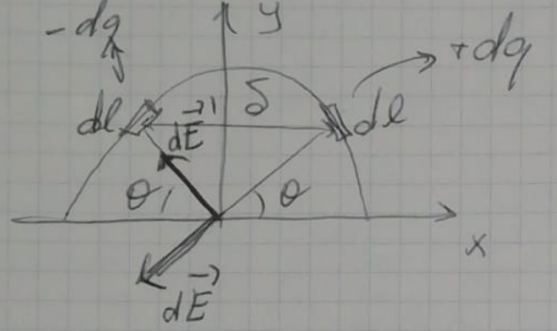
ESAME FISICA GENERALE 2 - IC1, I3S - 13/01/2021

ESERCIZIO 1



$$dq = \lambda(\theta)\, dl = \lambda_0 \cos\theta \underbrace{R\, d\theta}_{dl}$$

$$\delta = 2R\cos\theta$$

$$d\vec{E}_{COPPIA} = d\vec{E} + d\vec{E}' \parallel -\hat{x}$$

$$\Rightarrow \boxed{\vec{E} \parallel -\hat{x}}$$

$$dE = \frac{dq}{4\pi\varepsilon_0 R^2} = \frac{\lambda_0 \cos\theta\, R\, d\theta}{4\pi\varepsilon R^2} \qquad dE'_y = dE_y$$

$$dE_x = -dE\cos\theta = -\frac{\lambda_0 \cos\theta\, d\theta}{4\pi\varepsilon_0 R}\cos\theta$$

$$E_x = \int_0^{\pi} -\frac{\lambda_0 \cos^2\theta\, d\theta}{4\pi\varepsilon_0 R} = \frac{-\lambda_0}{4\pi\varepsilon_0 R}\int_0^{\pi}\overbrace{\left[\frac{1+\cos(2\theta)}{2}\right]}^{\cos^2\theta} d\theta = \frac{-\lambda_0}{8\varepsilon_0 R}$$

ESERCIZIO 2

$$V_P - V_A = \int_P^A \vec{E}\cdot d\vec{l} = \int_R^b E_0(r)\, dr + \int_b^a E(r)\, dr$$

$$D = D(r) = \frac{\lambda}{2\pi r} \qquad \text{con } D_n \text{ continua}$$

- Per $a < r < b$:

$$D(r) = \varepsilon_0 \varepsilon_r E(r) = \varepsilon E(r) \quad \text{(essendo il dielettrico lineare)}$$

$$\Rightarrow E(r) = \frac{D(r)}{\varepsilon_0 \varepsilon_r}$$

- Per $r > b$ sfruttando la continuità di $D(r)$:

$$D(r) = \varepsilon_0 E_0(r) \Rightarrow E_0(r) = \frac{D(r)}{\varepsilon_0}$$

$$\Rightarrow V_P - V_A = \frac{\lambda}{2\pi\varepsilon_0}\left[\ln\left(\frac{b}{R}\right) + \frac{1}{\varepsilon_r}\ln\left(\frac{a}{b}\right)\right]$$

$$\vec{P} = \varepsilon_0 \chi \vec{E} = \varepsilon_0 \chi \frac{\lambda}{2\pi\varepsilon r}\hat{r} = \left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)\frac{\lambda}{2\pi r}\hat{r}$$

$$\sigma_p(a) = -\frac{\varepsilon_r - 1}{\varepsilon_r}\frac{\lambda}{2\pi a}$$

$$\sigma_p(b) = \frac{\varepsilon_r - 1}{\varepsilon_r}\frac{\lambda}{2\pi b}$$

$$\rho_P = 0$$

La carica per unità di lunghezza è quindi:

$$q_{\sigma,h}(a) = \sigma_p(a) \cdot 2\pi a = -\frac{\varepsilon_r - 1}{\varepsilon_r}\lambda$$

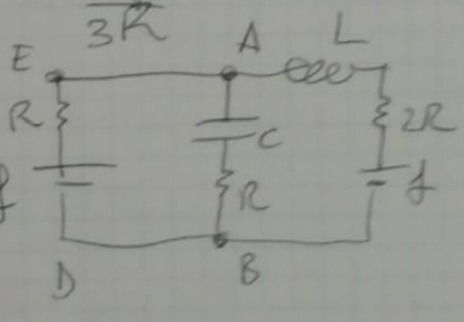
$$q_{\sigma,h}(b) = \sigma_p(b) \cdot 2\pi b \Rightarrow \frac{\varepsilon_r - 1}{\varepsilon_r}\lambda$$

$$\Rightarrow q_{TOT} = q_\sigma(a) + q_\sigma(b) = 0$$

Esercizio 3

La corrente circola solo nella maglia esterna

$$3f - f = (2R + R)\, i \Rightarrow i = \frac{2f}{3R}$$



$$U_L = \frac{1}{2}L i^2 = \frac{2}{9}\frac{L f^2}{R^2}$$

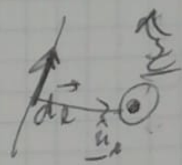
$$\Delta V_c = V_A - V_B = V_E - V_D =$$

$$= 3f - Ri = \frac{7}{3}f \Rightarrow U_c = \frac{1}{2}C\Delta V_c^2 = \frac{49}{18}C f^2$$

$$U_L = 2U_c \Rightarrow \boxed{R = \sqrt{\frac{2}{49}\frac{L}{C}} = 1.6\,k\Omega}$$

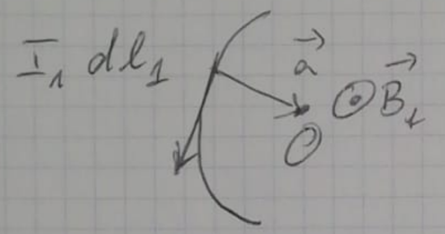
ESERCIZIO 4

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{u}_r}{r^2} = \frac{\mu_0}{4\pi} \frac{I \, dl \, \text{sen}\,\theta}{r^2} \hat{z}$$



1) Tratti rettilinei : $\text{sen}\,\theta = 0$

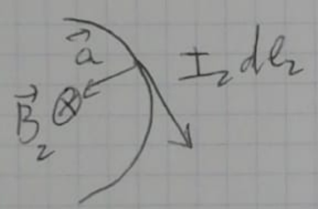
2)



$$B_1 = \frac{\mu_0}{4\pi} \frac{I_1}{a^2} \int_0^{\pi a} dl = \frac{\mu_0}{4a} I_1 =$$

$$= \frac{\mu_0}{4a} \left(\frac{R_2}{R_1 + R_2} \right) I$$

3)



$$B_2 = \frac{\mu_0}{4a} \left(\frac{R_1}{R_1 + R_2} \right) I > B_1$$

essendo $R_1 > R_2$

$$\Rightarrow |\vec{B}(0)| = \frac{\mu_0 I}{4a} \left(\frac{R_1 - R_2}{R_1 + R_2} \right)$$

con direzione ortogonale al piano e verso entrante (stesso verso di $\vec{B}_2$ essendo $B_2 > B_1$)

$$\vec{B}(0) = -\frac{\mu_0 I}{4a} \left(\frac{R_1 - R_2}{R_1 + R_2} \right) \hat{z}$$

ESERCIZIO 5

$$W_0 = \int_\Sigma \langle \vec{S} \rangle \cdot \hat{u}_n \, d\Sigma = \Sigma \cdot I = \Sigma \frac{E_0^2}{2Z_0} =$$

$$= (\Delta\Omega \cdot L^2) \frac{E_0^2}{2Z_0} \Rightarrow E_0 = \sqrt{\frac{2Z_0 W_0}{\Delta\Omega \cdot L^2}} = 6.9 \, \frac{V}{m}$$

$$\Rightarrow B_0 = \frac{E_0}{c} = 2.3 \cdot 10^{-8} \, T$$