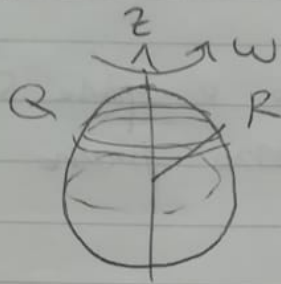


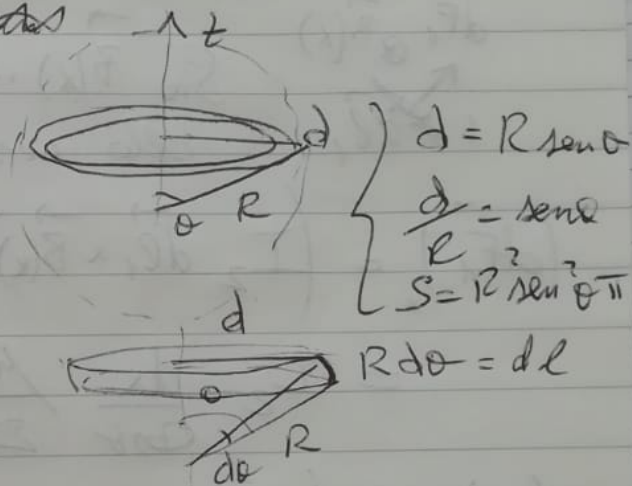
Esercizio 4 - 10/09/2021



$$\left\{ \begin{array}{l} T = \frac{2\pi}{\omega} \text{ è il periodo di rotazione} \\ Q = 4\pi R^2 \sigma \end{array} \right.$$

Prendo le spire infinitesime: ~~infinitesime~~

$$dI = \frac{dq}{T} \quad dq(\text{spira})$$



$$\Rightarrow dq(\text{spira}) = \sigma(2\pi d) d\ell = \sigma(2\pi d) R d\theta$$

$$\Rightarrow dI = \frac{dq}{T} = \frac{\sigma(2\pi d) R d\theta}{(2\pi/\omega)} = \frac{Q}{4\pi R^2} \frac{(2\pi d) R d\theta}{2\pi/\omega} \Rightarrow$$

$$\Rightarrow dI = \frac{Q R d d\theta \omega}{4\pi R^2} = \frac{Q \cdot d \cdot \omega \cdot d\theta}{4\pi R} = \frac{Q \omega}{4\pi} \sin \theta d\theta$$

$$d\vec{m} = dI \cdot \vec{z} = \frac{Q \omega}{4\pi} \sin \theta \pi R^2 \sin^2 \theta d\theta \vec{z}$$

$$d\vec{m} = \frac{Q \omega}{4\pi} \pi R^2 \sin^3 \theta d\theta \vec{z}$$

$$\begin{aligned} dx &= -\sin \theta d\theta \\ x &= \cos \theta \end{aligned}$$

$$\begin{aligned} |\vec{m}| &= \int \frac{Q \omega R^2}{4} \sin^3 \theta d\theta = \\ &= \frac{Q \omega R^2}{4} \int_{-1}^1 (1-x^2) (-dx) = \frac{Q \omega R^2}{4} \left(x^3 - \frac{x^5}{5} \right)_{-1}^1 = \frac{Q \omega R^2}{3} \end{aligned}$$