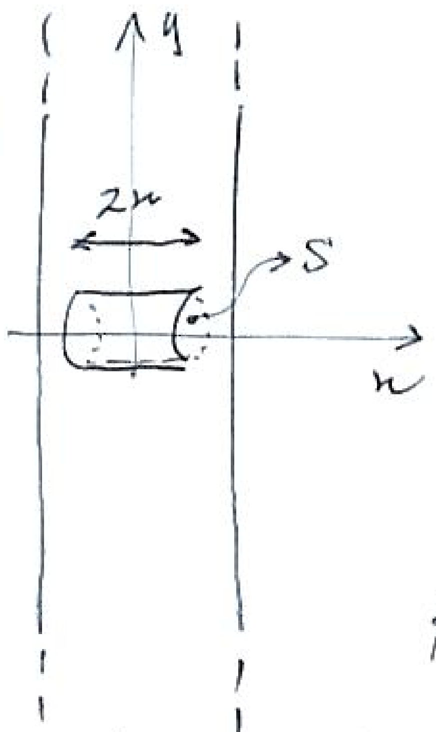
Soluzioni

(1)



Per il teorema di Gauss:

$$2ES = \frac{1}{\varepsilon_0}\rho S 2x$$

$$\Downarrow$$

$$E = \frac{\rho x}{\varepsilon_0}$$

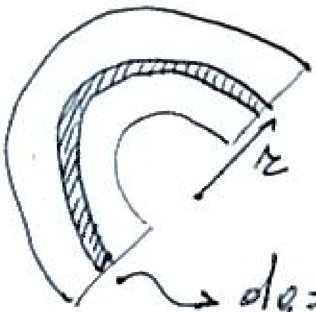
per $x>0$:

$$\Delta V = V_{p.m.} - V_{f.e.} = \int_0^{d/2} \vec{E}\cdot d\vec{x} = \frac{1}{8}\frac{\rho d^2}{\varepsilon_0}$$

$$\rho = \frac{8\varepsilon_0 \Delta V}{d^2} \cong \frac{8\cdot 8{,}854\cdot 10^{-12}\,F/m \cdot 5\,V}{(0{,}02)^2\,m^2} \Rightarrow$$

$$\rho = 8{,}85\cdot 10^{-7}\left[\frac{C}{m^3}\right]$$

(2)



$dq = \sigma \pi r\, dr$

$$di = \frac{dq}{T} = \frac{dq}{\frac{2\pi}{\omega}}$$

B elementare di spira con corrente elementare di nel centro vale: $dB = \frac{\mu_0\, di}{2r}$

$$B = \int_a^{3a} dB = \int_a^{3a} \frac{\mu_0}{2\cancel{\pi}} \frac{\sigma\cancel{\pi}\cancel{r}\,dr}{2\cancel{r}} \cdot \omega =$$

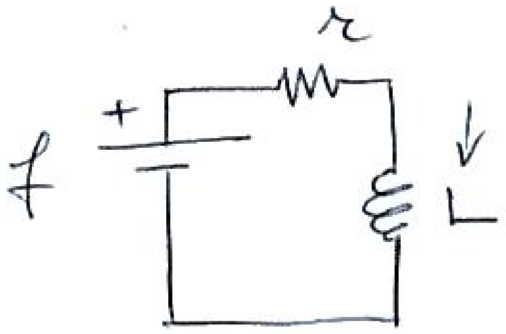
$$= \frac{\mu_0 \sigma \omega}{4}(3a - a)$$

$$B = \frac{\mu_0 \sigma \omega}{2\cancel{4}} \cancel{2}a = \frac{\mu_0 \sigma \omega}{2} \cdot a$$

$$\vec{B} = \frac{\mu_0 \sigma a}{2} \vec{\omega}$$

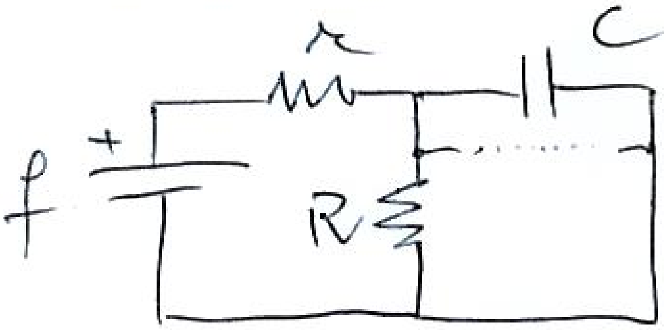
(3) $t=0$: interruttore aperto

$t<0$:



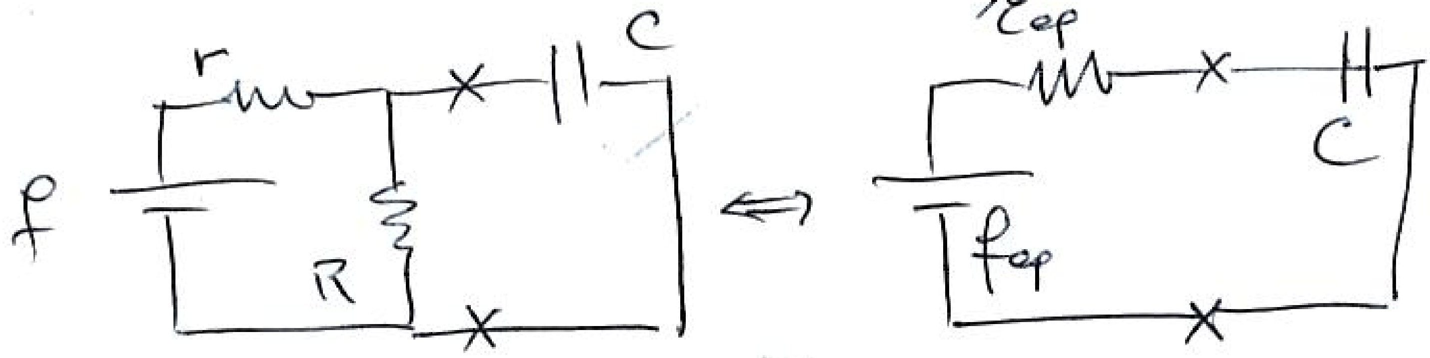
$\downarrow I_0 = \frac{f}{r} \Rightarrow f = I_0 \cdot r = 1\,V$

$t>0$ il condensatore inizia a caricarsi :



$$V_c(t>0) = V_0\left(1 - e^{-t/\tau}\right)$$

V_0 iniziale ai capi di C al momento di apertura dell'interruttore e la costante tempo τ si possono calcolare p.e. con Thevenin:



$$r_{eq} = \frac{r \cdot R}{r+R} \quad ; \quad f_{eq} = \frac{f\,R}{r+R} = 0{,}5\,V$$

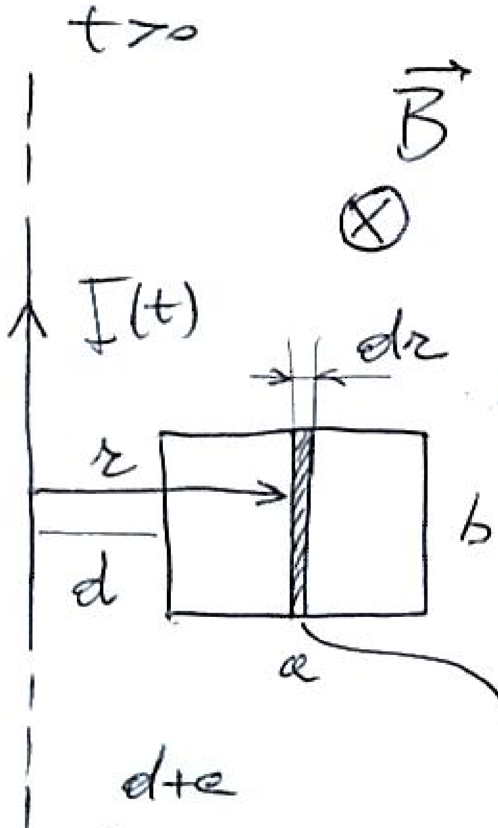
$$\Downarrow$$

$$V_0 = f_{eq}$$

$$\tau = r_{eq} \cdot C = \frac{rR}{r+R} \cdot C = 25\,\mu s$$

$$U_c = \frac{1}{2} C V_c^2$$

4

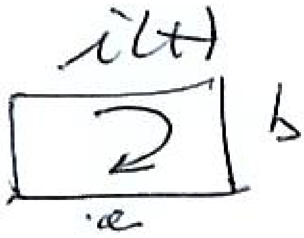


$$\Phi_B(t) = \int_{\square} \vec{B}\cdot d\vec{S} = \int_{\square} B\,dS \doteq$$

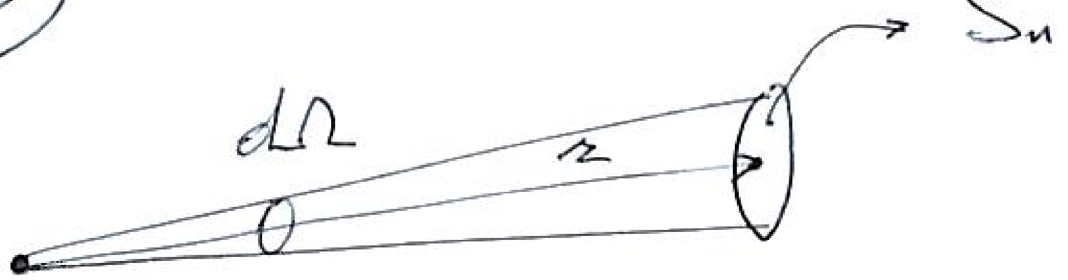
$$\doteq \int_d^{d+a} \frac{\mu_0 I(t)}{2\pi r}\cdot \overbrace{b\,dr}^{dS} = \frac{\mu_0 I(t)\, b}{2\pi}\int_d^{d+a}\frac{dr}{r} \doteq$$

$$\doteq \frac{\mu_0 b I_0 e^{-t/\tau}}{2\pi} \ln\left(\frac{d+a}{d}\right)$$

$$i(t) = -\frac{d\Phi/dt}{R} = +\frac{\mu_0 b I_0 e^{-t/\tau}}{2\pi R \tau}\ln\left(\frac{d+a}{d}\right)$$



5



$$I = \frac{E_{eff}^2}{Z_0} \quad ; \quad \Omega = \frac{S_n}{r^2}$$

$$I = \frac{W}{S_n} = \frac{W}{r^2 \Omega}$$

$$E_{eff} = \sqrt{\frac{Z_0 W}{\Omega}} \cdot \frac{1}{r} = \sqrt{\frac{377 \cdot 10^4}{10^{-2}}} \cdot \frac{1}{10^4} \left[\frac{V}{m}\right]$$

$$E_{eff} \simeq 1{,}9 \left[\frac{V}{m}\right]$$

$$B_{eff} \simeq \frac{E_{eff}}{c} = 0{,}64 \cdot 10^{-8} \, [T]$$