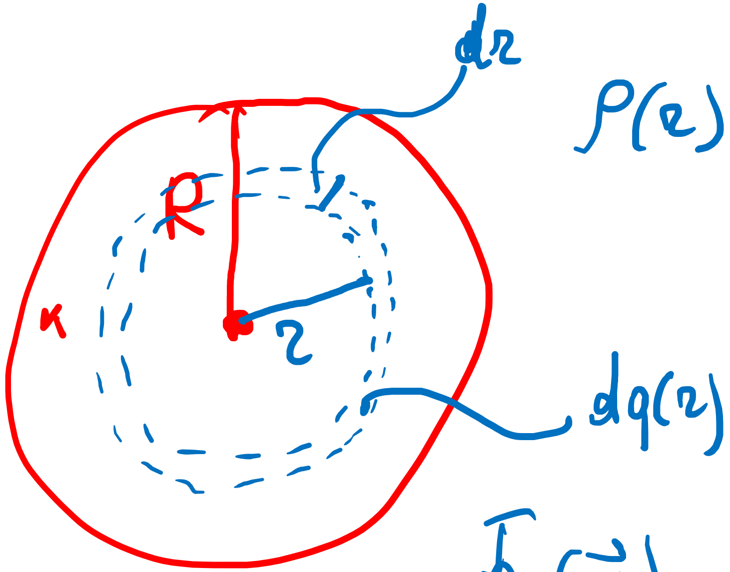
12/07/2023

1)



$$\rho(r) = A r^2 + B r^3 \qquad dq = \rho\, d\tau = \rho(r)\, 4\pi r^2\, dr$$

USO GAUSS SU SFERA DI RAGGIO r

per $0 < r < R$

$$\Phi(\vec{E}) = 4\pi E(r)\, r^2 = \frac{1}{\varepsilon_0}\int_0^r dq(r') = \frac{1}{\varepsilon_0}\int_0^r \rho(r')\, 4\pi r'^2\, dr' =$$

$$= \frac{1}{\varepsilon_0} 4\pi \left[\int_0^r A r'^4\, dr' + \int_0^r B r'^5\, dr'\right] = \frac{4\pi}{\varepsilon_0}\left[A\frac{r^5}{5} + \frac{B r^6}{6}\right]$$

da cui $\vec{E}(r) = \left(\frac{A}{\varepsilon_0}\frac{r^3}{5} + \frac{B}{\varepsilon_0}\frac{r^4}{6}\right)\hat{r}$ per $0 < r < R$

per $r > R$ $\quad \Phi(\vec{E}) = 4\pi E(r)\, r^2 = \frac{1}{\varepsilon_0}\int_0^R dq(r) = \frac{1}{\varepsilon_0}\int_0^R \rho(r)\, 4\pi r^2\, dr = \frac{4\pi}{\varepsilon_0}\left[\int_0^R A r^4\, dr + \int_0^R B r^5\, dr\right] =$

$$= \frac{4\pi}{\varepsilon_0}\left[A R^5/5 + B R^6/6\right] \longrightarrow$$

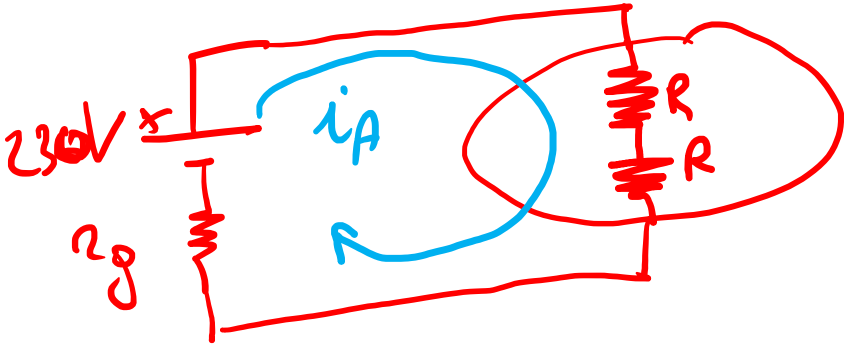
p.2

$$\vec{E}(r) = \hat{r}\left[\frac{1}{\varepsilon_0 r^2}\left(A\frac{R^5}{5} + B\frac{R^6}{6}\right)\right] \quad \text{pr} \quad r > R$$

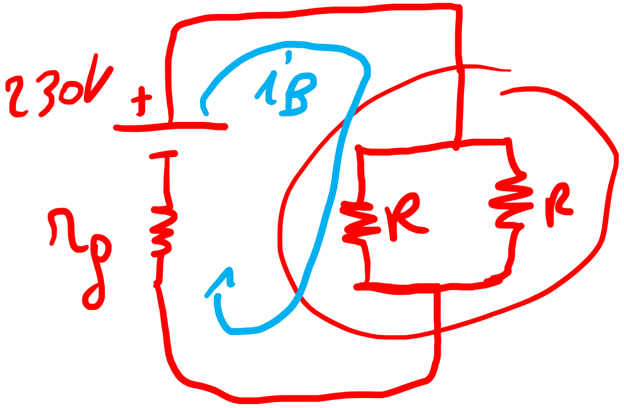
$$E(2R) = \frac{1}{\varepsilon_0 4 \cancel{R^2}}\left(A\frac{R^{\cancel{5}\,3}}{5} + B\frac{R^{\cancel{6}\,4}}{6}\right) = \frac{AR^3}{\varepsilon_0 20} + \frac{BR^4}{\varepsilon_0 24}$$

2)

(A) (B)



$R_S = 2R$



$R_P = \frac{R}{2}$

$$i_A = \frac{V}{r_g + 2R}$$

$$i_B = \frac{V}{r_g + \frac{R}{2}}$$

$$P_{(R_S)} = R_S\, i_A^2 = \frac{2R\,V^2}{(r_g + 2R)^2}$$

$$P_{(R_P)} = R_P\, i_B^2 = \frac{R}{2}\frac{V^2}{\left(r_g + \frac{R}{2}\right)^2}$$

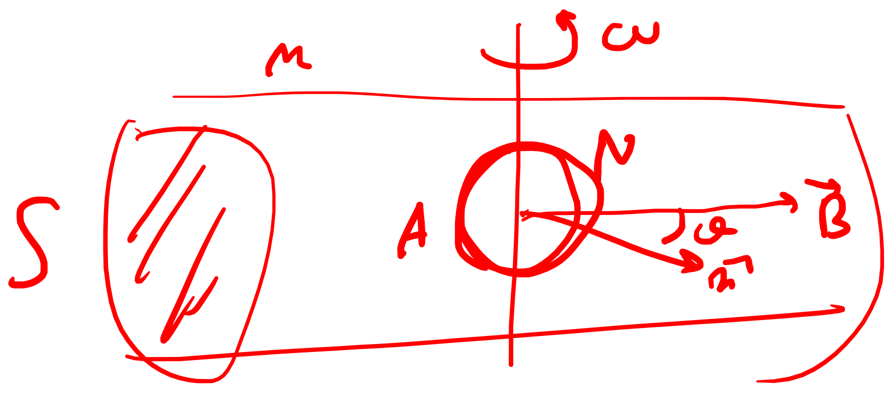
$$E_{m_S} = P_{R_S}\cdot \Delta t_S = E_{m_P} = P_{R_P}\cdot \Delta t_P \rightarrow \Delta t_P = \Delta t_S \cdot \frac{P_{R_S}}{P_{R_P}} = \Delta t_S \cdot \frac{\frac{2\cancel{R}\cancel{V^2}}{(r_g+2R)^2}}{\frac{\cancel{R}\cancel{V^2}}{2(r_g+R/2)^2}}$$

P. 4

$$\Delta t_p = \Delta t_s \cdot \frac{\frac{2}{(y+2R)^2}}{\frac{1}{2(y+R/2)^2}} = \frac{4\left(y+\frac{R}{2}\right)^2}{(y+2R)^2}$$

$$\Delta t_p = 5\,\text{min} \cdot \frac{4(75\cancel{R})^2}{(225\cancel{R})^2} = 5\,\text{min} \cdot \frac{4(3\cdot\cancel{5^2})^2}{(3^2\cdot\cancel{5^2})^2} = \frac{4\,\cancel{3^2}}{3^{\cancel{4}\,2}} \cdot 5\,\text{min} = \frac{4}{9}\cdot 5\,\text{min}$$

$$= \frac{20}{9}\,\text{min} = 2{,}22\,\text{min}$$

3) 

$\omega = \text{cost} \rightarrow \theta(t) = \theta_0 + \omega t$

PS

$M_{12} = M_{21} \rightarrow$ ipotizzo i nel solenoide

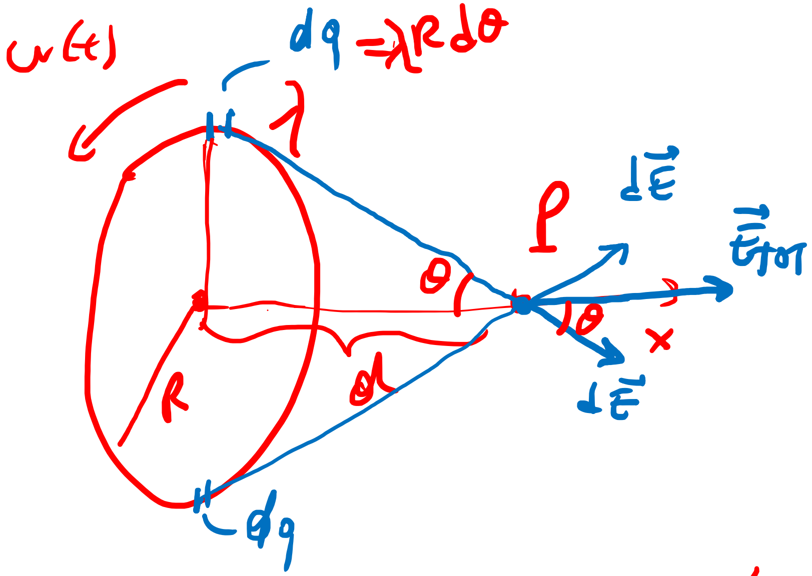
$B = \mu_0 n i$

$$\phi(B)_{su\ A} = \mu_0 n i A \cos\theta(t) \xrightarrow[N \text{ spire}]{} \phi(B)_N = \mu_0 N n i A \cos\theta(t)$$

$$M = \frac{\phi_A}{i} = \mu_0 N n A \cos[\theta(t)] = \mu_0 N n A \cos(\theta_0 + \omega t)$$

$$f_{em_S} = -\frac{d\phi_S}{dt} = -\frac{d}{dt} M I = -I \frac{dM}{dt} = +I \mu_0 N n A \omega \operatorname{sen}(\theta_0 + \omega t)$$

4)



$\omega(t) = At \qquad \alpha = \frac{d\omega}{dt} = A \qquad \theta(t) = \theta_0 + \frac{1}{2}At^2$

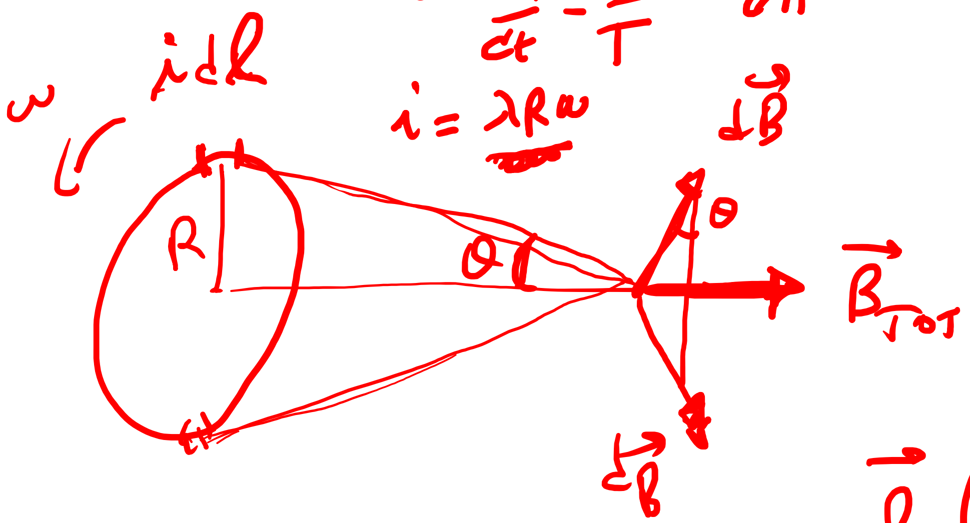
p.6

$$\vec{E}_{TOT} = \int_0^{2\pi} dE \cos\theta \, \hat{x} = \hat{x} \int_0^{2\pi} \frac{\lambda R \, d\theta \, d}{4\pi\varepsilon(R^2+d^2)^{3/2}} = \hat{x} \frac{\lambda R \cancel{2\pi} d}{2\cancel{4\pi}\varepsilon(R^2+d^2)^{3/2}}$$

$$\vec{E}_{TOT} = \hat{x} \frac{\lambda R \; d}{2\varepsilon(R^2+d^2)^{3/2}}$$

$$i = \frac{dq}{dt} = \frac{q}{T} = \frac{\lambda \cancel{2\pi} R\omega}{\cancel{2\pi}}$$

$$i = \lambda R \omega$$



$$d\vec{B} = \hat{x} \int_0^{2\pi R} \frac{\mu_0 \, i \, dl \, R}{4\pi(R^2+d^2)^{3/2}} = \hat{x} \frac{\mu_0 i \, 2\pi R^2}{4\pi(R^2+d^2)^{3/2}} =$$

$$\vec{B}_{TOT} = \hat{x} \, \mu_0 \frac{\lambda R \omega \cancel{2\pi} R^2}{2\cancel{4\pi}(R^2+d^2)^{3/2}} =$$

$$\vec{B}_{TOT}(t^*) = \hat{x} \frac{\mu_0 \lambda R^3 A t^*}{2(R^2+d^2)^{3/2}}$$