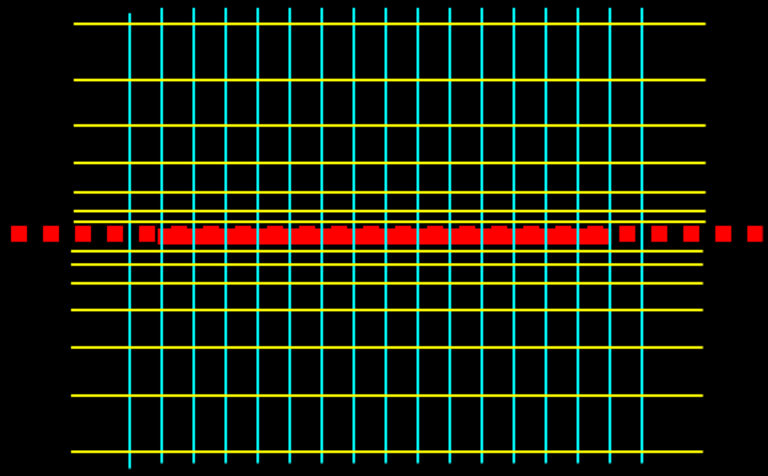
1) ELETTROSTATICA NEL VUOTO

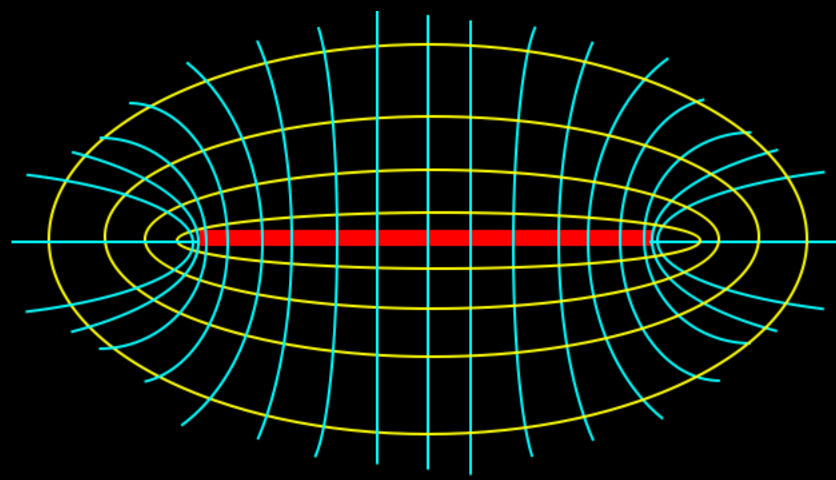
lezioni precedenti

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2} \qquad \mathrm{div}(\vec{E}) = \frac{\rho(\vec{r})}{\varepsilon_0} \qquad \overrightarrow{\mathrm{rot}}(\vec{E}) = 0$$

$$V(\vec{r}) = V(\overrightarrow{r_0}) + \int_{\overrightarrow{r_0}}^{\vec{r}} -\vec{E}\,\overrightarrow{dl} \qquad \vec{E} = -\overrightarrow{\mathrm{grad}}\,V \qquad dV(\vec{r}) = -\vec{E}\,\overrightarrow{dl}$$



filo rettilineo indefinito

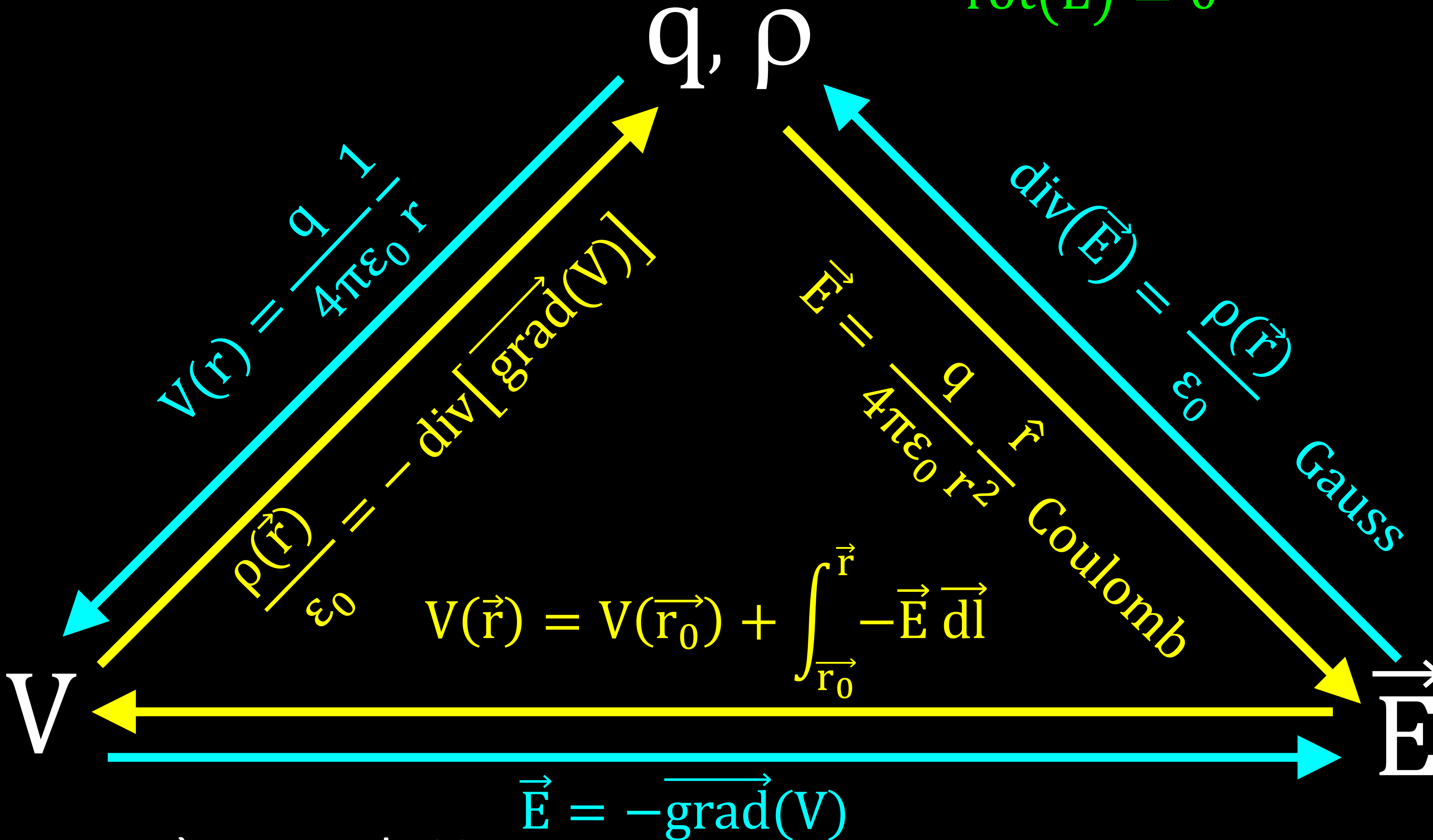


segmento rettilineo

1) ELETTROSTATICA NEL VUOTO

$\overrightarrow{\mathrm{rot}}(\vec{E}) = 0$

$\vec{E}, V, \rho$



spesso è noto solo V

1) ELETTROSTATICA NEL VUOTO esercizio

$\vec{E}$, V, ρ

in una regione di spazio è nota $V(\vec{r}) = \alpha\, r^2$ con $\alpha = 10^5$ V/m^2

Determinare $\vec{E}$ e la densità di carica in r = 1 cm

$$V(\vec{r}) = \alpha\, r^2 = \alpha\,(x^2+y^2+z^2)$$

$$\vec{E} = -\overrightarrow{\text{grad}}(V) = -\left[\hat{\imath}\left(\frac{\partial V}{\partial x}\right) + \hat{\jmath}\left(\frac{\partial V}{\partial y}\right) + \hat{k}\left(\frac{\partial V}{\partial z}\right)\right]$$

$$= -\left[\hat{\imath}\,2\alpha x + \hat{\jmath}\,2\alpha y + \hat{k}\,2\alpha z\right] = -2\alpha\,\vec{r}$$

$$E_r(1\text{ cm}) = -2\text{ kV/m}$$

$$\text{div}(\vec{E}) = \frac{\rho(\vec{r})}{\varepsilon_0} \qquad \rho(\vec{r}) = \varepsilon_0\,\text{div}(\vec{E}) = \varepsilon_0\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)$$

$$= -\varepsilon_0(2\alpha + 2\alpha + 2\alpha) = -6\alpha\,\varepsilon_0 = -5{,}3\ \mu\text{C/m}^3$$

1) ELETTROSTATICA NEL VUOTO esercizio $\vec{\mathbf{E}}, \mathbf{V}, \rho$

direttamente: $\dfrac{\rho(\vec{r})}{\varepsilon_0} = -\nabla^2 V$

$$V(\vec{r}) = \alpha\, r^2 = \alpha\,(x^2+y^2+z^2)$$

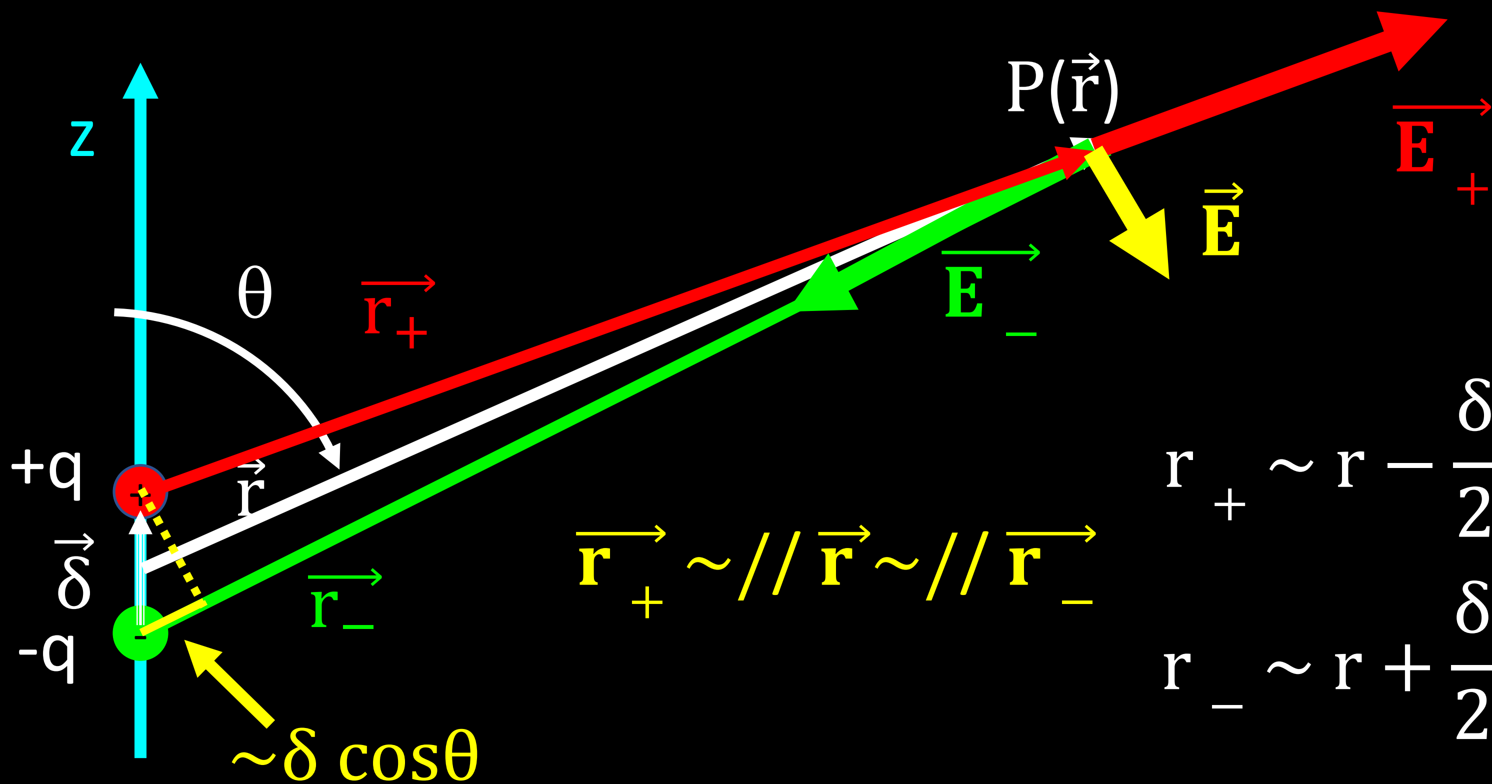
$$\Rightarrow \rho(\vec{r}) = -\varepsilon_0\, \nabla^2 V = -\,\varepsilon_0 \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right) =$$

$$= -\varepsilon_0 (2\alpha + 2\alpha + 2\alpha) = -6\alpha\, \varepsilon_0$$

1) ELETTROSTATICA NEL VUOTO potenziale **dipolo elettrico**

momento di dipolo elettrico: $\vec{p} = q\vec{\delta}$

approssimazione di dipolo: $r >> \delta$



$$\vec{r}_+ \sim // \vec{r} \sim // \vec{r}_-$$

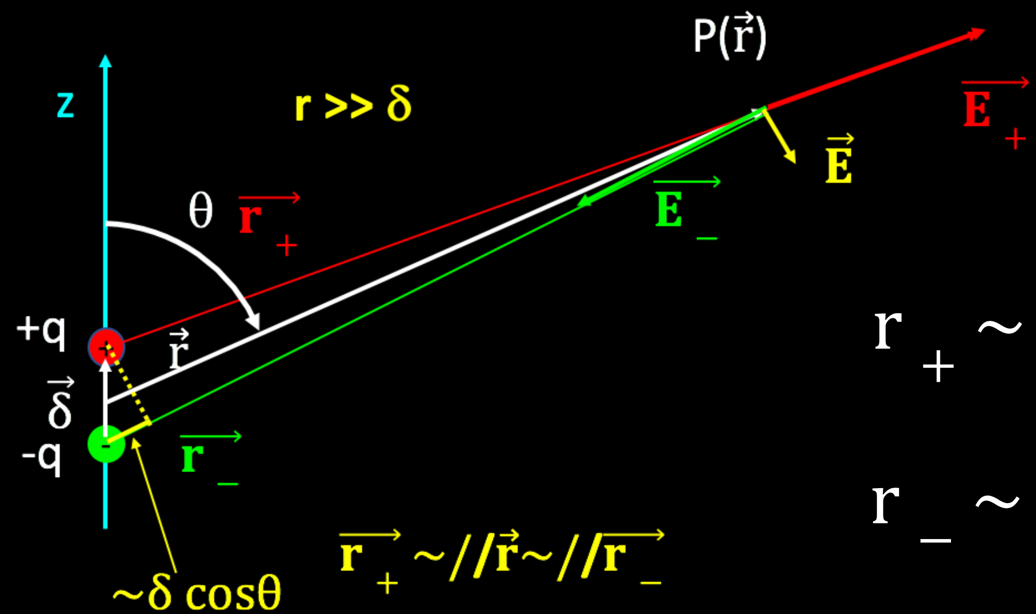
$$r_+ \sim r - \frac{\delta}{2} \cos\theta$$

$$r_- \sim r + \frac{\delta}{2} \cos\theta$$

1) ELETTROSTATICA NEL VUOTO potenziale **dipolo elettrico**

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_+} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_-} \sim \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r - \frac{\delta}{2}\cos\theta} - \frac{1}{r + \frac{\delta}{2}\cos\theta}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{\delta\cos\theta}{r^2 - \left(\frac{\delta}{2}\cos\theta\right)^2}\right) \sim \frac{q}{4\pi\varepsilon_0}\frac{\delta\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$



$$\vec{p} = q\vec{\delta}$$

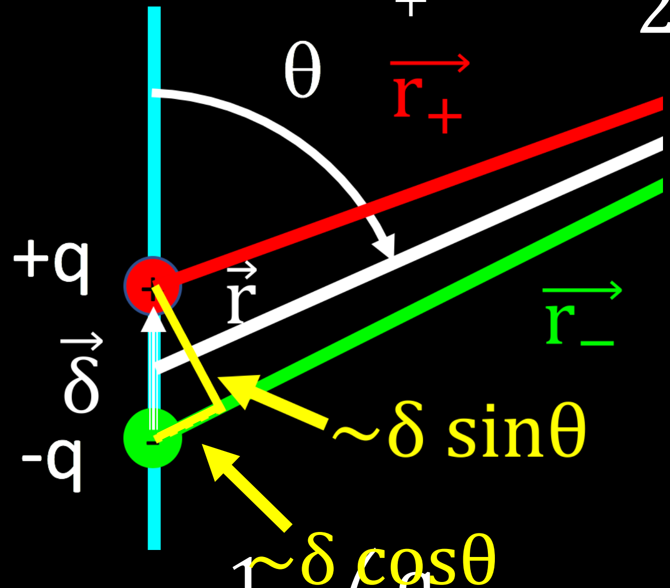
$$r_+ \sim r - \frac{\delta}{2}\cos\theta$$

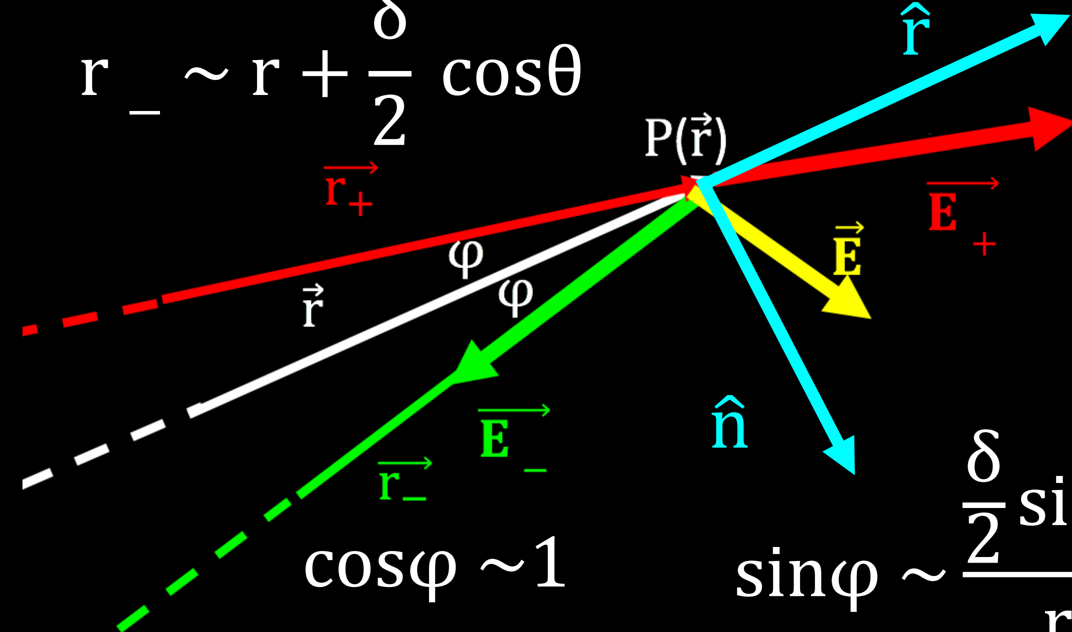
$$r_- \sim r + \frac{\delta}{2}\cos\theta$$

1) ELETTROSTATICA NEL VUOTO campo **dipolo elettrico**

$r >> \delta$

$$r_+ \sim r - \frac{\delta}{2}\cos\theta \qquad r_- \sim r + \frac{\delta}{2}\cos\theta$$





$$\cos\varphi \sim 1 \qquad \sin\varphi \sim \frac{\frac{\delta}{2}\sin\theta}{r}$$

$$E_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+^2}\cos\varphi - \frac{q}{r_-^2}\cos\varphi\right) \sim \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+^2} - \frac{1}{r_-^2}\right) \sim$$

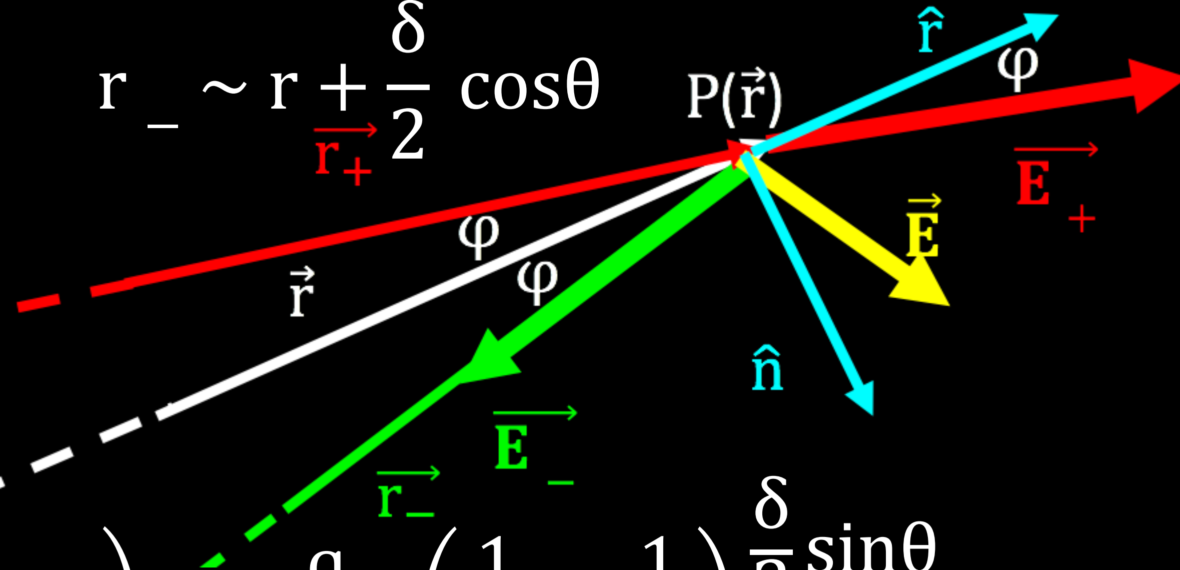
$$\sim \frac{q}{4\pi\varepsilon_0}\frac{\left(r+\frac{\delta}{2}\cos\theta\right)^2 - \left(r-\frac{\delta}{2}\cos\theta\right)^2}{\left(r-\frac{\delta}{2}\cos\theta\right)^2\left(r+\frac{\delta}{2}\cos\theta\right)^2} \sim \frac{q}{4\pi\varepsilon_0}\frac{2r\delta\cos\theta}{r^4} = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

$$\vec{p} = q\vec{\delta}$$

1) ELETTROSTATICA NEL VUOTO

campo **dipolo elettrico**

$$\vec{p} = q\vec{\delta} \qquad r_+ \sim r - \frac{\delta}{2}\cos\theta \qquad r_- \sim r + \frac{\delta}{2}\cos\theta$$



$$\sin\varphi \sim \frac{\frac{\delta}{2}\sin\theta}{r}$$

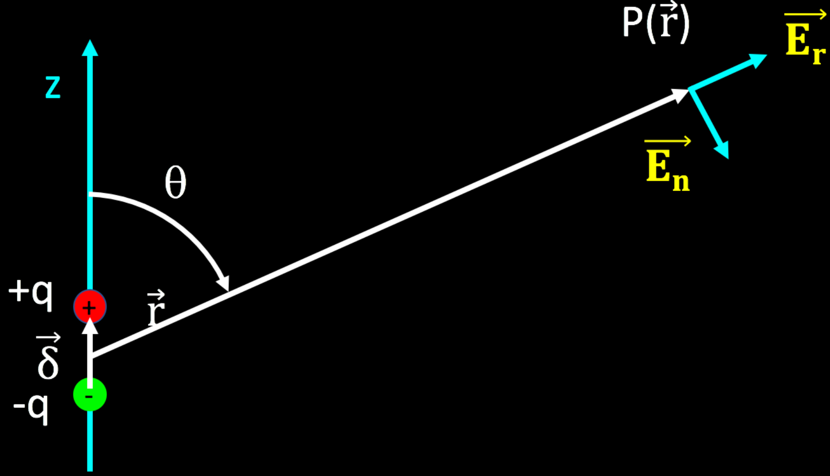
$$E_n = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+^2}\sin\varphi + \frac{q}{r_-^2}\sin\varphi\right) \sim \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+^2} + \frac{1}{r_-^2}\right)\frac{\frac{\delta}{2}\sin\theta}{r} \sim$$

$$\sim \frac{q}{4\pi\varepsilon_0}\frac{\left(r+\frac{\delta}{2}\cos\theta\right)^2 + \left(r-\frac{\delta}{2}\cos\theta\right)^2}{\left(r-\frac{\delta}{2}\cos\theta\right)^2\left(r+\frac{\delta}{2}\cos\theta\right)^2}\frac{\frac{\delta}{2}\sin\theta}{r} \sim \frac{q}{4\pi\varepsilon_0}\frac{2r^2}{r^4}\frac{\frac{\delta}{2}\sin\theta}{r} = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

$$\vec{p} = q\vec{\delta}$$

dipolo elettrico

$$\vec{p} = q\vec{\delta}$$

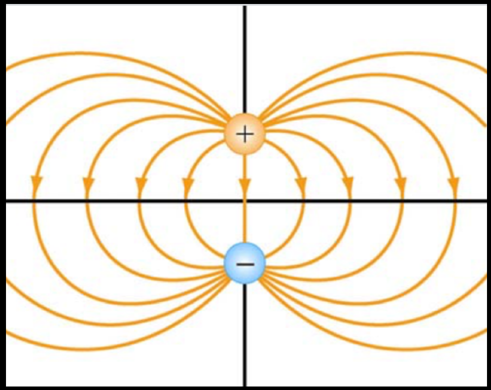


$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

$$E_r(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

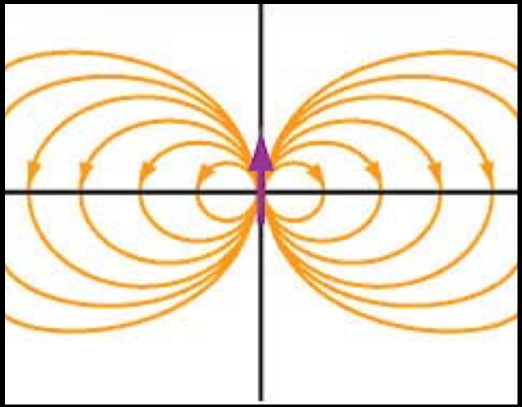
$$E_n(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

$r >> \delta$



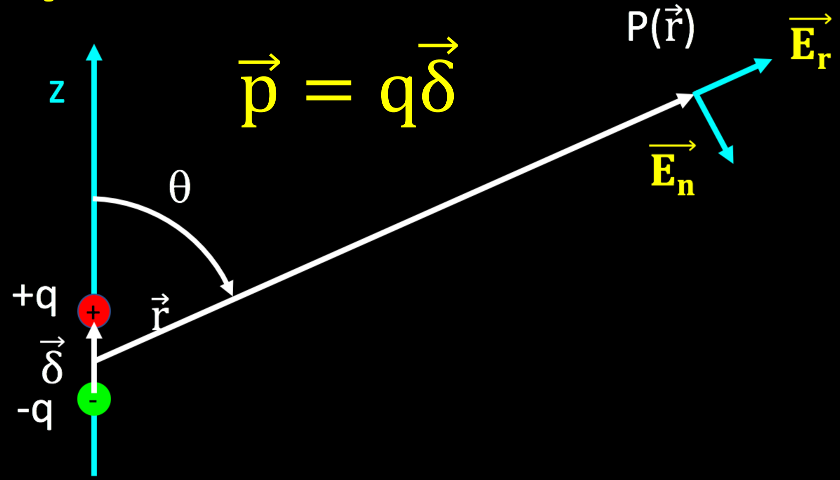
dipolo puntiforme

$\lim_{\delta\to 0}$



1) ELETTROSTATICA NEL VUOTO

dipolo elettrico

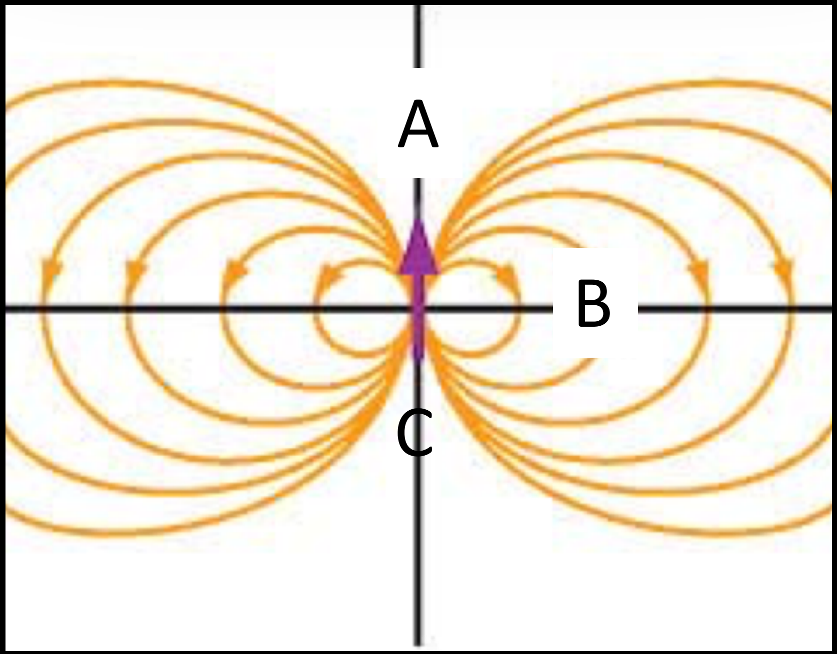


$r >> \delta$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

$$E_r(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

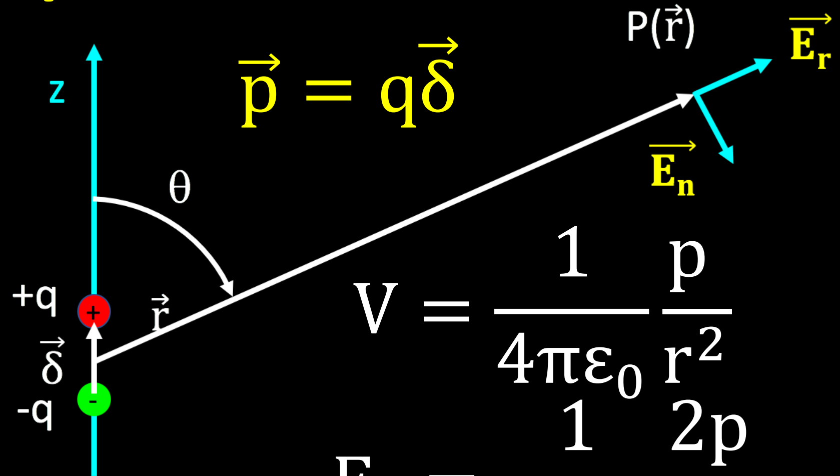
$$E_n(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$



angoli notevoli: 0, $\pi/2$, π

1) ELETTROSTATICA NEL VUOTO

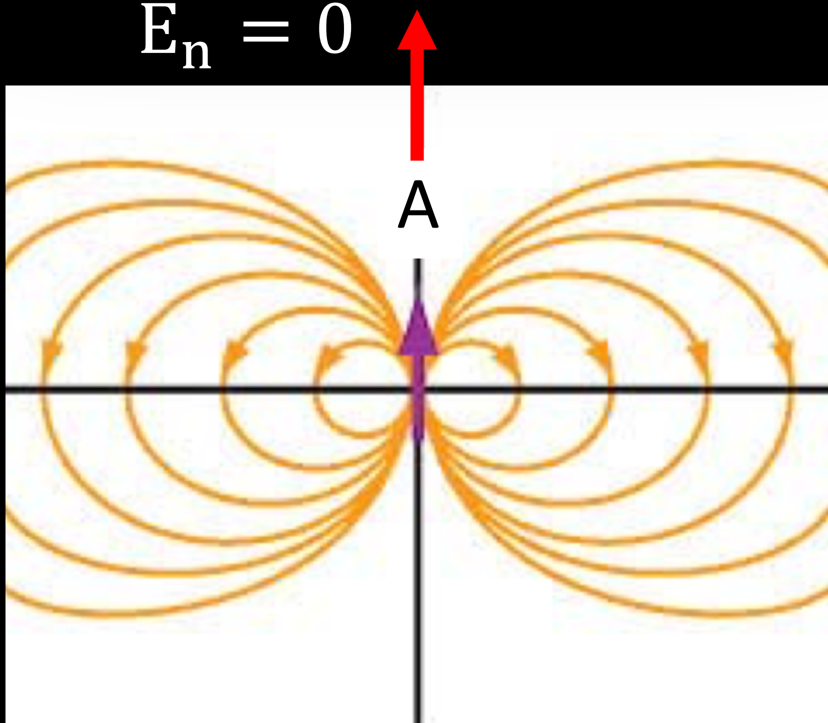
dipolo elettrico



$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

$$E_r = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

$$E_n = 0$$

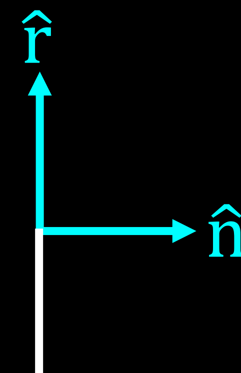


$r >> \delta$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

$$E_r(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

$$E_n(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

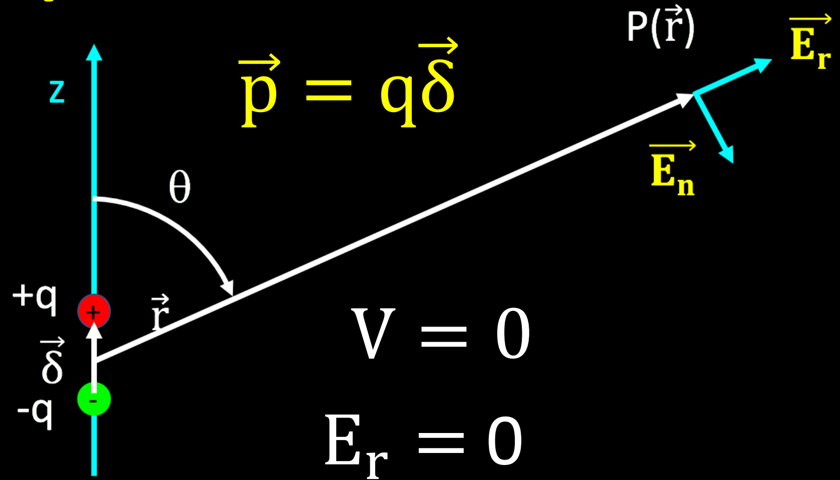


angoli notevoli: 0, $\pi/2$, π

1) ELETTROSTATICA NEL VUOTO

dipolo elettrico

$$\vec{p} = q\vec{\delta}$$



$r >> \delta$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

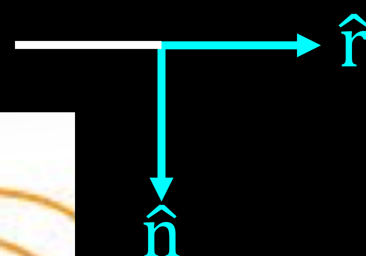
$$E_r(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

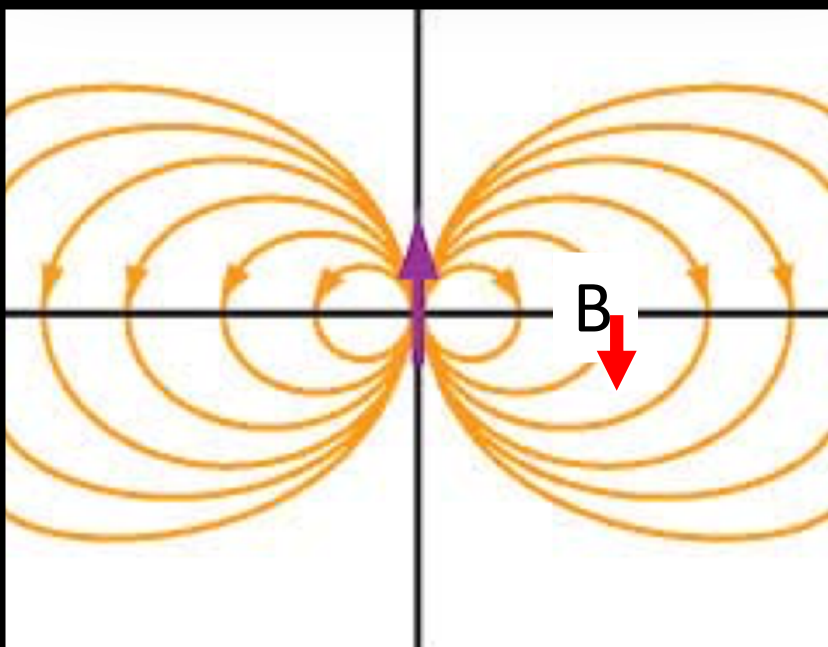
$$E_n(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

$$V = 0$$

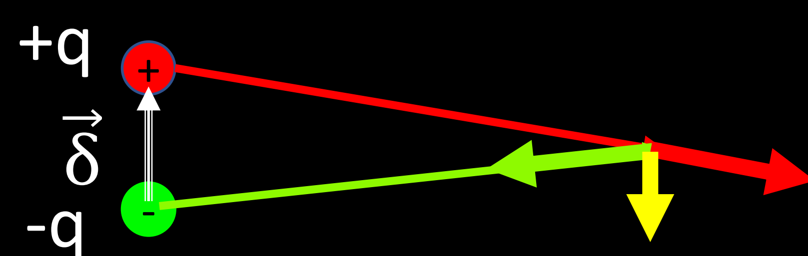
$$E_r = 0$$

$$E_n = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$





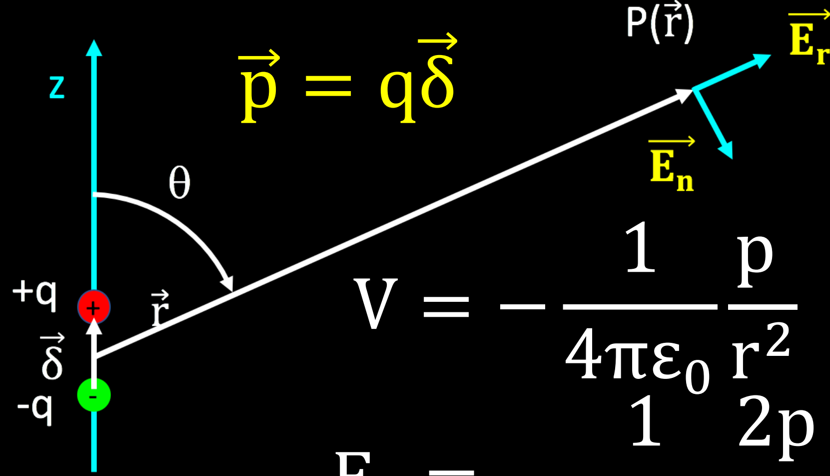
angoli notevoli: 0, $\pi/2$, π



1) ELETTROSTATICA NEL VUOTO

dipolo elettrico

$$\vec{p} = q\vec{\delta}$$



$$V = -\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

$$E_r = -\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

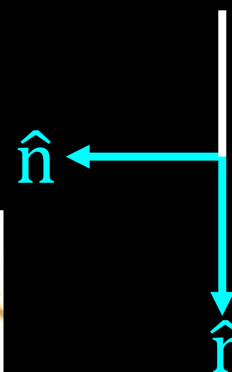
$$E_n = 0$$

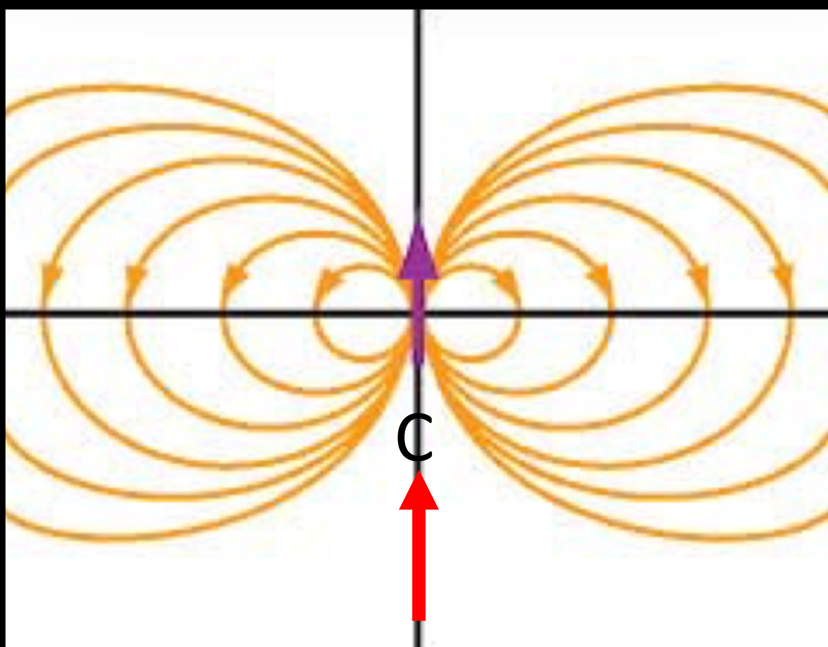
$r >> \delta$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

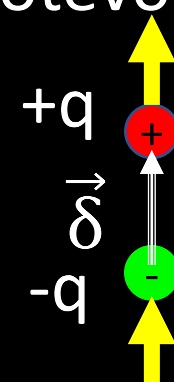
$$E_r(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

$$E_n(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$





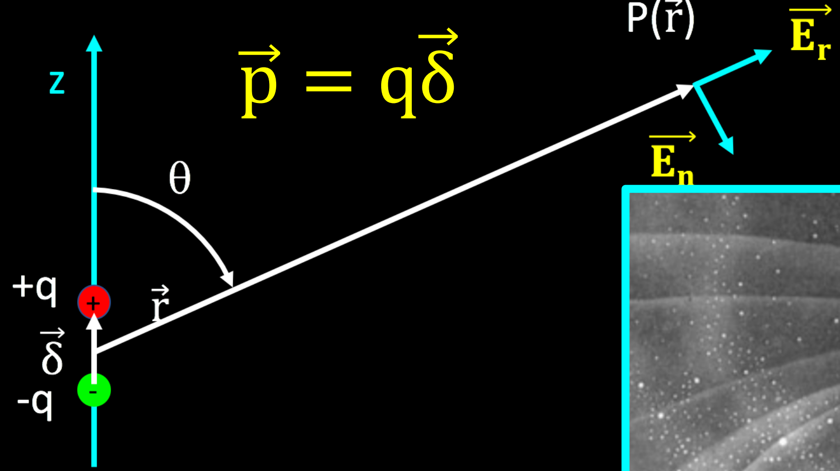
angoli notevoli: 0, $\pi/2$, π



1) ELETTROSTATICA NEL VUOTO

dipolo elettrico

$$\vec{p} = q\vec{\delta}$$



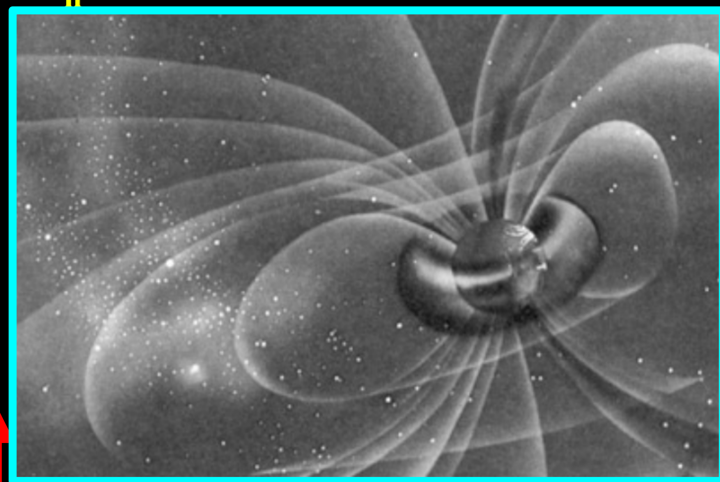
$r >> \delta$

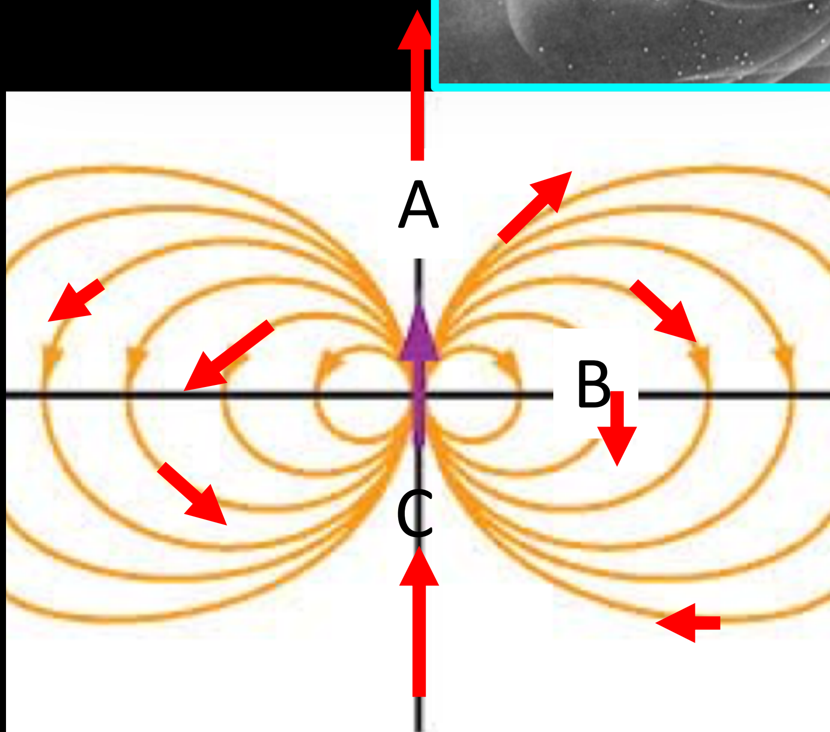
$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

$$E_r(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$$

$$E_n(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$$

$\forall \varphi$





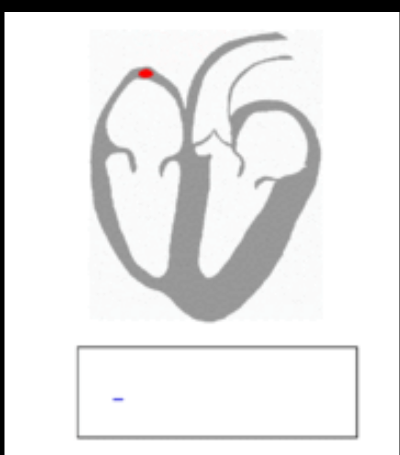
angoli notevoli: $0, \pi/2, \pi$

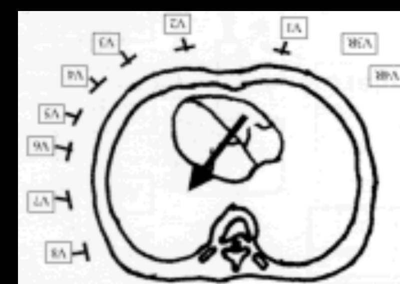
1) ELETTROSTATICA NEL VUOTO ECG applicazione

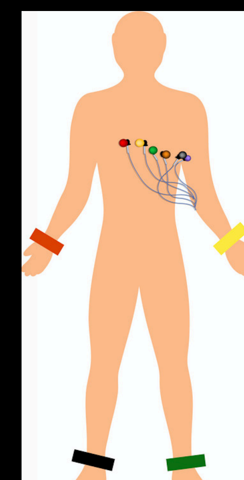
un elettrodo applicato sul torace è in grado di percepire il campo elettrico dipolare generato dal cuore.
Stimare il momento di dipolo cardiaco ipotizzando che a ~3 cm di distanza si misuri una differenza di potenziale di ~1 mV

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \sim \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

$$\rightarrow p \sim 4\pi\varepsilon_0 V(r) r^2 = \frac{1}{9\cdot 10^9}10^{-3}(3\cdot 10^{-2})^2 = 10^{-16}\,\mathrm{Cm}$$







EEG ~ 0,1 mV