$$V(\vec{r}) = \sum_{i=1,N} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i}$$

$$V(\vec{r}) = \int_\tau \frac{1}{4\pi\varepsilon_0} \frac{\varrho(\overrightarrow{r'})d\tau}{|\vec{r} - \overrightarrow{r'}|}$$

$$\boxed{V(\vec{r}) = V(\overrightarrow{r_0}) + \int_{\overrightarrow{r_0}}^{\vec{r}} -\vec{E}\,\overrightarrow{dl}}$$

$V(\infty) = 0$

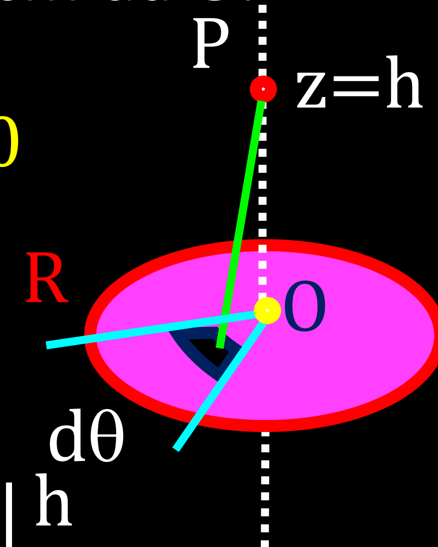
$$V(\vec{r}) = \int_{\overrightarrow{r_\infty}}^{\vec{r}} -\vec{E}\,\overrightarrow{dl}$$

$$V(\vec{r}) = \int_S \frac{1}{4\pi\varepsilon_0} \frac{\sigma(\overrightarrow{r'})dS}{|\vec{r} - \overrightarrow{r'}|}$$

$$V(\vec{r}) = \int_l \frac{1}{4\pi\varepsilon_0} \frac{\lambda(\overrightarrow{r'})dl}{|\vec{r} - \overrightarrow{r'}|}$$

1) ELETTROSTATICA NEL VUOTO esercizio **potenziale**

Una carica è uniformemente distribuita (σ = 1 nC/m^2) su un disco di raggio R = 30 cm. Calcolare la d.d.p. fra il punto O al centro del disco e un punto P sull'asse del disco distante h = 40 cm da O.



$$E_z(z) = \frac{\sigma}{2\varepsilon_0}\frac{z}{|z|}\left(1 - \frac{1}{\sqrt{1+(R/z)^2}}\right) \qquad V(O) = 0$$

$$V(P) = V(O) + \int_0^h -\frac{\sigma}{2\varepsilon_0}\frac{z}{|z|}\left(1 - \frac{1}{\sqrt{1+(R/z)^2}}\right)dz$$

$$= -\frac{\sigma}{2\varepsilon_0}\int_0^h \left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)dz = -\frac{\sigma}{2\varepsilon_0}\left(z - \sqrt{z^2+R^2}\right)\Big|_0^h$$

$$\Delta V = V(O) - V(P) = \frac{\sigma}{2\varepsilon_0}\left[h - \left(\sqrt{h^2+R^2} - R\right)\right] = +11{,}3\ V$$

$$V(\vec{r}) = V(\vec{r_0}) + \int_{\vec{r_0}}^{\vec{r}} -\vec{E}\,\vec{dl}$$

1) ELETTROSTATICA NEL VUOTO esercizio **potenziale**

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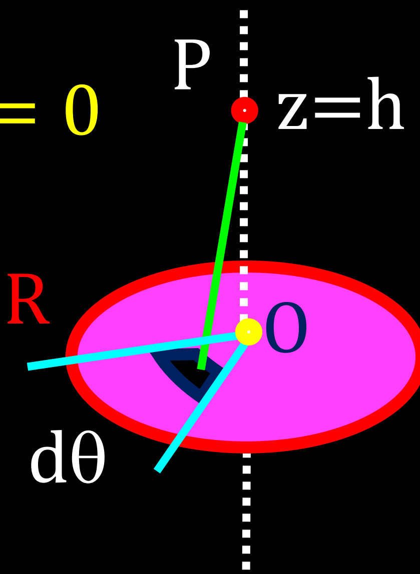
$$V(\infty) = 0$$



$$V(z) = \int_0^{2\pi}\int_0^R \frac{1}{4\pi\varepsilon_0}\frac{\sigma\, r d\theta\, dr}{\sqrt{r^2+z^2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{r\,dr}{\sqrt{r^2+z^2}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - |z|\right)$$

$$\Delta V = V(O) - V(P) = \frac{\sigma}{2\varepsilon_0}\left[R - \left(\sqrt{R^2+h^2} - h\right)\right] = +11{,}3\ \mathrm{V}$$

$$V(\vec{r}) = \int_S \frac{1}{4\pi\varepsilon_0}\frac{\sigma\left(\vec{r'}\right)dS}{|\vec{r}-\vec{r'}|}$$

1) ELETTROSTATICA NEL VUOTO esempio $\vec{E}, V, \rho$

campo elettrostatico e potenziale
di una sfera uniformemente carica

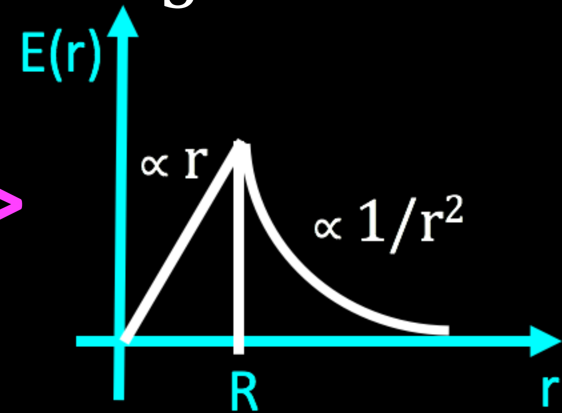
$$\rho = \frac{q}{\frac{4\pi}{3} R^3}$$

se $r \le R$ $\quad E(r) = \frac{\rho\, r}{3\varepsilon_0} = \frac{q}{4\pi\varepsilon_0}\frac{r}{R^3}$

\>>>

se $r \ge R$ $\quad E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

\>>>



al solito, ponendo $V(\infty) = 0$ partiamo da $r \ge R$ $\rightarrow E(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}$

$$\rightarrow V(r) = V(\infty) + \int_\infty^r -E(r')\, dr' = \int_\infty^r -\frac{q}{4\pi\varepsilon_0 r'^2}\, dr' = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$$

$$V(R) = \frac{q}{4\pi\varepsilon_0}\frac{1}{R}$$ \>>>

poi, fra 0 e R

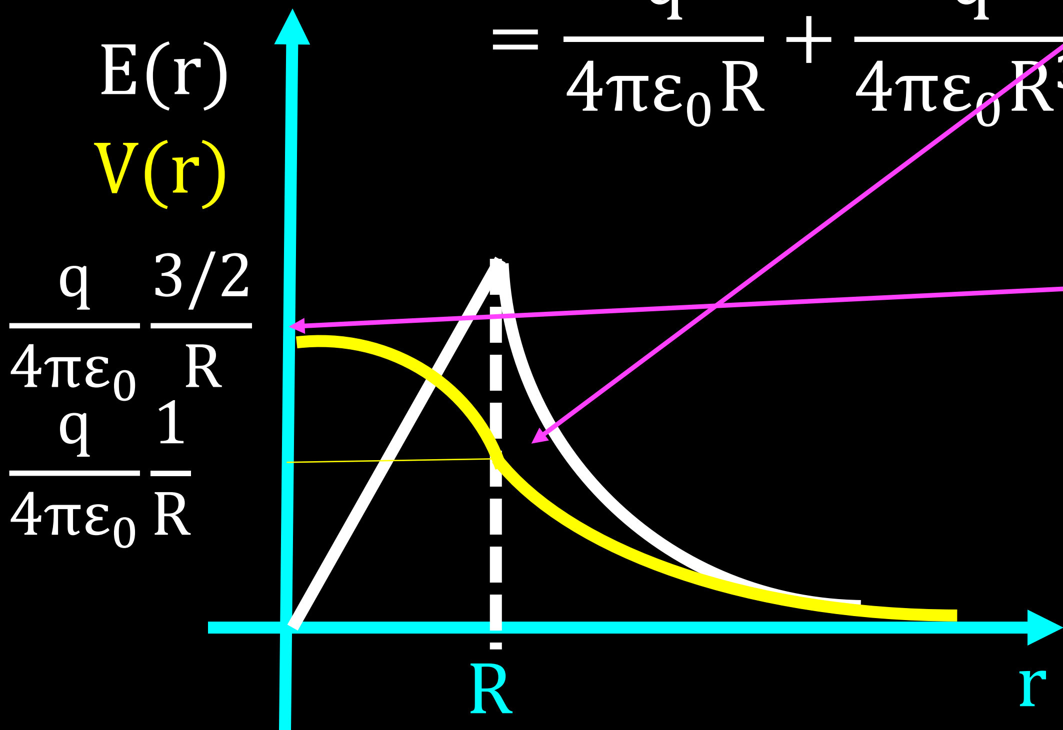
$$\rightarrow E(r) = \frac{q}{4\pi\varepsilon_0}\frac{r}{R^3}$$

$$\rightarrow V(r) = V(R) + \frac{q}{4\pi\varepsilon_0 R^3}\int_R^r -r\,dr$$

$$= \frac{q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R^3}\frac{R^2 - r^2}{2} = \frac{q}{4\pi\varepsilon_0 R}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right)$$

\>>> $V(R) = \dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{R}$

$$V(0) = \frac{q}{4\pi\varepsilon_0 R}\frac{3}{2}$$



1) ELETTROSTATICA NEL VUOTO nabla ⇔ gradiente **matematica**

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}\,\hat{\imath} + \frac{\partial}{\partial y}\,\hat{\jmath} + \frac{\partial}{\partial z}\,\hat{k}\right)$$ operatore differenziale lineare nabla

un altro impiego:

$$\boxed{\overrightarrow{\text{grad}}\,U = -\vec{F}}$$

$\{$dU differenziale esatto: $dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz\,\}$

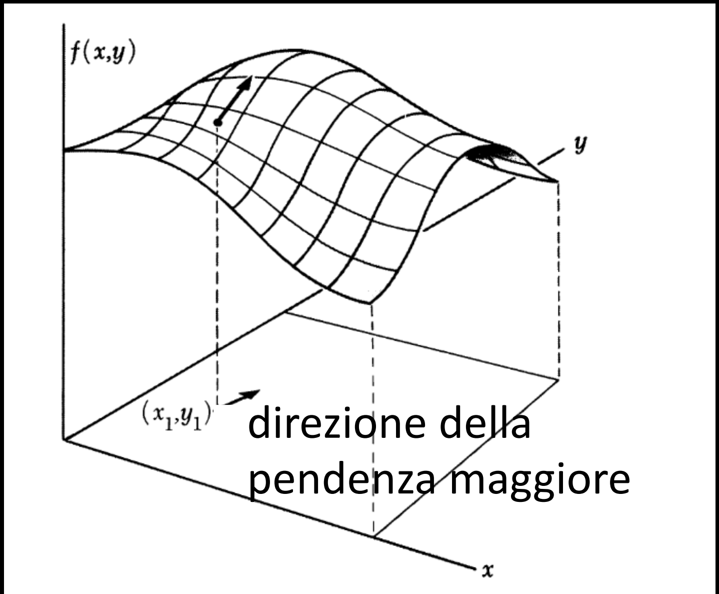
$$\overrightarrow{\textbf{grad}}[U(x,y,z)] = \frac{\partial U(x,y,z)}{\partial x}\,\hat{\imath} + \frac{\partial U(x,y,z)}{\partial y}\,\hat{\jmath} + \frac{\partial U(x,y,z)}{\partial z}\,\hat{k}$$

$$\boxed{\vec{\nabla}\,U = \;-\vec{F}}$$

$$\frac{\vec{\nabla}\,U}{q} = \frac{-\vec{F}}{q} \rightarrow \vec{\nabla}\,\frac{U}{q} = \vec{\nabla}\,V = -\vec{E}$$

$$\boxed{\vec{\nabla}\,V = \;-\vec{E}}$$

$$\overrightarrow{\textbf{grad}}[V(x,y,z)] = -\vec{E}\,(x,y,z)$$



potenziale e campo e.s.

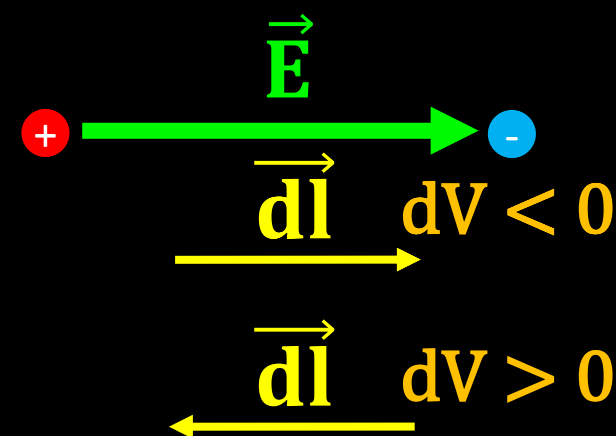
\>>>

$$V(\vec{r}) = V(\overrightarrow{r_0}) + \int_{\overrightarrow{r_0}}^{\vec{r}} -\vec{E}\,\overrightarrow{dl} \qquad \vec{E} = -\overrightarrow{\mathrm{grad}}\,V$$

$$dV(\vec{r}) = \frac{\partial V(x,y,z)}{\partial x}dx + \frac{\partial V(x,y,z)}{\partial y}dy + \frac{\partial V(x,y,z)}{\partial z}dz =$$

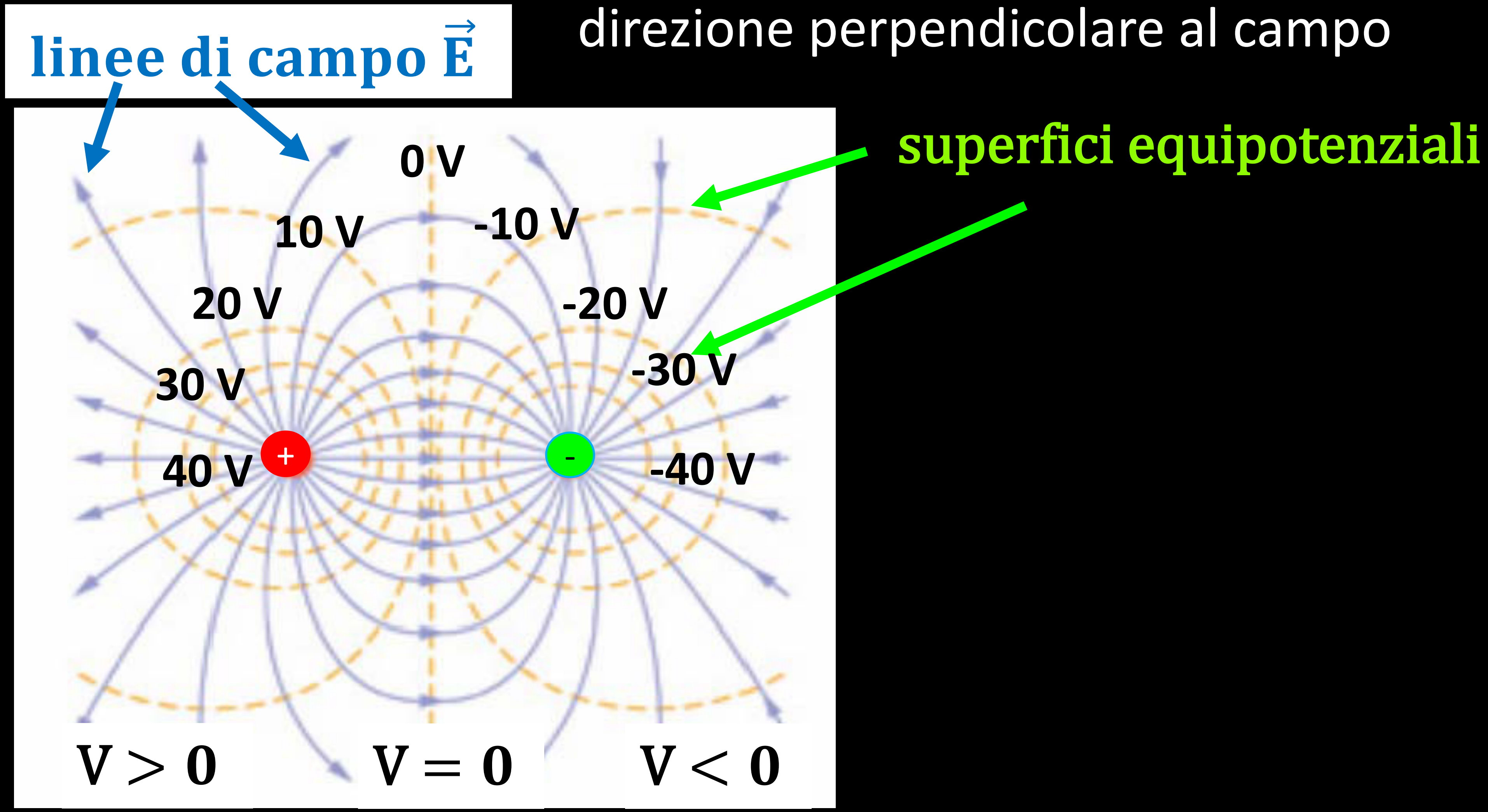
$$= \hat{\imath}\,\frac{\partial V(x,y,z)}{\partial x}\hat{\imath}dx + \hat{\jmath}\,\frac{\partial V(x,y,z)}{\partial y}jdy + \hat{k}\,\frac{\partial V(x,y,z)}{\partial z}\hat{k}dz =$$

$$= \overrightarrow{\mathrm{grad}}\,V \cdot \overrightarrow{dl} = -\vec{E}\,\overrightarrow{dl}$$



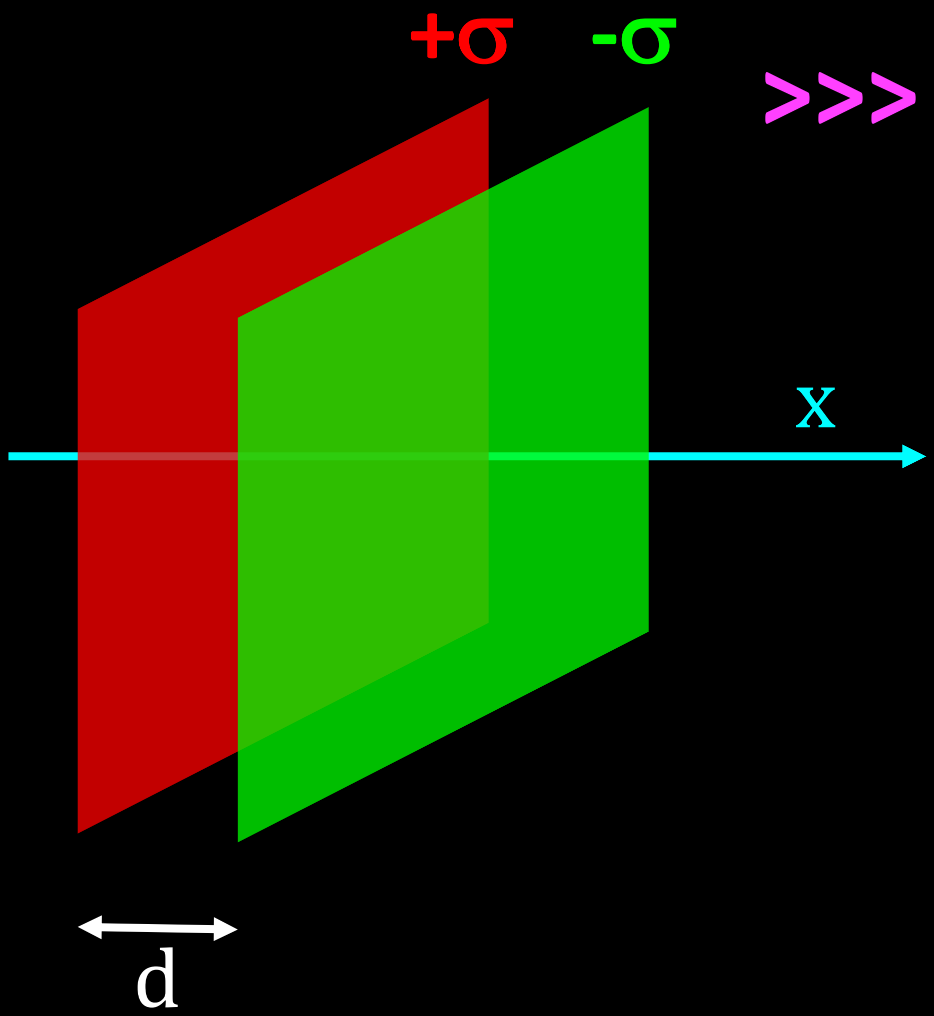
>>> $dV(\vec{r}) = -\vec{E}\,\vec{dl}$

se $\vec{E} \perp \vec{dl}$ allora $dV(\vec{r}) = 0$ cioè il potenziale non cambia muovendosi in direzione perpendicolare al campo

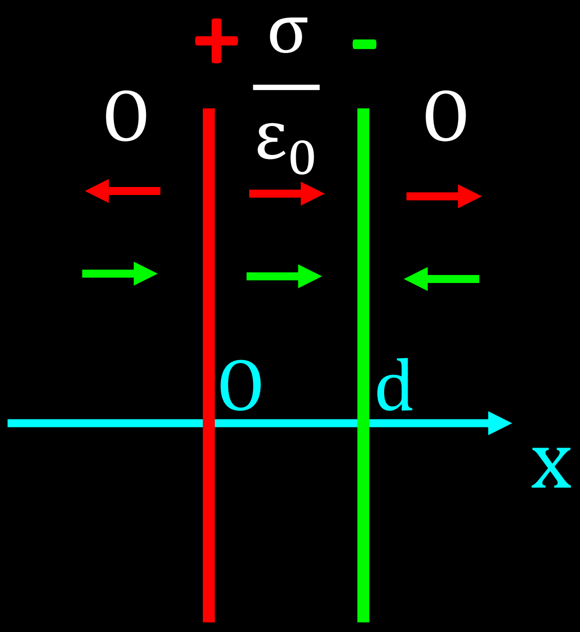


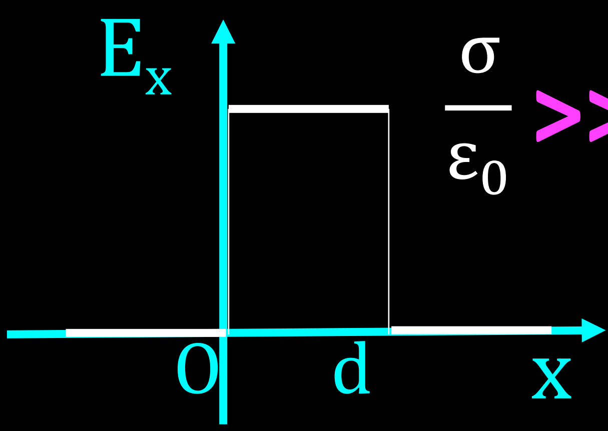
1) ELETTROSTATICA NEL VUOTO doppio strato $\vec{E}, V, \rho$

campo elettrostatico del doppio strato di carica



>>> $E_x = \dfrac{\sigma}{2\varepsilon_0} \dfrac{x}{|x|}$

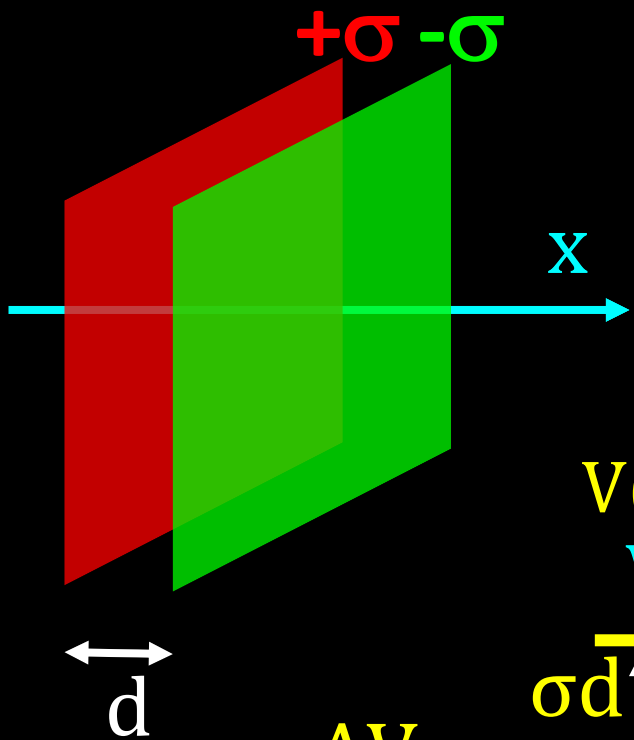


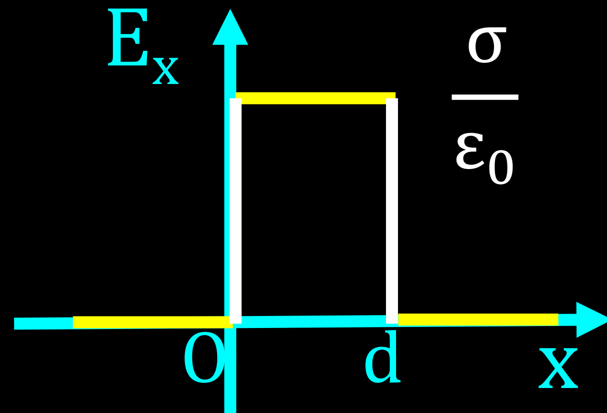


>>>

1) ELETTROSTATICA NEL VUOTO doppio strato $\vec{E}$, V, ρ

campo elettrostatico e potenziale del doppio strato di carica



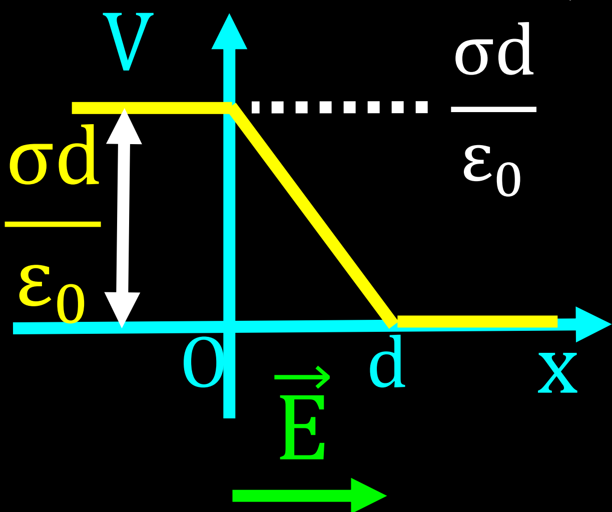


$$\overrightarrow{\text{grad}}(V) = -\vec{E}$$

$$\frac{dV}{dx} = -E_x$$

$$dV = -E_x\,dx$$

$$V(\infty) = 0 \qquad V(d) = 0$$



$$\Delta V = \frac{\sigma d}{\varepsilon_0}$$

$$V(-\infty) = \frac{\sigma d}{\varepsilon_0}$$

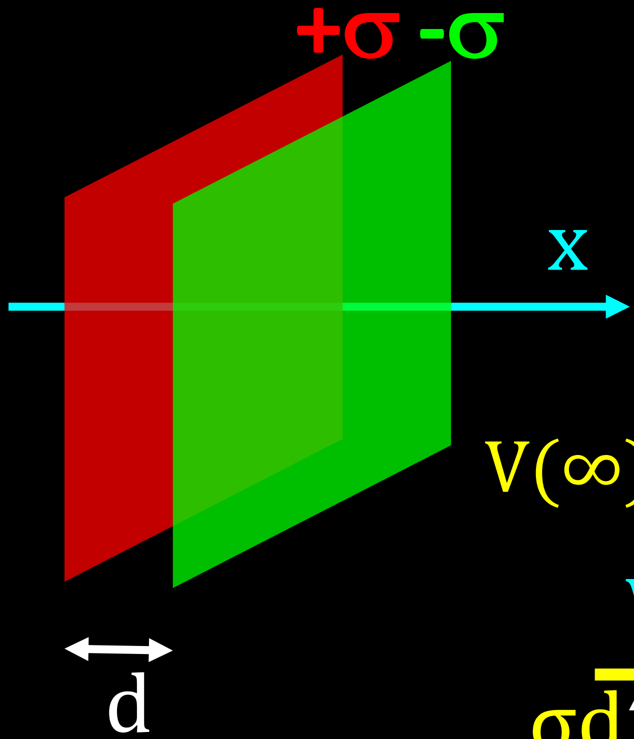
$$V(d) - V(x) = -\frac{\sigma}{\varepsilon_0}(d-x)$$

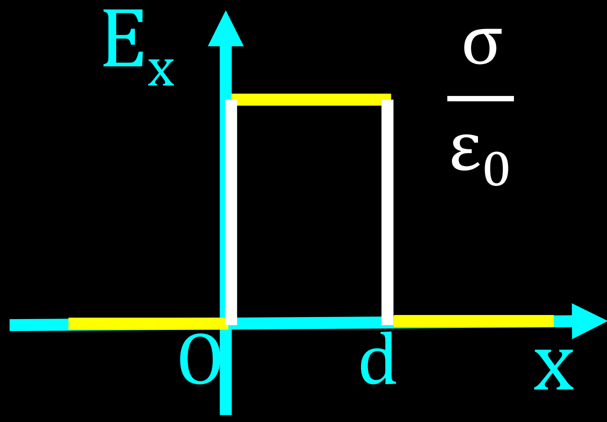
$$V(x) = V(d) + \frac{\sigma}{\varepsilon_0}(d-x)$$

$$V(x) = \frac{\sigma}{\varepsilon_0}(d-x)$$

1) ELETTROSTATICA NEL VUOTO doppio strato $\vec{E}$, V, ρ

potenziale del doppio strato di carica



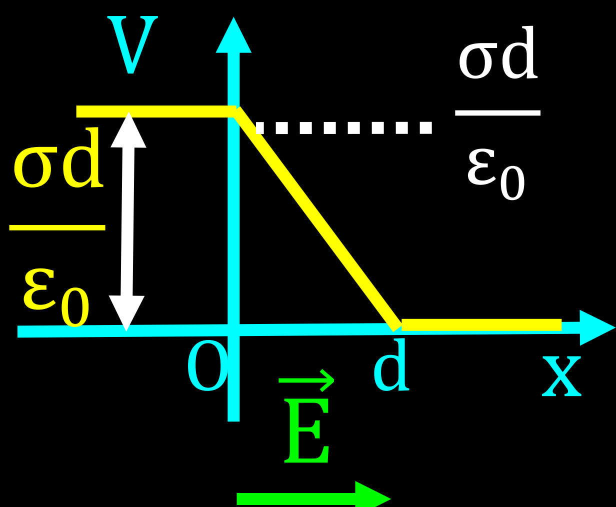


$$\overrightarrow{\text{grad}}(V) = -\vec{E}$$

$$\frac{dV}{dx} = - E_x$$

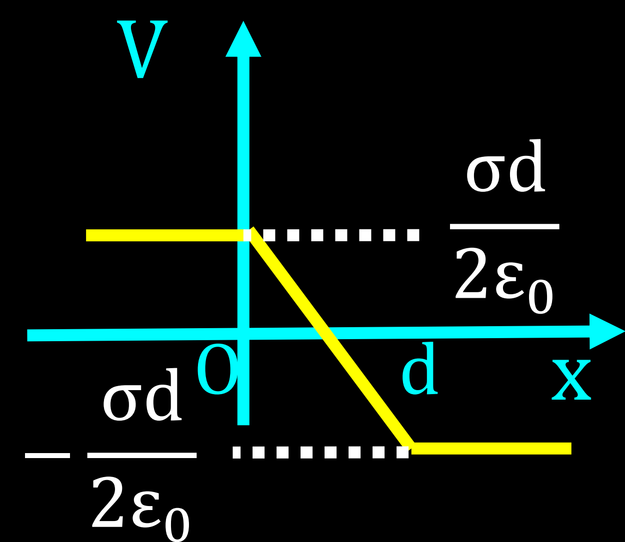
$$V(\infty) = 0$$

$$V(d/2) = 0$$

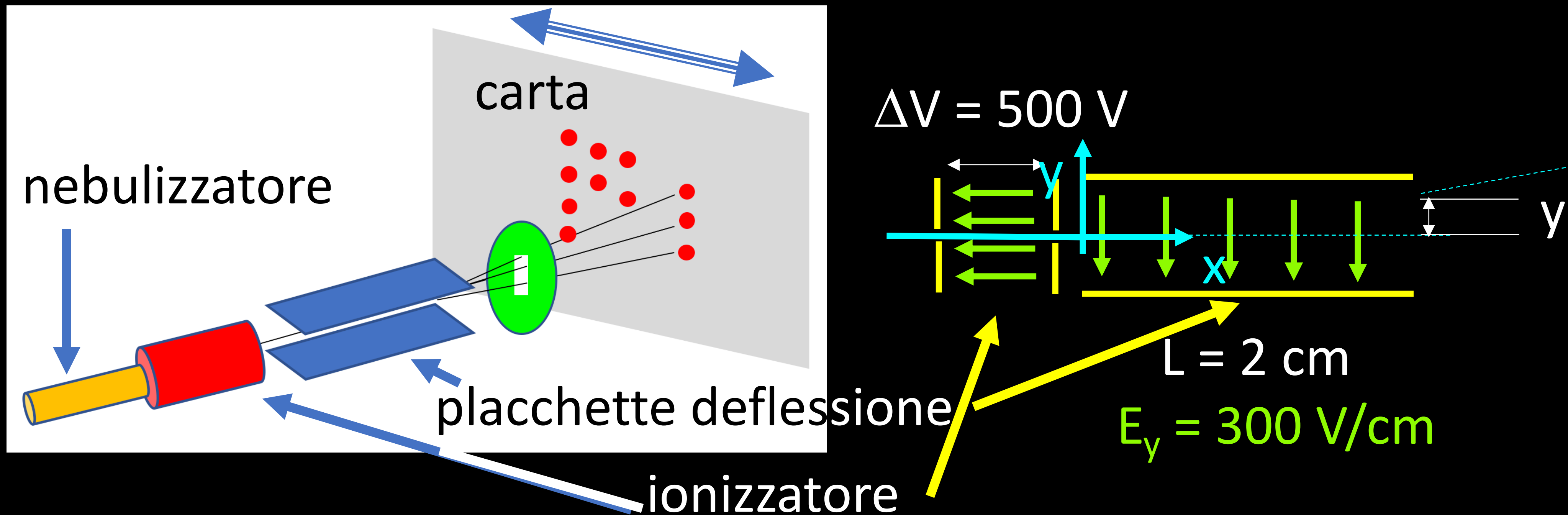


$$\Delta V = \frac{\sigma d}{\varepsilon_0}$$

$$V(-\infty) = \frac{\sigma d}{\varepsilon_0}$$



1) ELETTROSTATICA NEL VUOTO ink jet printer **applicazione**

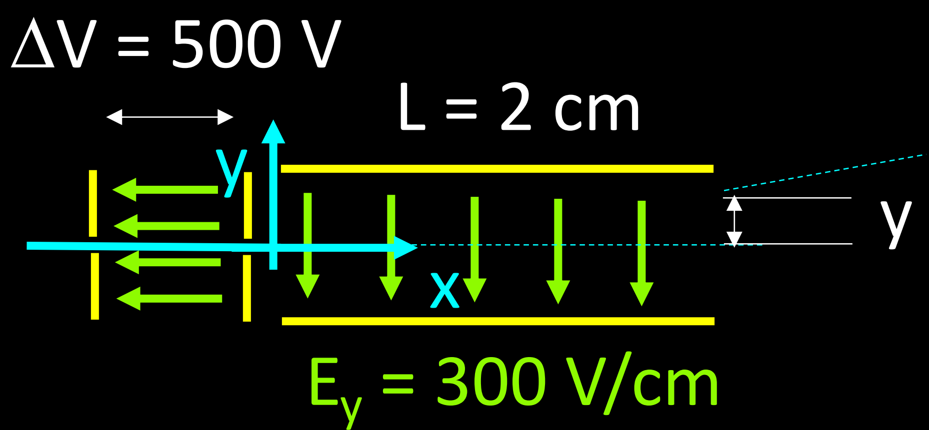


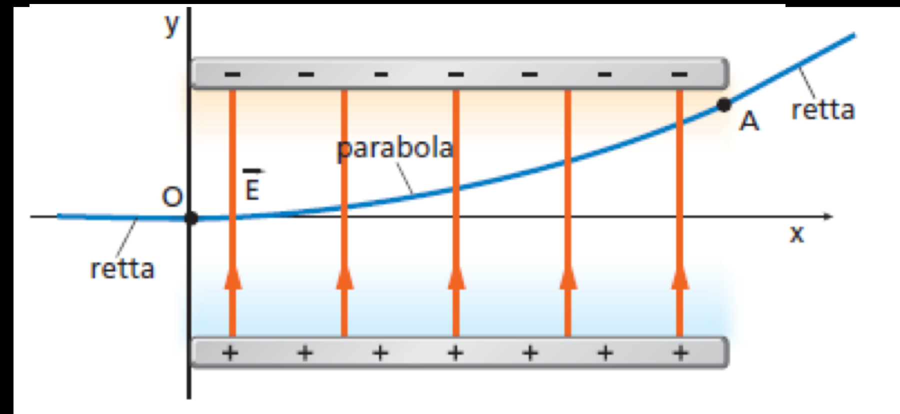
1) le gocce di inchiostro vengono caricate negativamente nello ionizzatore che provvede anche ad accelerarle verso la carta

2) entrano nella regione delle placchette dove è presente un campo verticale uniforme

Determinare il tipo di traiettoria nelle placchette e lo spostamento verticale

1) ELETTROSTATICA NEL VUOTO ink jet printer applicazione





1) ionizzatore: $U = q\,\Delta V$; $K = \frac{1}{2}\, m\, v_x^2 \rightarrow \frac{1}{2}\, m\, v_x^2 = q\,\Delta V$
2) placchette: v_x = costante; $a_y = qE_y/m \rightarrow$ parabola
3) $t = L/v_x$
4) $y = \frac{1}{2}\, a_y\, t^2 = \frac{1}{2}\, q\, E_y/m\, (L/v_x)^2 = \frac{1}{2}\, q\, E_y L^2/2K = \frac{1}{4}\, E_y\, L^2/\Delta V$
5) $y = \frac{1}{4}\; 3\; 10^4\ \text{V/m} \times 4\; 10^{-4}\ \text{m}^2/500\ \text{V} = 3/500\ \text{m} = 6\ \text{mm}$