Soluzioni

(1) $\vec{E}_{\text{CILINDRO CON TAGLIO}} = \vec{E}_{\text{CIL}} + \vec{E}_{\text{TAGLIO}}$

$$\vec{E}_{\text{CIL}} = \frac{\sigma\, 2\pi R}{2\pi \varepsilon_0 (D+R)}\,\hat{r}$$

$$\vec{E}_{\text{TAGLIO}} \cong \vec{E}_{\text{FILO}} \quad \text{CON} \quad \lambda_{\text{TAGLIO}} = -\sigma h$$

$$\vec{E}_{\text{TAGLIO}} = \frac{-\sigma h}{2\pi \varepsilon_0 D}\,\hat{r}$$

Considerando un elemento dl ovvero l della distribuzione lineare λ:

$$dF = dq\,E = \lambda dl \left(\frac{\sigma R}{\cancel{2\pi}\varepsilon_0 (D+R)} - \frac{\sigma h}{2\pi \varepsilon_0 D} \right)$$

$$dF/dl = \frac{F}{l} = \frac{\lambda\sigma}{\varepsilon_0}\left(\frac{R}{D+R} - \frac{h}{2\pi D} \right)$$

(2) $$f_{em}(t) = -\frac{d\Phi(t)}{dt}$$

$$q(t) = \int_0^t i(t)\,dt = \frac{1}{R}\int_0^t f_{e.m.}(t)\,dt = \frac{1}{R}\left[\Phi(0) - \Phi(t)\right]$$

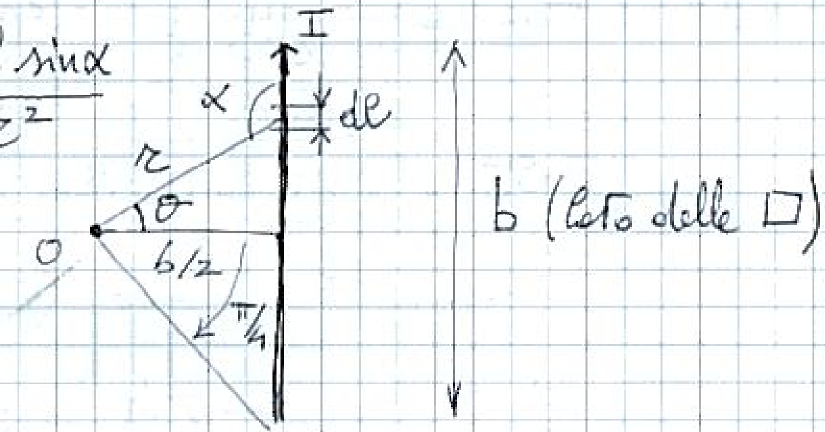
$$Q = \frac{\Phi_{\text{iniziale}} - \Phi_{\text{finale}}}{R} \underset{\uparrow}{=} \frac{2\,\Phi_{\text{iniziale}}}{R}$$

$$\Phi_{\text{finale}} = -\Phi_{\text{iniziale}}$$

B al centro della spira quadrata $= B_\square$

$$B_\square = 4\, B_{LATO}$$

$$dB_{LATO} = \frac{\mu_0 I}{4\pi}\,\frac{dl \sin\alpha}{r^2}$$



$$\sin\alpha = \cos\theta$$

$$dB_{LATO} = \frac{\mu_0 I}{\cancel{4}_{2}\pi}\,\frac{\cos\theta\, d\theta}{b/\cancel{2}}$$

$$B_\square = 4\, B_{LATO} = 4\int_{-\pi/4}^{\pi/4} dB_{LATO} = 2\sqrt{2}\,\frac{\mu_0 I}{\pi b}$$

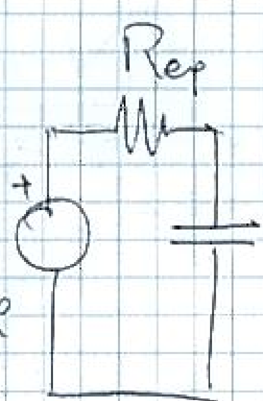
$$\left[\,Q = \frac{4\sqrt{2}\,\mu_0 I a^2}{R b}\,\right]$$

(3) $\cancel{R_{eq} = \left(R_1 // \{R_2 + R_3\}\right)}$

$$R_{eq} = R_1 // R_2 + R_3 = 250\,\Omega$$

$$\tau = R_{eq} C = 500\,\mu s$$

$$f_{eq} = \frac{R_1}{R_1+R_2}\, f = \frac{f}{2} = 50\,V$$



$$U_C = \frac{1}{2} C f_{eq}^2 = 25\cdot 10^{-4}\,J$$

(4) A $t>0$ nelle spire $i(t) = -\frac{d\Phi}{dt}$

$$i(t) = -\frac{1}{R}\frac{d}{dt}\int_{\Sigma S} \vec{B}\cdot d\vec{S} \simeq$$

$$\simeq -\frac{S}{R}\frac{d}{dt}B \simeq -\frac{S}{R}\frac{\mu}{2\pi a}\frac{dI}{dt}$$

Considero B uniforme su S.

Prendo il valore di B nel toro pari a quello sull'asse del toro

$$i(t) \simeq -\frac{S}{R}\frac{\mu}{2\pi a}\frac{dI}{dt}$$

$$I = I_0 e^{-t/\tau}$$

$$i(t) = \frac{S}{R}\frac{\mu I_0}{2\pi a \tau} e^{-t/\tau}$$

$$U = \int_0^\infty i^2(t) R\, dt = \frac{1}{R}\left(\frac{S\mu I_0}{2\pi a \tau}\right)^2 \int_0^\infty e^{-\frac{2t}{\tau}} dt \Rightarrow$$

$$U = \frac{1}{2R\tau}\left(\frac{S\mu I_0}{2\pi a}\right)^2$$

(5)

$$f = -\frac{d\Phi}{dt} = -\ell^2\frac{dB_x}{dt} = -\ell^2\cos(30^\circ)\frac{d}{dt}B =$$

$$= -\ell^2\cos(30^\circ)\frac{d}{dt}B_0\cos\omega t =$$

$$= -\underbrace{\left[\ell^2\cos(30^\circ)\,\omega\, B_0\right]}_{\hat{f}}\sin(\omega t)$$

$$f_{eff} = \frac{|\hat{f}|}{\sqrt{2}} = \ell^2\cos(30^\circ)\,\omega\,\underbrace{\frac{B_0}{\sqrt{2}}}_{B_{eff}}$$

$$E_{eff} = \frac{B_{eff}}{c}\ ; \quad \omega = 2\pi f = 2\pi\frac{1}{T}$$

$$f_{eff} = \ell^2\cos(30^\circ)\cdot\frac{2\pi}{cT}E_{eff}\ ; \quad \lambda = cT$$

$$E_{eff} = \frac{f_e\,\lambda}{\ell^2\,2\pi\cos 30^\circ} \Rightarrow I = \frac{E_{eff}^2}{Z_0}$$

$$I = \left(\frac{5\cdot 4\cdot 10^{-3}}{25\cdot 10^{-4}\cdot 2\pi\cdot 0{,}9}\right)^2\cdot\frac{1}{377}\left[\frac{(m\,V)^2}{m^4\,\frac{V}{A}}\right]$$

$$I = 5{,}3\,10^{-3}\,\frac{W}{m^2} = 5{,}3\ \underbrace{m}_{\text{milli}}\left[\frac{W}{m^2}\right]$$