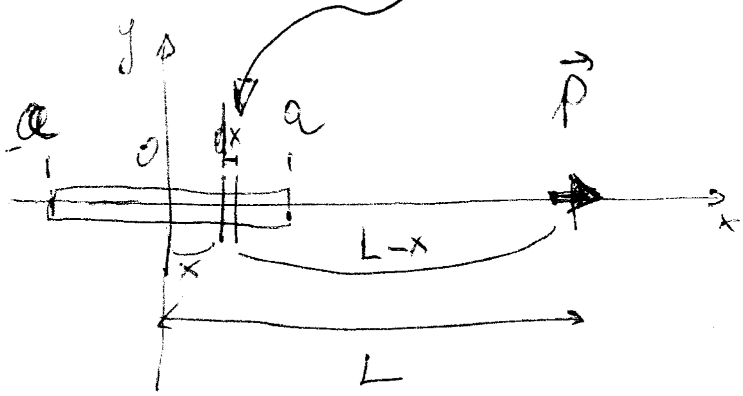
10/7/2018 p.1

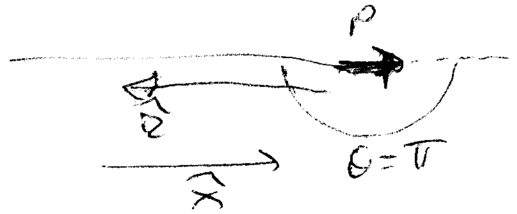
1)



$L > a$

$$\lambda = \frac{Q}{2a} \qquad dq = \lambda\, dx$$

Campo $\vec{E}$ di un dipolo alla distanza $(L-x)$:



$$E_r = \frac{2P\cos\theta}{4\pi\varepsilon(L-x)^3} = -\frac{2P}{4\pi\varepsilon(L-x)^3}$$

$$E_\theta = 0$$

$-\hat{r} = \hat{x}$ nel nostro caso

$$\vec{E} = \frac{2P}{4\pi\varepsilon(L-x)^3}\hat{x}$$

$$d\vec{F} = dq\,\vec{E} \qquad d\vec{F}(L-x) = \hat{x}\,\frac{\lambda\, 2P}{4\pi\varepsilon(L-x)^3}\,dx$$

$$\vec{F} = \int_{-a}^{a} d\vec{F} = \hat{x}\int_{-a}^{a}\frac{\lambda 2P}{4\pi\varepsilon}\frac{dx}{(L-x)^3} = \hat{x}\int_{a}^{-a}\frac{\lambda 2P}{4\pi\varepsilon}\frac{dx}{(x-L)^3}$$

$$\vec{F} = \hat{x}\,\frac{\lambda 2p}{4\pi\varepsilon}\int_{a}^{-a}\frac{dx}{(x-L)^3} \qquad \begin{aligned} x' &= x-L \\ dx' &= dx \end{aligned}$$

$$\downarrow$$

$$\int \frac{dx'}{x'^3} \rightarrow \int x'^{-3}\,dx' \rightarrow \frac{x'^{-3+1}}{-3+1} \rightarrow \frac{x'^{-2}}{-2}$$

$$\hookrightarrow \frac{-1}{2x'^2} \rightarrow -\frac{1}{2(x-L)^2}$$

$$= -\hat{x}\,\frac{\lambda \cancel{2} p}{4\pi\varepsilon}\,\frac{1}{\cancel{2}}\left[\frac{1}{(-a-L)^2}-\frac{1}{(a-L)^2}\right] =$$

$$= \hat{x}\,\frac{\lambda p}{4\pi\varepsilon}\left[\frac{1}{(L-a)^2}-\frac{1}{(a+L)^2}\right]$$

questa è la forza del dipolo sulla sbarra, la forza della sbarra sul dipolo è uguale e opposta

$$\vec{F}_{s\to d} = -\vec{F} = \hat{x}\,\frac{\lambda p}{4\pi\varepsilon}\left[\frac{1}{(L+a)^2}-\frac{1}{(L-a)^2}\right]$$

Campo $d\vec{E}$ ~~p~~rodotto dalla carica dq alla distanza $(L-x)$

$$d\vec{E} = \frac{dq}{4\pi\varepsilon(L-x)^2}\hat{x} = \frac{\lambda\, dx}{4\pi\varepsilon_0(L-x)^2}\hat{x}$$

Campo $\vec{E}$ prodotto da tutte le sbarrette sul dipolo distante L da (0)

$$\vec{E} = \int_{-e}^{e} d\vec{E} = \hat{x}\int_{-e}^{e}\frac{\lambda}{4\pi\varepsilon(L-x)^2}dx =$$

$x' = x-L$
$dx' = dx$

$$= \hat{x}\frac{\lambda}{4\pi\varepsilon}\int\frac{dx}{(x-L)^2} \rightarrow \int\frac{dx'}{x'^2}$$

$$\rightarrow \int x'^{-2}dx' \rightarrow \frac{x'^{-2+1}}{-2+1} = -\frac{1}{x'} = -\frac{1}{(x-L)}$$

$$= \hat{x}\frac{\lambda}{4\pi\varepsilon}\left[\frac{1}{(-e-L)} - \frac{1}{(e-L)}\right]_{0}^{e} = \hat{x}\frac{\lambda}{4\pi\varepsilon}\left[\frac{1}{L-e} - \frac{1}{L+e}\right]$$

Forza su un dipolo p in E della sbarra

$$\vec{F}_{s\to d} = p \frac{\partial E}{\partial x} \hat{x} = p \frac{\partial E}{\partial L} \hat{x}$$

$$\vec{F}_{s\to d} = \hat{x}\, p \frac{\partial}{\partial L} \frac{\lambda}{4\pi\varepsilon_0} \left(\frac{1}{L-e} - \frac{1}{L+e} \right) =$$

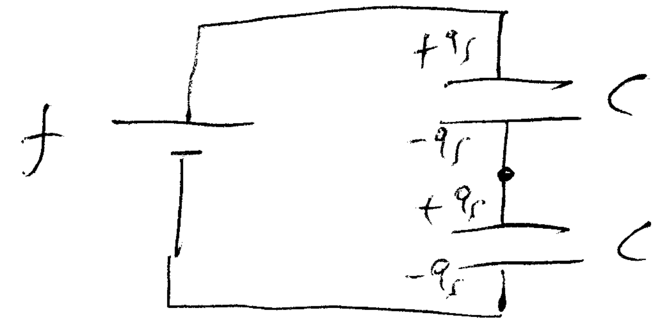
$$= \hat{x} \frac{p\lambda}{4\pi\varepsilon} \left(\frac{\partial}{\partial L} (L-e)^{-1} - \frac{\partial}{\partial L} (L+e)^{-1} \right) =$$

$$= \hat{x} \frac{p\lambda}{4\pi\varepsilon} \left(\frac{-1}{(L-e)^2} + \frac{1}{(L+e)^2} \right) =$$

$$= \hat{x} \frac{p\lambda}{4\pi\varepsilon} \left(\frac{1}{(L+e)^2} - \frac{1}{(L-e)^2} \right)$$

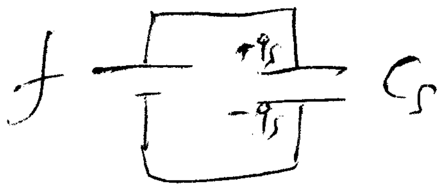
2) in condizioni di regime (all'equilibrio) non scorre corrente su C.

Caso (A)



Non scorre corrente i C sono carichi ed in serie

$\parallel$



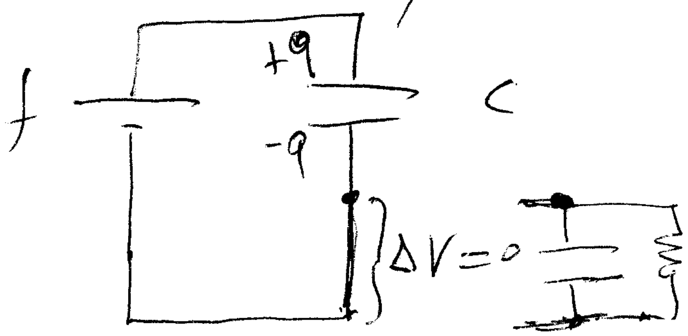
$$C_s = \frac{1}{\frac{1}{C}+\frac{1}{C}} = \frac{C}{2}$$

$$q_s = C_s f = \frac{C}{2} f$$

$$U_A = \frac{1}{2} C_s f^2 =$$

$$= \frac{1}{2}\frac{C}{2} f^2 = \frac{1}{4} C f^2$$

Caso (B) c è carico alla tensione f



Su R non scorre corrente percui

$$\Delta V_R = R \overset{0}{i} = 0$$

questo c è scarico

$$+q = Cf$$

$$U_B = \frac{1}{2} C f^2$$

il lavoro fatto dal generatore

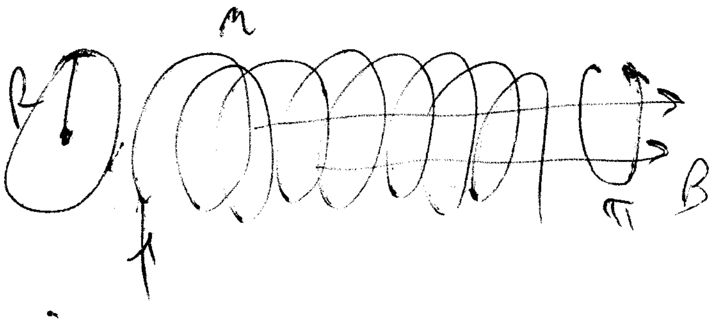
è $L_{gen} = f \cdot \Delta q = f(q - q_i)$

$$= f\left(Cf - \frac{C}{2} f\right) =$$

$$= \frac{1}{2} C f^2$$

ATTENZIONE

$$\Delta U = U_B - U_A = \frac{1}{2} Cf^2 - \frac{1}{4} Cf^2 = \frac{1}{4} Cf^2 = \frac{1}{2} L_{gen}$$

3) 

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$i(t) = i_0 \operatorname{sen}(\omega t)$

$i(t) = i_0 \operatorname{sen}(2\pi f t)$

$\omega = 2\pi f$

$$B(t) = \mu_0 n i(t) \rightarrow \mu_0 n i_0 \operatorname{sen}(\omega t)$$

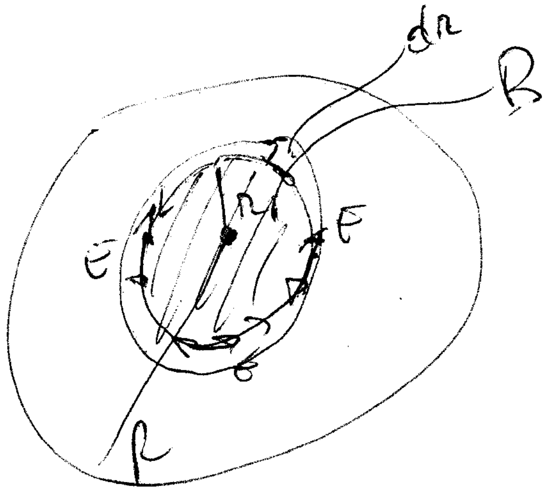
$$u_B = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\frac{\mu_0^{\cancel{2}} n^2 i_0^2 \operatorname{sen}^2(\omega t)}{\cancel{\mu_0}}$$

$$U_B = u_B \, Vol = u_B \pi R^2 L = \frac{1}{2}\pi R^2 L n^2 \mu_0 i_0^2 \operatorname{sen}^2(\omega t)$$

$$U_{B\,max} \rightarrow (\operatorname{sen}^2 = 1) \rightarrow \frac{1}{2}\pi R^2 L n^2 \mu_0 i_0^2$$

$$f_{em} = \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\Phi(B)$$

prendiamo una circonferenza di raggio r dentro al solenoide $r < R$



$$\oint \vec{E}\cdot d\vec{l} = 2\pi r E(r) =$$

$$= -\frac{d}{dt} B\pi r^2$$

$$2\cancel{\pi}\cancel{r}E(r) = -\cancel{\pi} r^{\cancel{2}} \mu_0 n i_0 \omega \cos(\omega t)$$

$$E(r) = -\frac{1}{2}\mu_0 n i_0 \omega r \cos(\omega t)$$

$$u_e = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \frac{1}{4}\mu_0^2 n^2 i_0^2 \omega^2 r^2 \cos^2(\omega t)$$

$$dU_e = u_e\, d\tau = u_e\, 2\pi r L\, dr = \frac{1}{\cancel{8}4}\varepsilon_0 \mu_0^2 n^2 i_0^2 \omega^2 \cancel{2}\pi r^3 L \cos^2(\omega t)\, dr$$

$$U_e = \int_0^R u_e\, d\tau = \int_0^R \frac{1}{4}\varepsilon_0 \mu_0^2 n^2 i_0^2 \omega^2 \pi L \cos^2(\omega t)\, r^3\, dr =$$

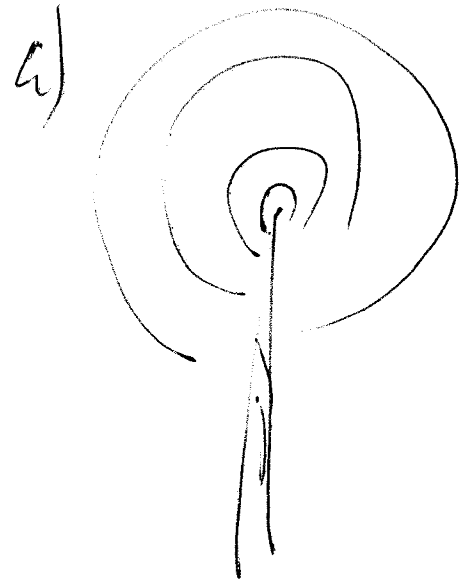
$$= \frac{\varepsilon_0 \mu_0^2 n^2 i_0^2 \omega^2 \pi L \cos^2(\omega t)}{4}\int_0^R r^3\, dr = \frac{\varepsilon_0 \mu_0^2 n^2 i_0^2 \omega^2 \pi L \cos^2(\omega t) R^4}{16}$$

$$U_{e\,max} = (\cos\omega t \to 1) = \frac{\varepsilon_0 \mu_0^2 n^2 i_0^2 \omega^2 \pi L R^4}{16}$$

$$\frac{U_{e\,max}}{U_{B\,max}} = \frac{\cancel{2}\,\varepsilon_0\,\mu_0^{\cancel{2}}\,\cancel{N}^{\cancel{2}}\,\cancel{I_0}^{\cancel{2}}\,\omega^2\,\cancel{\pi}\,\cancel{L}\,R^{\cancel{4}\,2}}{\cancel{16}_{\,8}\,\cancel{\pi}\,\cancel{R^2}\,\cancel{L}\,\cancel{N^2}\,\cancel{\mu_0}\,\cancel{I_0^2}} =$$

$$= \frac{1}{8}\,\varepsilon_0\,\mu_0\,\omega^2 R^2 = \frac{1}{8}\,\varepsilon_0\,\mu_0\,(2\pi f)^2 R^2$$

4)



$f = \nu = 1\cdot 10^6\,Hz$

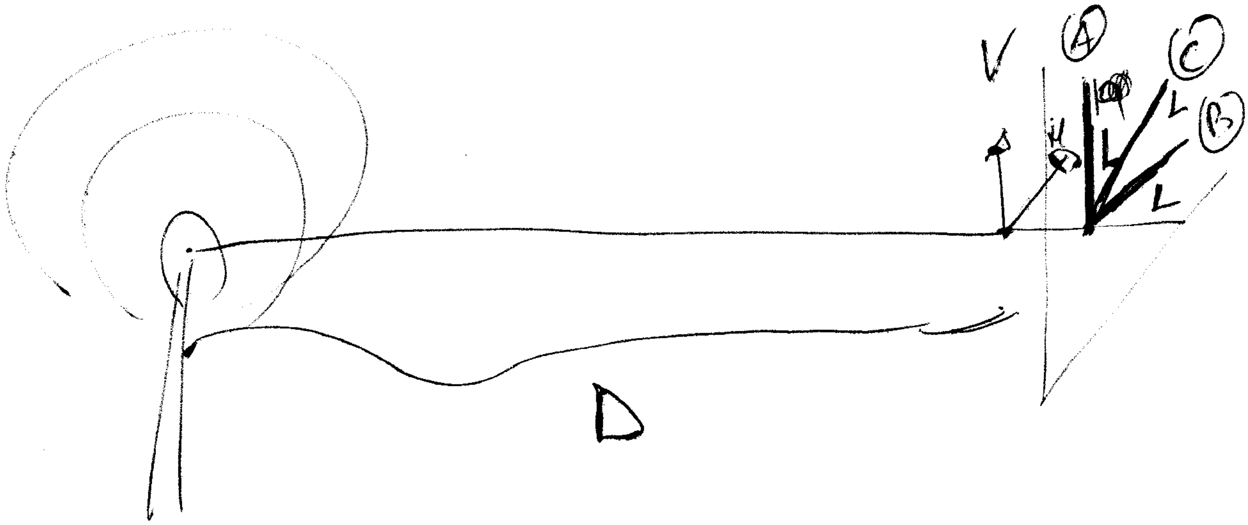
N. 10

$\lambda = \frac{c}{\lambda} = \frac{3\cdot 10^8\,m}{1\cdot 10^6} = 300\,m$

$\lambda >> L$

$P = 500\,W$

$$\boxed{\begin{aligned} P_V &= \frac{3}{4}P \\ P_H &= \frac{1}{4}P \end{aligned}}$$

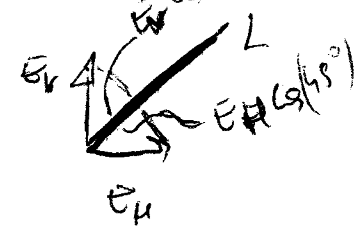


$I_V = \frac{P_V}{4\pi D^2}$ $\qquad I_H = \frac{P_H}{4\pi D^2}$

$E_V = \sqrt{\frac{2 I_V}{c\varepsilon_0}}$ $\qquad E_H = \sqrt{\frac{2 I_H}{c\varepsilon_0}}$ $\qquad \mathcal{E} = \int_0^L \vec{E}\cdot d\vec{l}$

$\mathcal{E}_V = E_V L$ (caso A) $\qquad \mathcal{E}_H = E_H L$ (caso B)

(caso C)



$$\mathcal{E}_{45°} = \left[E_V\cos(45°) + E_H\cos(45°)\right]L = \left(\frac{\sqrt{2}}{2}E_V + \frac{\sqrt{2}}{2}E_H\right)L$$