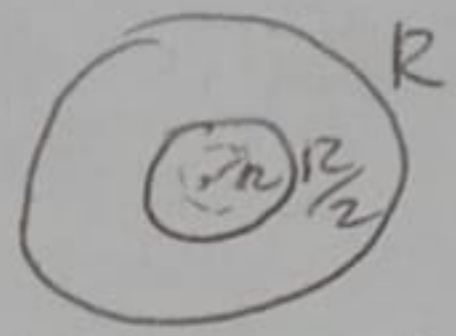
ESERCIZIO 1

1) $Q_{TOT} = \int_{\tau} \rho \, d\tau = k\int_0^{R/2} r(4\pi r^2 dr) + \frac{kR}{2}\int_{R/2}^{R} 4\pi r^2 dr =$

$$= 4\pi k \frac{r^4}{4}\Big|_0^{R/2} + \frac{2\pi R k}{3} r^3\Big|_{R/2}^{R} = \boxed{\frac{31}{48}\pi k R^4 = Q_{TOT}} = 2\times 10^{-14}\ C$$

2) $\vec{E} = E(r)\hat{r}$ → Uso Gauss:



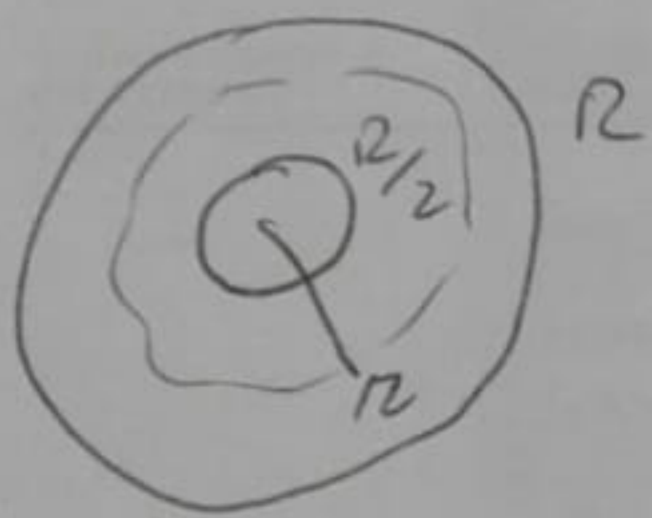
- $r < R/2$

$$\phi(\vec{E}) = 4\pi r^2 E_1 = \frac{1}{\varepsilon_0}\int_0^r kr(4\pi r^2 dr) = \frac{4\pi k}{\varepsilon_0}\frac{r^4}{4} \Rightarrow$$

$$\Rightarrow \boxed{E_1\left(r < \frac{R}{2}\right) = \frac{kr^2}{4\varepsilon_0}}$$

- $\frac{R}{2} < r < R$

$$\phi(\vec{E}) = 4\pi r^2 E_2 = \frac{1}{\varepsilon_0}\int \rho \, dV = \frac{1}{\varepsilon_0}\int_0^r \rho \, 4\pi r^2 dr$$

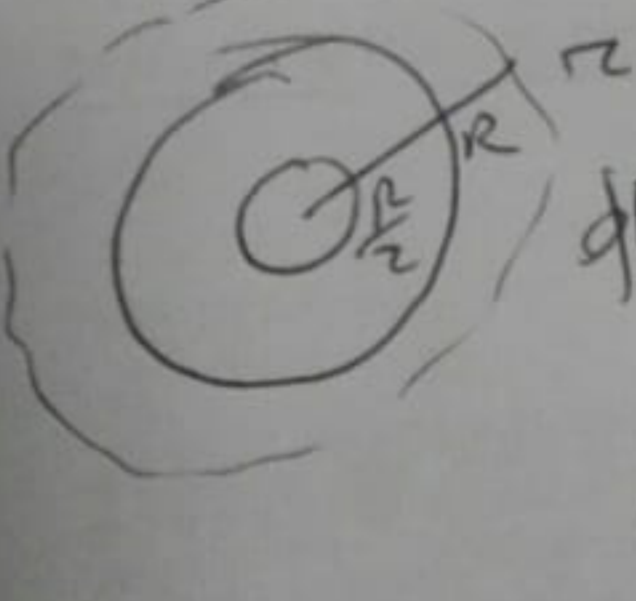


$$= \frac{1}{\varepsilon_0}\int_0^{R/2} kr \, 4\pi r^2 dr + \frac{1}{\varepsilon_0}\int_{R/2}^{r}\frac{kR}{2} 4\pi r^2 dr =$$

$$= \frac{4\pi k}{\varepsilon_0}\frac{(R/2)^4}{4} + \frac{2\pi k R}{\varepsilon_0}\frac{1}{3}\left[r^3 - \left(\frac{R}{2}\right)^3\right]$$

$$\Rightarrow \boxed{E_2(r) = \frac{kR^4}{64\varepsilon_0 r^2} + \frac{kR}{6\varepsilon_0}r - \frac{kR^4}{48\varepsilon_0 r^2}}$$

- $r > R$ la carica vista in questo caso è Q_{TOT} calcolata nel punto 1):

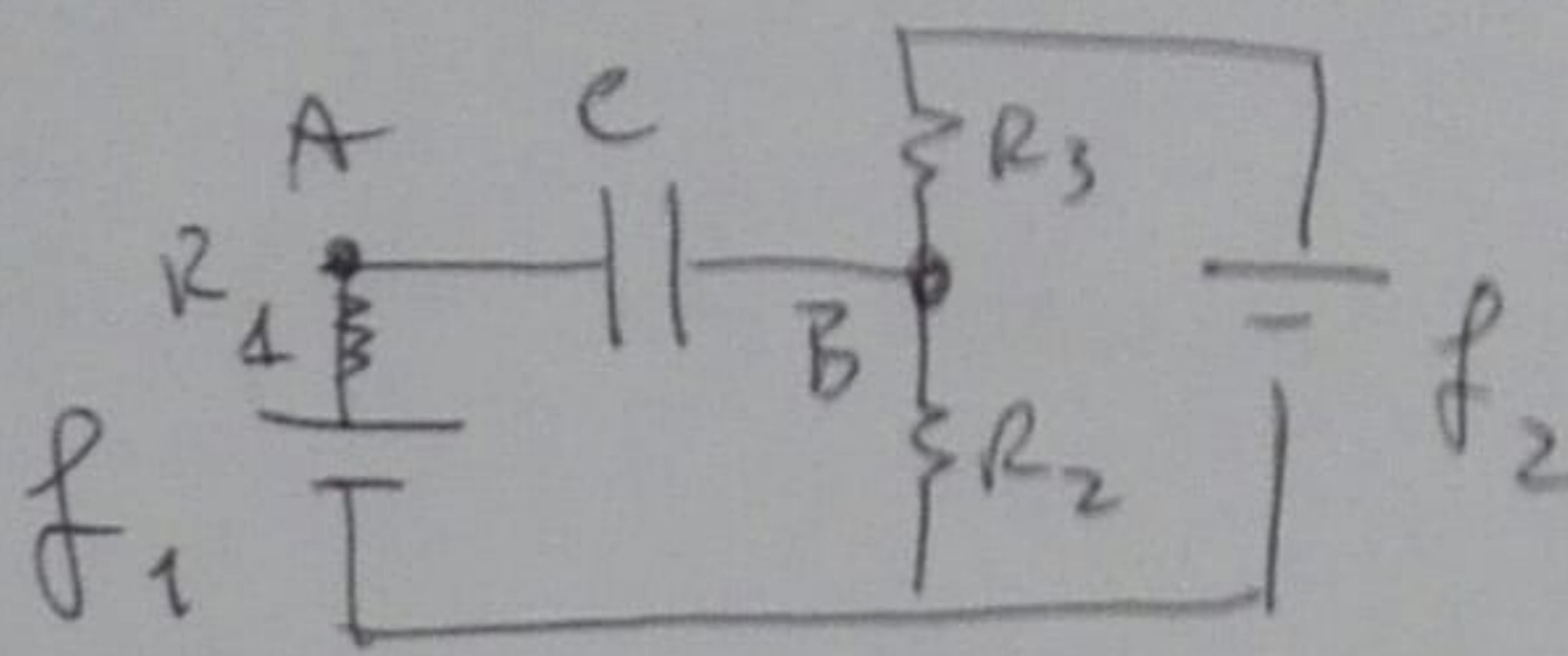


$$\phi(\vec{E}) = 4\pi r^2 E_3(r) = \frac{Q_{TOT}}{\varepsilon_0} \Rightarrow$$

$$\Rightarrow \boxed{E_3(r) = \frac{Q_{TOT}}{4\pi\varepsilon_0 r^2} = \frac{31}{192}\frac{kR^4}{\varepsilon_0 r^2}}$$

ESERCIZIO 2

T APERTO:

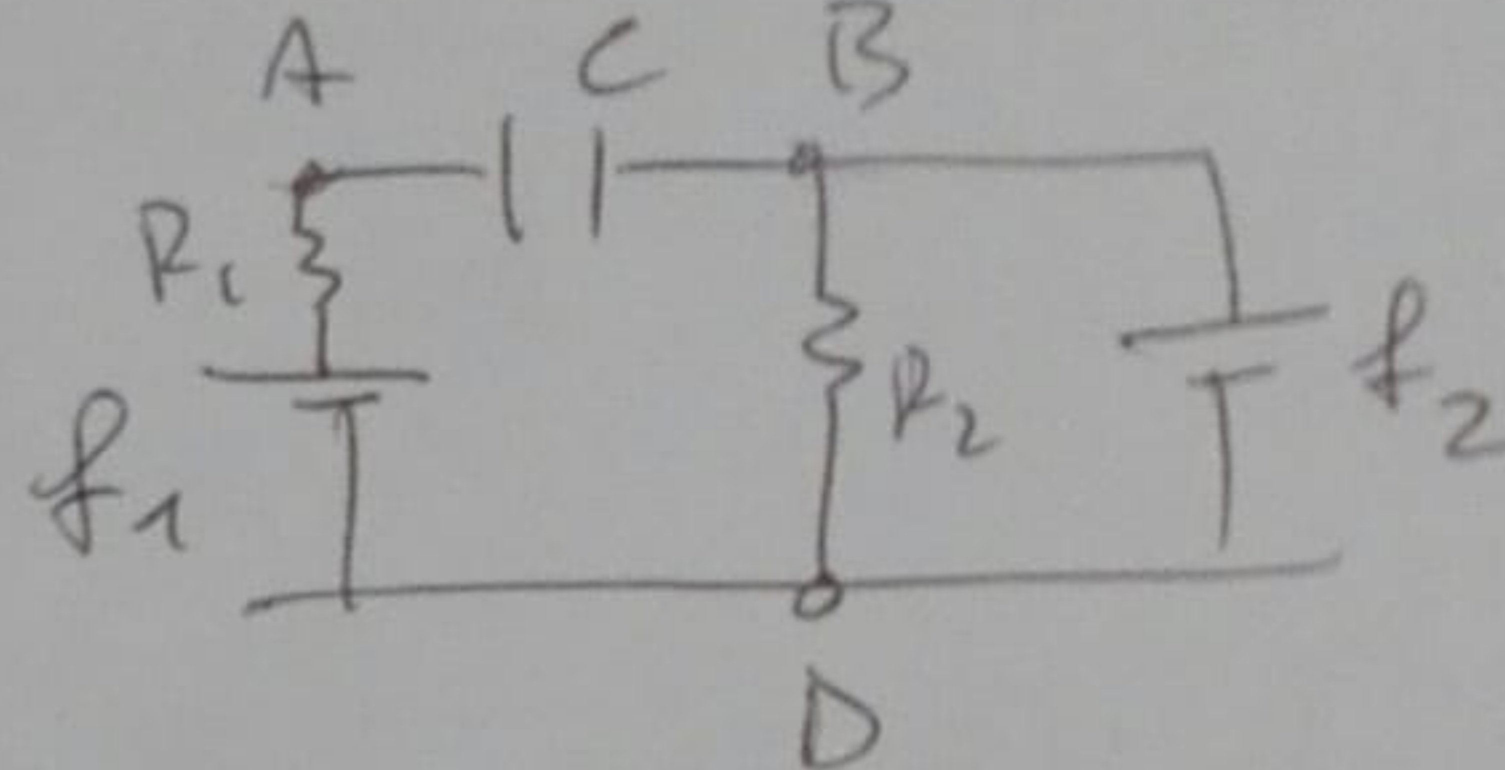


Calcolando ai capi di C:

$$\Delta V_c = V_A - V_B = f_1 - f_2 \frac{R_2}{R_2 + R_3} \Rightarrow 9V - \frac{16}{4}V = 5V$$

$$\Rightarrow Q_{IN} = C\Delta V_c = 10\,\mu F \times 5\,V = 5 \times 10^{-5}\,C$$

T CHIUSO:

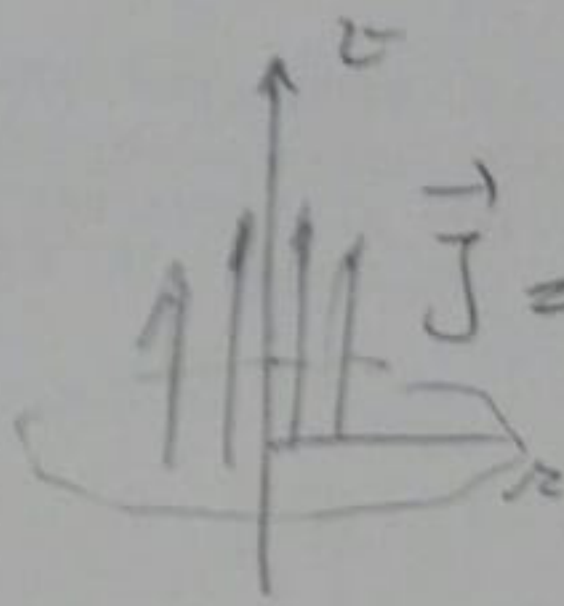


$$\Delta V_c = f_1 - f_2 = (9-16)V = -7V$$

$$\Rightarrow Q_{FIN} = C \cdot \Delta V_c = (10\,\mu F) \cdot (-7V) = -7 \times 10^{-5}\,C$$

$$\boxed{W_1 = f_1 \cdot \Delta Q = (Q_{FIN} - Q_{IN})\, f_1 = -1{,}08 \cdot 10^{-3}\,J}$$

Esercizio 3



$$\vec{J} = J_0\, e^{-kr^2}\, \hat{z}$$

Applico l. Ampere:

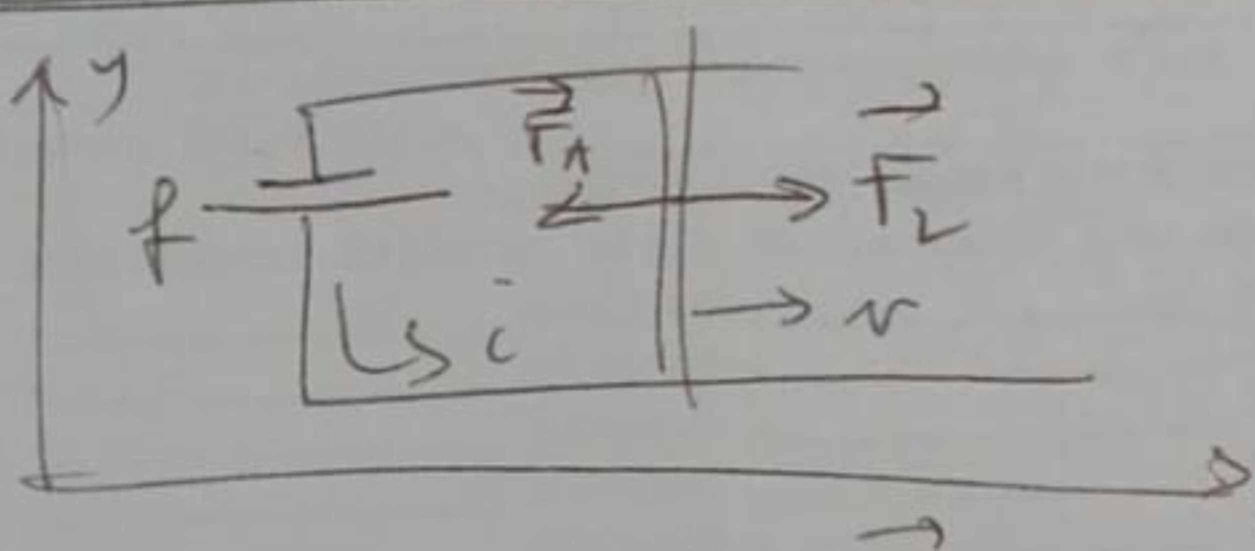
$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i = \mu_0 \int_\Sigma \vec{J}\cdot \hat{u}_n\, \underbrace{d\Sigma}_{2\pi r\, dr}$$

$$2\pi r\, B(r) = \mu_0 \int_0^r J_0 e^{-kr^2} (2\pi r\, dr) =$$

$$= \mu_0 J_0 2\pi \int_0^r r e^{-kr^2} dr = -\frac{\mu_0 J_0 \pi}{k} e^{-kr^2}\Big|_0^r$$

$$\Rightarrow \boxed{B(r) = \frac{\mu_0 J_0}{2k} \frac{1}{r} \left(1 - e^{-kr^2}\right)}$$

Esercizio 5



A regime: $F_A = F_L$

$$\begin{cases} \vec{F}_L = i L B\, \hat{x} \\ \vec{F}_A = -mg\mu_D\, \hat{x} \end{cases}$$

$$i_{REGIME} = \frac{mg\mu_D}{LB} = 7.85\,A = i_R$$

Per ottenere v_{REGIME}:

$$f + f_{em} = R i_R \qquad f_{em} = -\frac{d\phi(\vec{B})}{dt} = -BL$$

$$\Rightarrow v_{REGIME} = \frac{f - R i_R}{LB} = 0.46\ m/s$$

Esercizio 4

Applico Ampere per il campo H:

$$\oint \vec{H} \cdot d\vec{l} = Ni \qquad B = \mu H \Rightarrow$$

$$\Rightarrow \frac{B}{\mu_0 \mu_r} L + \frac{B}{\mu_0} 2d = Ni \Rightarrow$$

$$\Rightarrow B = \frac{\mu_0 N i}{\frac{L}{\mu_r} + 2d}$$

$$u_B = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0 \mu_r}$$ densità di energia magnetica

$$U_B = \frac{1}{2} \frac{B^2}{\mu_0 \mu_r} L \cdot S + \frac{1}{2} \frac{B^2}{\mu_0} 2d \cdot S =$$

$$= \frac{B^2}{2\mu_0} \left[\frac{L}{\mu_r} + 2d \right] \cdot S =$$

$$= \left(\frac{\mu_0 N i}{\frac{L}{\mu_r} + 2d} \right)^2 \frac{S}{2\mu_0} \left[\frac{L}{\mu_r} + 2d \right] \Rightarrow$$

$$\Rightarrow \boxed{U_B = \frac{\mu_0 S (Ni)^2}{2\left(\frac{L}{\mu_r} + 2d\right)} = 0.5 \text{ J}}$$