SAPIENZA
UNIVERSITÀ DI ROMA

Ph.D. in Information and Communication Technologies ICT
Ph.D. Course:

Perturbative and qualitative methods: tools to solve nonlinear problems

Edition `2020`(30 hours, 6 credits)

Instructor: Sandra Carillo

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Course Description: This Course provides some methods to study nonlinear problems focussing mainly on cases which are modelled via nonlinear ordinary differential equations. The aim is to construct solutions to nonlinear problems which arise in applications via *qualitative* and *perturbation* methods. The first ones study the analytical consequences of conservation laws and their implications as far as the solutions admitted by such problems are concerned. The second ones are concerned with *ad hoc* methods developed when in the nonlinear a *small* parameter appears.

Class Schedule

The lectures of the course are scheduled in the period `March 2020, 24 Lecturing hours` in the seminar room at the second floor of the DIET department, Via Eudossiana 18, 00184

The arguments can be schematically listed in:

a) Qualitative methods: an introduction.

b) Straightforward Perturbation Method.

c) Multiple Scale Method.

d) Singular Perturbation Method.

e) Boundary Layer Method.

f) Visualisation of the results via computer algebra manipulation (MATLAB' Toll-box).

Some background notions on ordinary differential equations open the course. In particular, on *qualitative* and *perturbation* methods are introduced as tools to study nonlinear ordinary differential equations. Then, some ideas on qualitative methods (conservation of energy, phase plane) are briefly illustrated and applied to physically significative cases.

Then, various *perturbation* methods are presented and illustrative examples are studied in detail. Critic aspects as well as advantages of each method are pointed to the students' attention. In addition, via computer algebra methods, the solutions of the problems are constructed and plotted. Cauchy and boundary value problems are both treated. As a first *toy problem*, the Cauchy problem in the case of a linear weakly damped oscillator is studied. Then, nonlinear o.d.es, such as Duffing equation, are studied. Also the Van der Pol equation, which can be used to model the cardiac cycle, is analysed. In most of the provided examples, various methods are applied and a comparison among the different approximations obtained and the related region of validity (in time or space) is given. A variety of examples of application is provided and the students are invited to actively participate developing a personal project with applicative meaning.

If there is interest in the audience, an overview on how to apply Perturbation Methods in the case of partial differential equations closes the course.

Texts: Selected Chapters from

- M.H.Holmes, *Introduction to Perturbation Methods*, M.H.Holmes, Introduction to Perturbation Methods, Springer, New York, 1995; **Author(s):** M.H.Holmes; **ISBN-13:** 978-0000000000
- M. Lo Schiavo, *Note di sistemi dinamici*, SIMAI e-Lecture Notes, Vol 12 (2013) `http://cab.unime.it/journals/index.php/lecture/article/view/928`, **ISBN-13:**978-88-905708-5-8
- D. W. Jordan and P. Smith, *Nonlinear Ordinary Differential Equations An introduction for Scientists and Engineers*, FOURTH EDITION Oxford University Press, 1999.
- Further material provided by the Instructor.

Final Exam The students is asked for prepare a personally developed example similar to those studied during the course. Problems originated from scientific interests of the candidate are welcome.