On the identification of constant coefficients in a model of linear anisotropic diffusion

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Let $(H, \langle\cdot,\cdot\rangle)$ be a complex Hilbert space and $A_i : D(A) \to H$ $(i = 1, \cdots, n)$ be a family of nonnegative and self-adjoint operators mutually commuting. We study the inverse problem consisting in the identification of the function $u : [0, T] \to H$ and n constants $\alpha_1, \cdots, \alpha_n > 0$ (*diffusion coefficients*) that fulfill the initial-value problem

$$u'(t) + \alpha_1 A_1 u(t) + \cdots + \alpha_n A_n u(t) = 0, \quad t \in (0, T), \quad u(0) = x,$$

and the additional conditions

$$\langle A_1 u(T), u(T)\rangle = \varphi_1, \quad \cdots \quad , \langle A_n u(T), u(T)\rangle = \varphi_n.$$

Under suitable assumptions on the operators A_i and on the data $x \in H$ and $\varphi_1, \cdots, \varphi_n > 0$, we shall prove that the solution of such a problem is unique and depends continuously on the data. Applications are considered and a counterexample is displayed in the case when the operators are non commuting.

This extends previous results achieved by the author(s) in the cases when $n = 1$ and $n = 2$ (joint works with Alfredo Lorenzi, Noboru Okazawa and Tomomi Yokota).