

SVILUPPI DI McLAURIN (i.e. $x \rightarrow 0$). $O(\bullet)$ = o-grande, $o(\bullet)$ =o-piccolo:

$$\begin{aligned}\sin x &= x - \frac{1}{6}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + O(x^{2n+3}) \\ \cos x &= 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{6!}x^6 + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + O(x^{2n+2}) \\ \tan x &= x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + O(x^{11}) \\ \cot x &= x^{-1} - \frac{1}{3}x - \frac{1}{45}x^3 - \frac{2}{945}x^5 - \frac{1}{4725}x^7 - \frac{2}{93555}x^9 + O(x^{11}) \\ (1+x)^\alpha &= 1 + \alpha x + \binom{\alpha}{2}x^2 + \binom{\alpha}{3}x^3 + \dots + \binom{\alpha}{n}x^n + O(x^{n+1})\end{aligned}$$

dove $\alpha \in R$ e $\binom{\alpha}{n} = \frac{\alpha(\alpha-1)(\alpha-2)\dots(\alpha-n+1)}{n!}$

CASI PARTICOLARI:

$$\begin{aligned}\sqrt{1+x} &= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \frac{7}{256}x^5 - \frac{21}{1024}x^6 + O(x^7) \\ (1+x)^{\frac{1}{3}} &= 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 - \frac{10}{243}x^4 + \frac{22}{729}x^5 - \frac{154}{6561}x^6 + O(x^7) \\ (1+x)^{-1} &= 1 - x + x^2 - x^3 + x^4 - x^5 + \dots + (-1)^n x^n + O(x^{n+1}) \\ (1+x)^{-2} &= 1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5 + O(x^6) \\ (1+x)^{-\frac{1}{2}} &= 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \frac{35}{128}x^4 - \frac{63}{256}x^5 + O(x^6) \\ (1+x)^{-\frac{1}{3}} &= 1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \frac{35}{243}x^4 - \frac{91}{729}x^5 + O(x^6) \\ (1-x)^{-1} &= 1 + x + x^2 + x^3 + x^4 + x^5 + \dots + x^n + O(x^{n+1}) \\ (1-x)^{-2} &= 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + O(x^6) \\ \cot x &= x^{-1} - \frac{1}{3}x - \frac{1}{45}x^3 - \frac{2}{945}x^5 + O(x^7) \\ \sinh x &= x + \frac{1}{6}x^3 + \frac{1}{5!}x^5 + \frac{1}{7!}x^7 + \dots + \frac{x^{2n+1}}{(2n+1)!} + O(x^{2n+3}) \\ \cosh x &= 1 + \frac{1}{2}x^2 + \frac{1}{4!}x^4 + \frac{1}{6!}x^6 + \dots + \frac{x^{2n}}{(2n)!} + O(x^{2n+2}) \\ \tanh x &= x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{62}{2835}x^9 + O(x^{11}) \\ \coth x &= x^{-1} + \frac{1}{3}x - \frac{1}{45}x^3 + \frac{2}{945}x^5 - \frac{1}{4725}x^7 + \frac{2}{93555}x^9 + O(x^{11}) \\ \arcsin x &= x + \frac{1}{6}x^3 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5}x^5 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7}x^7 + O(x^9) \\ \arccos x &= \frac{1}{2}\pi - x - \frac{1}{6}x^3 - \frac{1 \cdot 3}{2 \cdot 4 \cdot 5}x^5 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7}x^7 + O(x^9) \\ \arctan x &= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + O(x^{2n+3}) \\ \operatorname{arccot} x &= \frac{1}{2}\pi - x + \frac{1}{3}x^3 - \frac{1}{5}x^5 + \frac{1}{7}x^7 + O(x^9) \\ \ln(1+x) &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 + \dots + \frac{(-1)^n}{n+1}x^{n+1} + O(x^{n+2}) \\ \ln(1-x) &= -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5 + \dots - \frac{1}{n+1}x^{n+1} + O(x^{n+2}) \\ \ln\left(\frac{1+x}{1-x}\right) &= 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \frac{2}{7}x^7 + O(x^9) = 2 \operatorname{sett} \sinh x \\ e^x &= 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{4!}x^4 + \dots + \frac{x^n}{n!} + O(x^{n+1}) \\ e^{-x} &= 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 + \dots + (-1)^n \frac{x^n}{n!} + O(x^{n+1}) \\ a^x &= 1 + (\ln a)x + \left(\frac{1}{2} \ln^2 a\right)x^2 + \left(\frac{1}{6} \ln^3 a\right)x^3 + \left(\frac{1}{4!} \ln^4 a\right)x^4 + \dots + \frac{(\ln a)^n x^n}{n!} + O(x^{n+1}) \\ \ln(n!) &= n \ln n - n + \frac{1}{2} \ln(n) + \ln(\sqrt{2\pi}) + \sigma_n, \quad 0 < \sigma_n < \frac{1}{12n} \\ n! &= \left(\frac{n}{e}\right)^n \sqrt{2\pi n} \left[1 + \frac{1}{12n} + O\left(\frac{1}{n^2}\right)\right] \\ 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots - \ln(n) &= \gamma + \varepsilon_n, \quad 0 < \varepsilon_n < \frac{1}{n}, \\ \gamma &\sim 0,5772 \quad \text{costante di Eulero-Mascheroni}\end{aligned}$$