Soluzioni

(1)

$$\Phi_{cil}(\vec{D}) = Q_{cil}^{int}$$

$$\vec{D} = D(r)\,\hat{r}$$

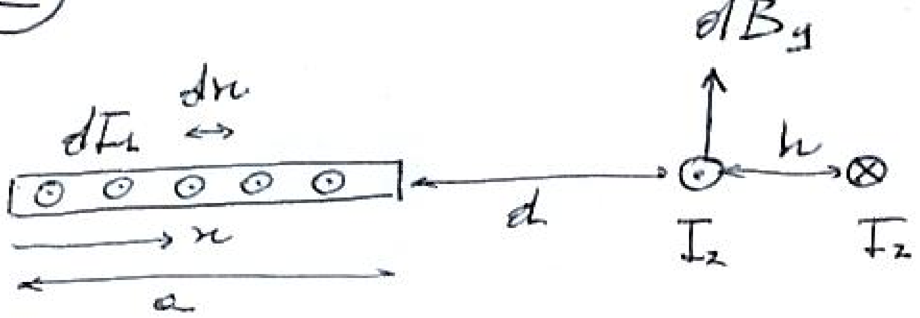
$$\Phi_{cil}(\vec{D}) = 2\pi r \ell D(r)$$

$$Q_{cil}^{int} = \int_0^r K r' \ell\, 2\pi r'\, dr' = \frac{K 2\pi r^3 \ell}{3}$$

$$D(r) = \frac{K r^2}{3} \quad \Rightarrow \quad \vec{E} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r}$$

$$V_A - V_B = -\int_B^A \vec{E}\, dr = \int_0^R \frac{K r^2}{3 \varepsilon_0 \varepsilon_r}\, dr = \frac{K R^3}{9 \varepsilon_0 \varepsilon_r} \simeq 12\,V$$

(2)



$$dI_1 = \frac{I_1\, dx}{a}$$

$$dB_y = \frac{\mu_0\, dI_1}{2\pi(d + a - x)} = \frac{\mu_0 I_1\, dx}{2\pi a (d + a - x)}$$

$$B_y(d+a) = \int dB_y = \frac{\mu_0 I_1}{2\pi a} \int_0^a \frac{dx}{(a + d - x)} = \frac{\mu_0 I_1}{2\pi a} \ln\left(\frac{d+a}{d}\right)$$

Le forze sui lati delle spire ortogonali si annullano

$$F_x = I_2 h B_y(d+a+h) - I_2 h B_y(d+a) =$$

$$= \frac{I_2 h \mu_0 I_1}{2\pi a} \ln\left[\frac{(d+a+h)\, d}{(d+a)\,(d+h)}\right]$$

(3) $Q(t) = Q_0 e^{-\frac{t}{\tau}}$ $\quad I(t) = \frac{Q_0}{\tau} e^{-t/\tau}$ $\quad \tau = RC$

$$U_c(t) = \frac{Q_0^2}{2C} e^{-\frac{2t}{\tau}}$$

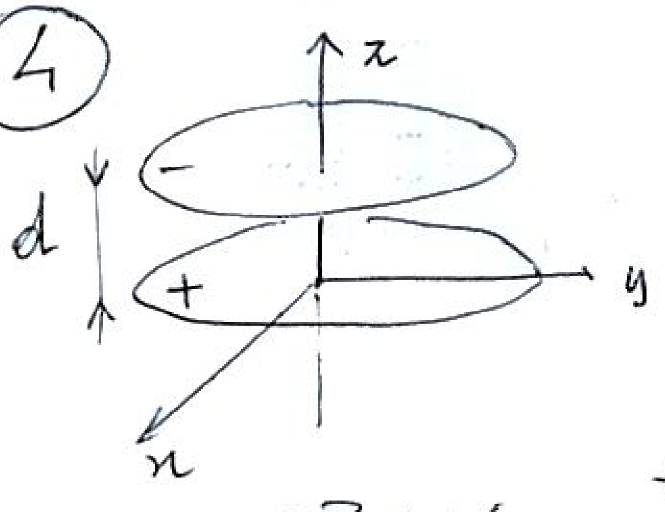
$$U_J(t) = \int_0^t R I^2(t')\,dt' = \frac{RQ_0^2}{\tau^2}\int_0^t e^{-\frac{2t'}{\tau}}\,dt' = \frac{RQ_0^2}{2\tau}\left(1-e^{-\frac{2t}{\tau}}\right)$$

$$U_J(t^*) = U_c(t^*)$$

$$\frac{RC}{\tau}\left(1-e^{-\frac{2t^*}{\tau}}\right) = e^{-\frac{2t^*}{\tau}}$$

$$t^* = \frac{\tau}{2}\ln(3)$$

(4)



$$\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$2B_0 e^{-t/\tau} = \mu_0\varepsilon_0 \frac{\partial E_z}{\partial t}$$

$$E_z(t) = \frac{2B_0\tau}{\mu_0\varepsilon_0}\left(1-e^{-t/\tau}\right) \Rightarrow V(t) = -\frac{2B_0\tau d}{\mu_0\varepsilon_0}\left(1-e^{-t/\tau}\right)$$

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Intensità media che entra nel materiale:

$$I_1 = I_0\left(1 - 1/5\right) = \frac{4}{5} I_0$$

$$I_1 = \frac{E_{eff}^2}{Z} \Rightarrow E_{eff} = \sqrt{I_1 Z} = \sqrt{\frac{4}{5} I_0 \frac{Z_0}{\sqrt{\varepsilon_r}}}$$

$$E_{eff} = \sqrt{\frac{2}{5} I_0 Z_0} \simeq 12,3 \ \frac{V}{m}$$