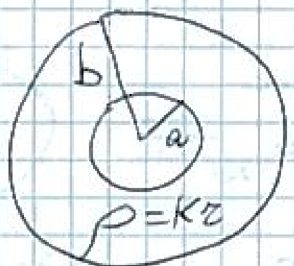
Soluzioni

(1)



Gauss: $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q_{int}(r)}{r^2} \hat{r}$

$r < a \quad \vec{E} = 0$

$a < r < b \quad Q_{int} = \int_a^r kr\, 4\pi r^2 dr = \pi k (r^4 - a^4)$

$$\vec{E}(r) = \frac{k(r^4 - a^4)}{4\varepsilon_0 r^2} \hat{r}$$

$r > b \quad Q_{int} = \int_a^b kr\, 4\pi r^2 dr = \pi k (b^2 - a^2)$

$$\vec{E}(r) = \frac{k(b^4 - a^4)}{4\varepsilon_0 r^2} \hat{r}$$

(2) Dal teorema di Ampere:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{conc}$$

$$\Rightarrow \quad B(r) = \frac{\mu_0}{2\pi r} I_{conc}$$

$r < c$ $\quad$ I è uniforme

$$I = J \pi c^2 \Rightarrow J = \frac{I}{\pi c^2}$$

$$I_{conc}(r) = J \cdot \pi r^2 = I \frac{r^2}{c^2}$$

$$\Rightarrow B(r) = \frac{\mu_0 I}{2\pi c^2} r$$

$c < r < b$

$$I_{conc}(r) = I \Rightarrow B(r) = \frac{\mu_0}{2\pi r} I$$

$b < r < a$ $\qquad J_{est} = \frac{I}{\pi (a^2 - b^2)}$

$$I_{conc}(r) = I - J_{est} \pi \cdot (r^2 - b^2) = I \frac{a^2 - r^2}{a^2 - b^2}$$

$$B(r) = \frac{\mu_0 I}{2\pi r} \cdot \frac{a^2 - r^2}{a^2 - b^2}$$

$r > a$

$$I_{conc}(r) = I - I = \emptyset$$

$$\Rightarrow B(r) = 0$$

③ $E_{in} = \frac{1}{2} C_1 V_0^2$

All'equilibrio finale $V_1 = V_2$

$$\Rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \quad e \quad Q_1 + Q_2 = Q_0$$

$$\begin{cases} Q_1 = \dfrac{C_1}{C_1 + C_2} Q_0 \\ Q_2 = \dfrac{C_2}{C_1 + C_2} Q_0 \end{cases}$$

$$E_{fin} = \frac{1}{2}\frac{Q_1^2}{C_1} + \frac{1}{2}\frac{Q_2^2}{C_2} = \frac{1}{2}\frac{Q_0^2}{C_1 + C_2} \Rightarrow$$

$$E_{fin} = \frac{1}{2}\frac{C_1^2 V_0^2}{C_1 + C_2}$$

$$E_{dis} = E_{in} - E_f = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2} V_0^2$$

è indipendente dal valore di R.

(4) $$q=\int_0^{\infty} i\,dt=\int_0^{\infty}-\frac{d\Phi/dt}{R}\,dt=$$

$$=-\frac{1}{R}\int_{\Phi_{in}}^{\Phi_{fin}} d\Phi=\frac{\Phi_{in}-\Phi_{fin}}{R}=\frac{\Phi_{in}}{R}$$

$$\begin{cases} R=\rho\dfrac{\ell}{S}=\rho\dfrac{2\pi r}{S} \\ \Phi_{in}=B\pi r^2 \end{cases} \Rightarrow q=\frac{B\pi r^2}{2\rho\pi r}S=\frac{BrS}{2\rho}=1{,}47\,C$$

(5) Legge di F-N-L;

$$\Phi(\vec{B})=L\int_0^{x=\lambda/2} B_0\cos(kx-\omega t)\,dx=\frac{LB_0}{k}\Big[\sin(kx-\omega t)\Big]_0^{\lambda/2}=$$

$$=\frac{LB_0}{k}\left[\sin\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}-\omega t\right)-\sin(-\omega t)\right]=$$

$$=\frac{2LB_0}{k}\sin\omega t=\frac{\lambda B_0}{k}\sin\omega t$$

$$f_i=-\frac{\lambda B_0}{k}\omega\cos\omega t=-\lambda B_0 c\cos(\omega t)$$

$$i_i=\frac{f_i}{R}=-i_{max}\cos(\omega t)$$

$$i_{max}=\frac{B_0 c\lambda}{R}=1\,mA.$$

$\square$