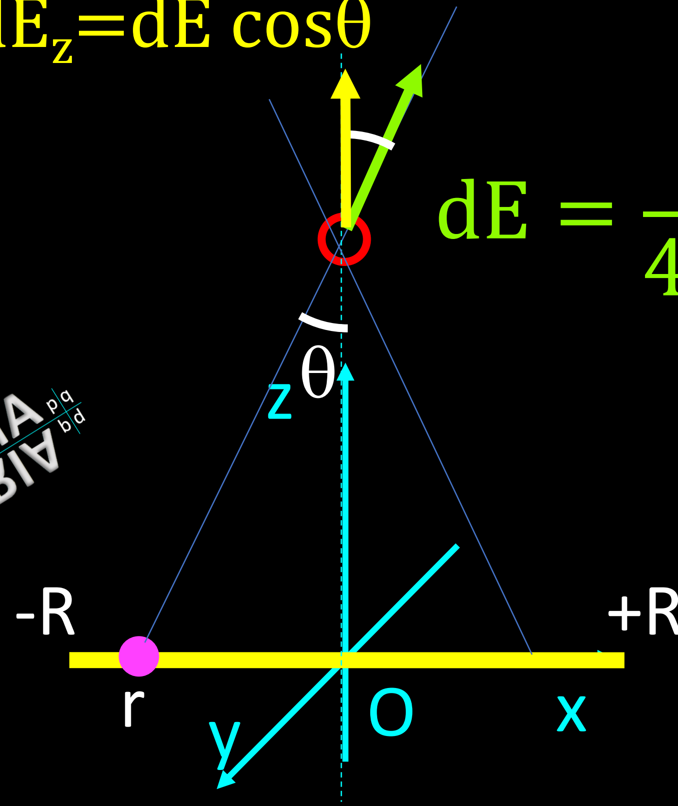


$$\overrightarrow{E(P)} = ???$$

$$dq = \sigma r d\varphi dr$$

$$\sigma = \frac{q}{\pi R^2}$$

$$dE_z = dE \cos\theta$$



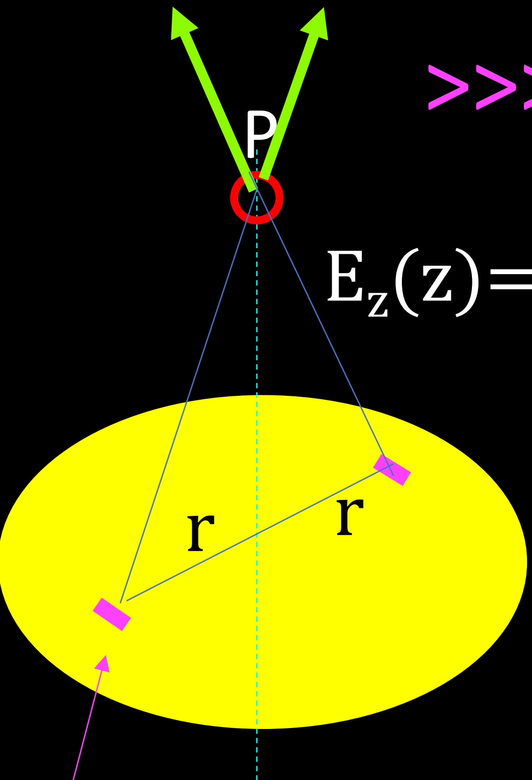
$$dE = \frac{1}{4\pi\varepsilon_0} \frac{\sigma r d\varphi dr}{z^2 + r^2}$$

$$dE_z = \frac{1}{4\pi\varepsilon_0} \frac{\sigma r d\varphi dr}{z^2 + r^2} \frac{z}{\sqrt{z^2 + r^2}}$$

>>>

1) ELETTROSTATICA NEL VUOTO

esempio: disco carico (su asse)



$dq = \sigma\, r d\varphi\, dr$

>>> $$dE_z = \frac{1}{4\pi\varepsilon_0}\,\frac{\sigma\, r d\varphi\, dr}{z^2+r^2}\,\frac{z}{\sqrt{z^2+r^2}}$$

$$E_z(z) = \int_0^{2\pi}\int_0^R \frac{1}{4\pi\varepsilon_0}\,\frac{\sigma\, r d\varphi\, dr\, z}{(z^2+r^2)^{3/2}} = \frac{\sigma z}{2\varepsilon_0}\int_o^R \frac{r\, dr}{(z^2+r^2)^{3/2}} =$$

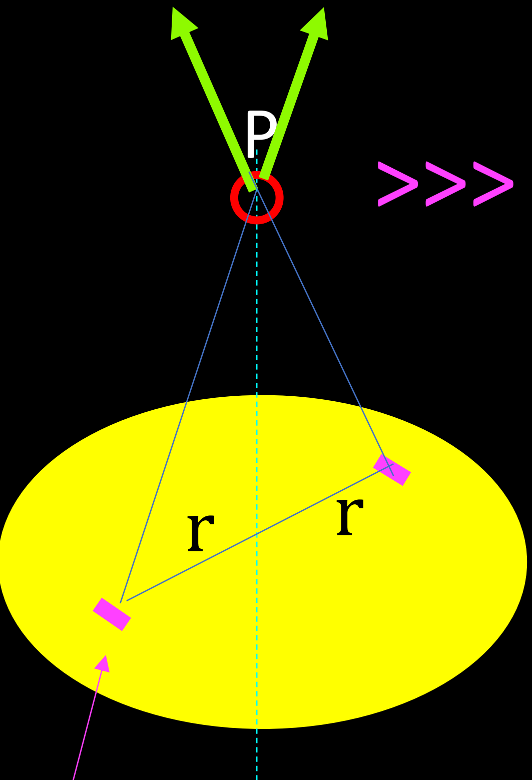
$$= \frac{\sigma z}{2\varepsilon_0}\left(-\frac{1}{\sqrt{z^2+R^2}} + \frac{1}{\sqrt{z^2+0^2}}\right) =$$

$$= \frac{\sigma z}{2\varepsilon_0}\left(\frac{1}{|z|} - \frac{1}{\sqrt{z^2+R^2}}\right) =$$

$$= \frac{\sigma}{2\varepsilon_0}\frac{z}{|z|}\left(1 - \frac{1}{\sqrt{1+(R/z)^2}}\right)$$ >>>

1) ELETTROSTATICA NEL VUOTO

esempio: disco carico (su asse)



$dq = \sigma\, r d\varphi\, dr$

>>> $E_z(z) = \frac{\sigma}{2\varepsilon_0}\frac{z}{|z|}\left(1 - \frac{1}{\sqrt{1+(R/z)^2}}\right)$

se fosse z << R

vicinissimo al disco

$\rightarrow E_z(z) = \frac{\sigma}{2\varepsilon_0}\frac{z}{|z|}$

se fosse R >> z

piano infinito

$\rightarrow E_z(z) = \frac{\sigma}{2\varepsilon_0}\frac{z}{|z|}$

e... se fosse z >> R?

lontanissimo dal disco

1) ELETTROSTATICA NEL VUOTO

$$1-\frac{1}{\sqrt{1+(R/z)^2}} \sim \frac{1}{2}(R/z)^2$$

sviluppo in serie di Maclaurin

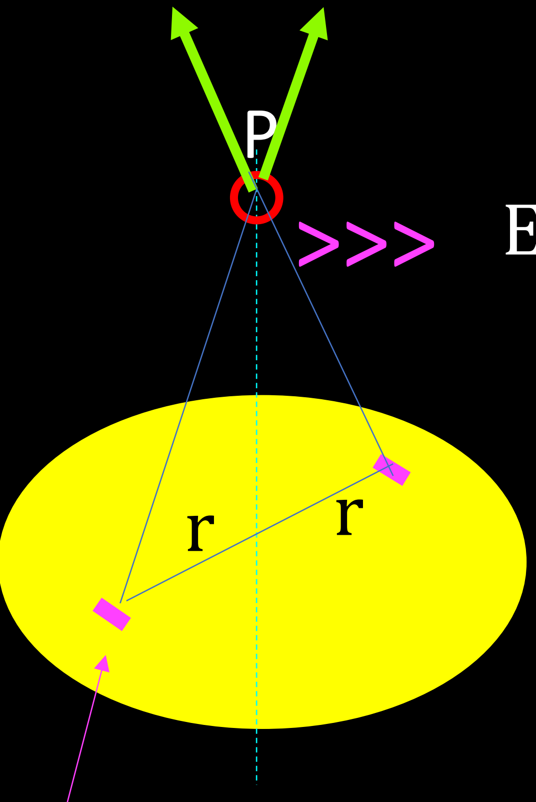
$$f(x)=f(0)+f'(0)\,x+\frac{1}{2}f''(0)\;x^2+\frac{1}{3!}f'''(0)\,x^3+\cdots$$

$$1-\frac{1}{\sqrt{1+x}}=\left(1-\frac{1}{\sqrt{1+0}}\right)+\left(0+\frac{1}{2}(1+0)^{-\frac{3}{2}}\right)x+\cdots$$

$$\sim \frac{1}{2}x$$

1) ELETTROSTATICA NEL VUOTO

esempio: disco carico (su asse)



$$dq = \sigma\, r d\varphi\, dr$$

\>>>

$$E_z(z) = \frac{\sigma}{2\varepsilon_0}\frac{z}{|z|}\left(1 - \frac{1}{\sqrt{1+(R/z)^2}}\right)$$

se fosse z >> R...

$$\sim \frac{\sigma}{2\varepsilon_0}\frac{z}{|z|}\quad \frac{1}{2}(R/z)^2$$

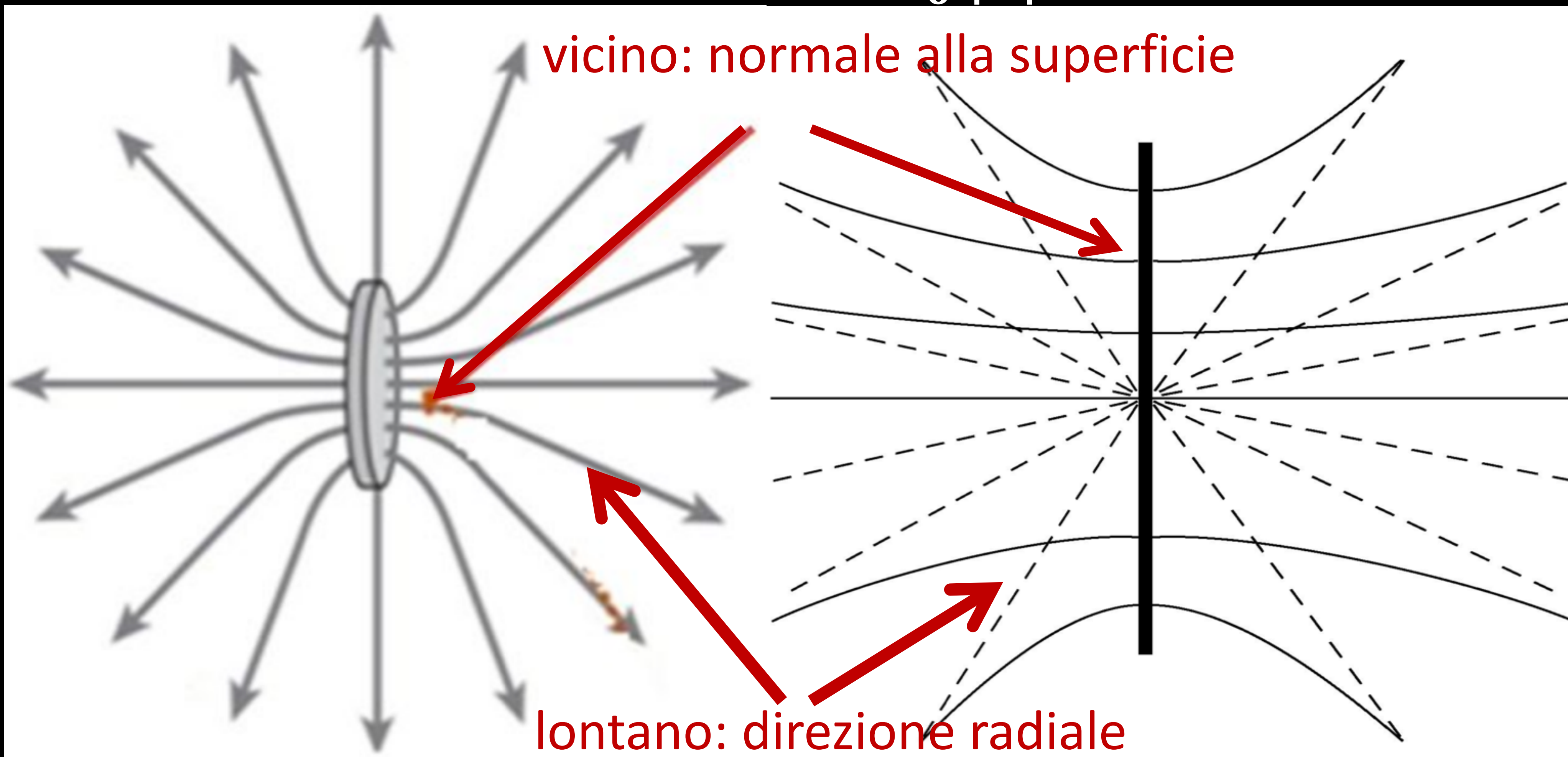
$$\text{ma } \frac{\sigma}{4\varepsilon_0}\frac{z}{|z|}\frac{R^2}{z^2}\frac{\pi}{\pi} = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}\frac{z}{|z|}$$

distante dal disco ~ carica puntiforme

1) ELETTROSTATICA NEL VUOTO

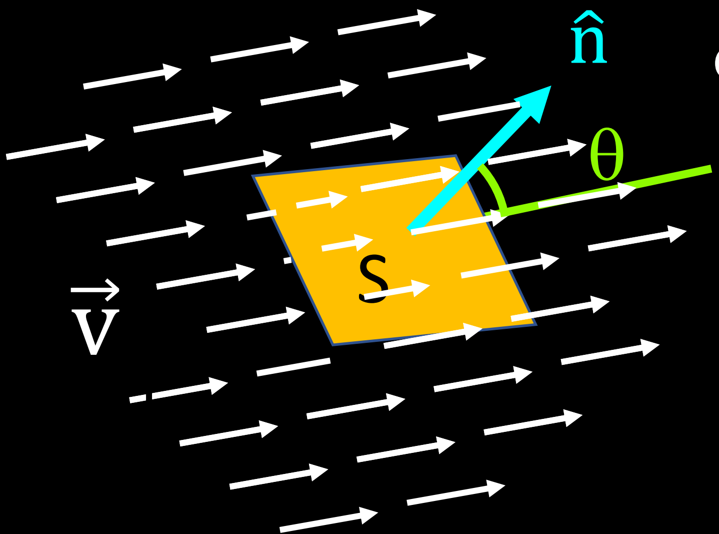
esempio: disco carico

$$\frac{\sigma}{2\varepsilon_0}\frac{z}{|z|}$$



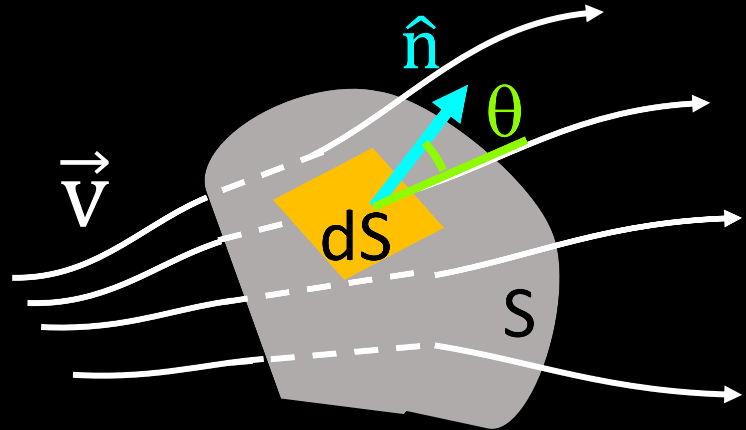
$$\frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}\frac{z}{|z|}$$

si definisce flusso del vettore $\vec{v}$ attraverso la superficie S:



$$\phi(\vec{v})_S = \vec{v}\,\hat{n}\,S = v\,S\cos\theta$$

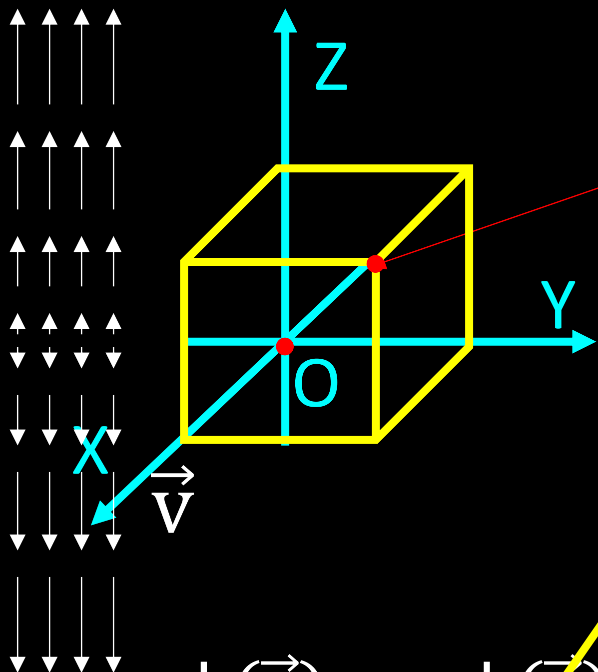
se $\vec{v}$ o $\hat{n}$ variano, la superficie S va suddivisa in elementi infinitesimi:



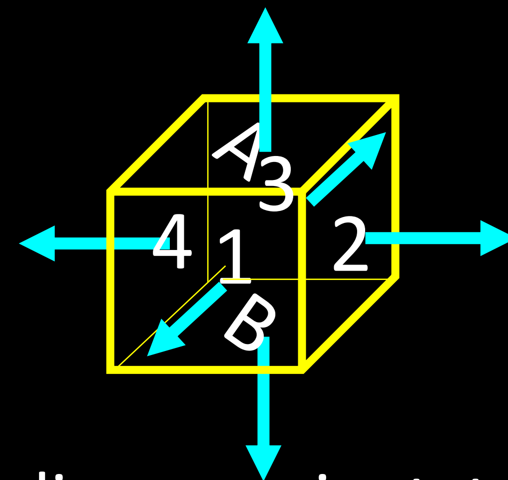
$$\phi(\vec{v})_S = \int_S \vec{v}\,\hat{n}\,dS$$

1) ELETTROSTATICA NEL VUOTO esercizio matematica

calcolare il flusso del vettore $\vec{v} = c\,\hat{k}\,z^3$ attraverso la superficie chiusa di un cubo (vertici 0, 0, 0 e L, L, L)



$$\phi(\vec{v})_S = \int_S \vec{v}\,\hat{n}\,dS$$

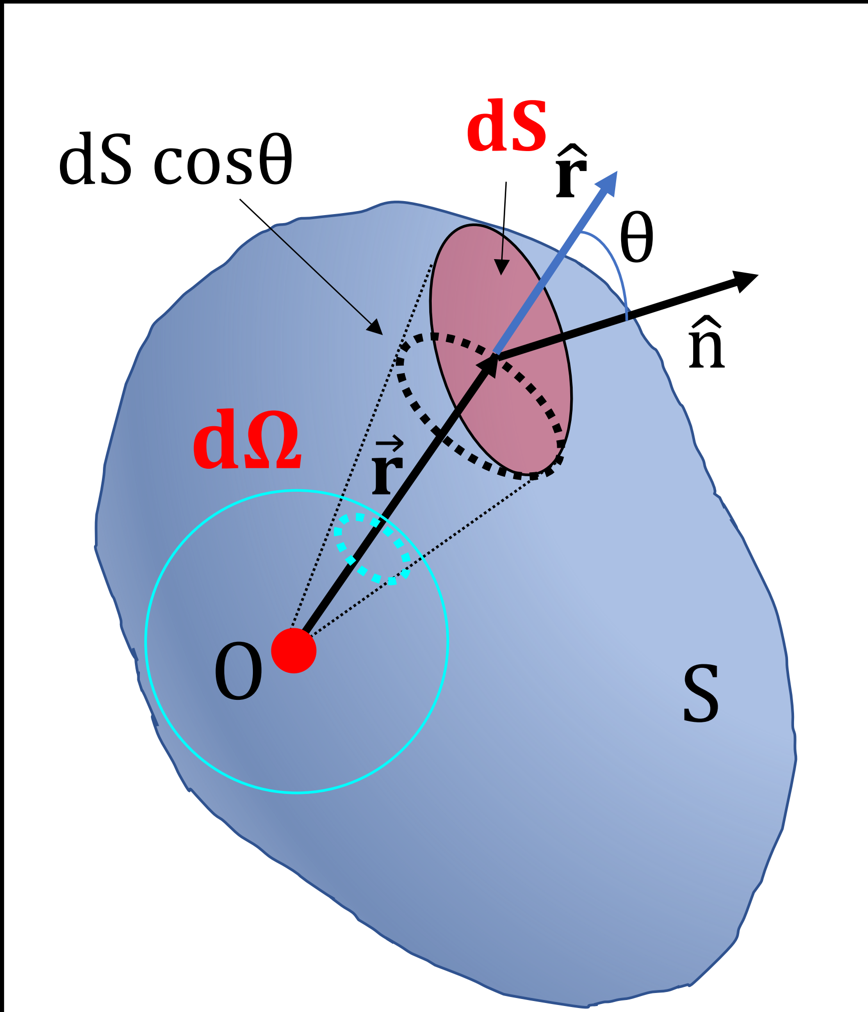


le normali sono orientate verso l'esterno delle superfici chiuse

$$\phi(\vec{v})_S = \cancel{\phi(\vec{v})_1} + \cancel{\phi(\vec{v})_2} + \cancel{\phi(\vec{v})_3} + \cancel{\phi(\vec{v})_4} + \phi(\vec{v})_A + \cancel{\phi(\vec{v})_B}$$

$\vec{v} \perp \hat{n}$ $\vec{v}\,(x, y, 0) = 0$

$$\phi(\vec{v})_S = \phi(\vec{v})_A = cL^3L^2 = cL^5$$

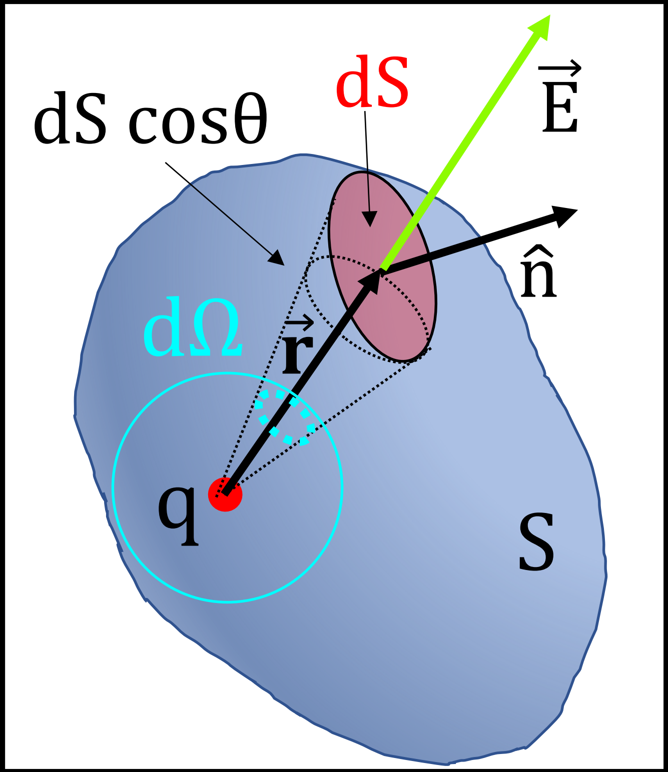


l'ANGOLO SOLIDO sotteso dalla superficie dS è pari al rapporto fra l'area proiettata e il quadrato della distanza da O

$$d\Omega = \frac{\hat{n} dS\, \hat{r}}{r^2} = \frac{dS\cos\theta}{r^2}$$

$$\Delta\Omega = \int_S \frac{dS\cos\theta}{r^2}$$

l'angolo solido massimo corrisponde alla superficie di una sfera ($4\pi r^2$) divisa per il quadrato del raggio: 4π srad (steradianti)



il flusso del campo elettrico attraverso una superficie CHIUSA che <u>racchiude</u> una carica puntiforme q è:

$$\phi(\vec{E})_S = \frac{q}{\varepsilon_0}$$

\>>>

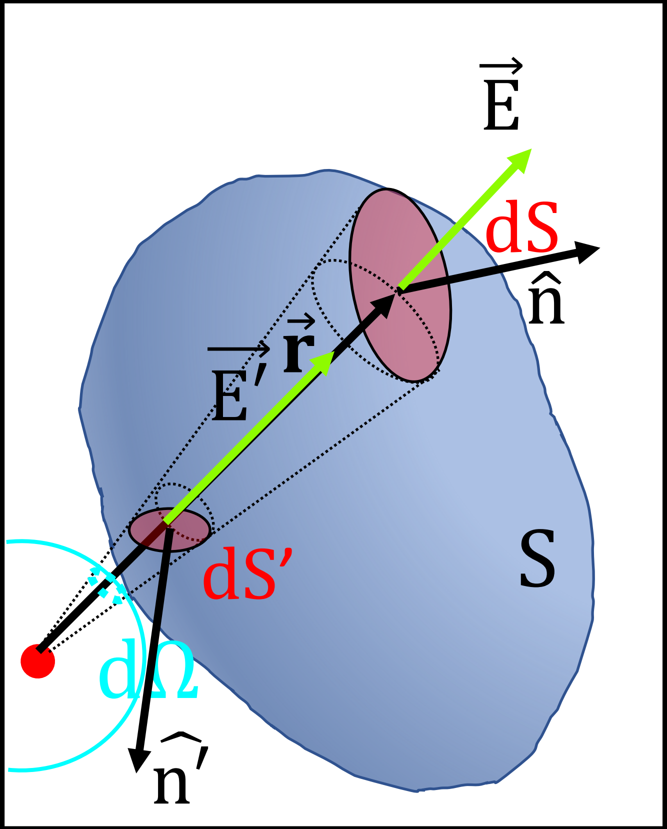
$$d\Omega = \frac{\hat{n} dS\, \hat{r}}{r^2} = \frac{dS\cos\theta}{r^2}$$

$$\Delta\Omega = \int_S \frac{dS\cos\theta}{r^2}$$

$$\phi(\vec{E})_S = \int_S \vec{E}\,\hat{n}\, ds = \int_S \frac{1}{4\pi\varepsilon_0}\frac{q\,\hat{r}}{r^2}\hat{n}\, ds =$$

$$= \frac{1}{4\pi\varepsilon_0}\, q \int_S \frac{\hat{r}}{r^2}\hat{n}\; ds = \frac{q}{4\pi\varepsilon_0}\int_S d\Omega = \frac{q}{4\pi\varepsilon_0}4\pi = \frac{q}{\varepsilon_0}$$

teorema di Gauss (II)



il flusso del campo elettrico attraverso una superficie CHIUSA dovuto a una carica puntiforme q esterna è nullo:

>>>

$$d\Omega = \frac{\hat{n} dS\, \hat{r}}{r^2} = \frac{dS\cos\theta}{r^2}$$

$$\Delta\Omega = \int_S \frac{dS\cos\theta}{r^2}$$

$$\phi(\vec{E})_S = \int_S \vec{E}\,\hat{n}\,ds = \int_S \frac{1}{4\pi\varepsilon_0}\frac{q\,\hat{r}}{r^2}\hat{n}\,ds =$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\int_S \frac{\hat{r}}{r^2}\hat{n}\;ds + \int_{S'} \frac{\widehat{r'}}{r'^2}\widehat{n'}\;ds'\right] = \frac{q}{4\pi\varepsilon_0}\left[\int_S d\Omega + \int_S -d\Omega\right] = 0$$

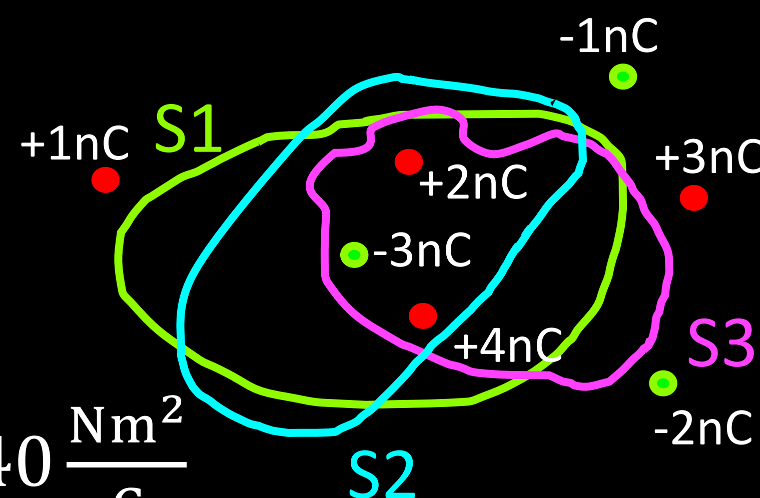
$$\cos\theta > 0 \qquad \cos\theta < 0$$

teorema di Gauss (III)

il flusso del campo elettrico attraverso una superficie CHIUSA dovuto a un insieme di cariche puntiformi è pari alla somma dei contributi (nulli) delle cariche esterne e di quelli ($\frac{q_i}{\varepsilon_0}$) dovuti alle cariche interne:

$$\phi(\vec{E})_S = \int_S \vec{E}\,\hat{n}\,ds = \int_S \sum_{i=1,N} \vec{E_i}\,\hat{n}\,ds = \sum_{int} \int_S \vec{E_i}\,\hat{n}\,ds + \sum_{est} \int_S \vec{E_i}\,\hat{n}\,ds$$

$$= \sum_{int} \frac{q_i}{\varepsilon_0} + 0 = \frac{q_{int}}{\varepsilon_0}$$



$$\phi(\vec{E})_{S1} = \phi(\vec{E})_{S2} = \phi(\vec{E})_{S3} = \frac{3\text{ nC}}{\varepsilon_0} = 340\,\frac{\text{Nm}^2}{\text{C}}$$