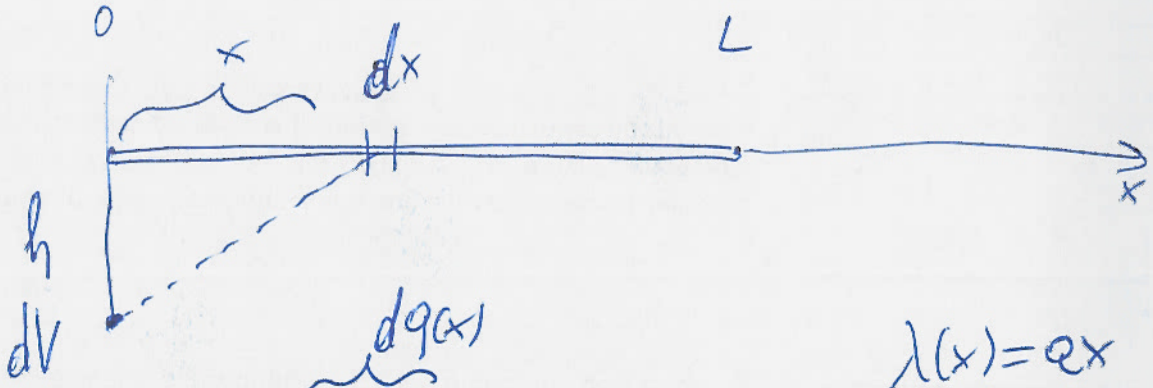
13/2/2019

P.1

1)



$$\lambda(x) = ax$$

$$dV(x) = \frac{\overbrace{\lambda(x)\,dx}^{dq(x)}}{4\pi\varepsilon\sqrt{x^2+h^2}}$$

$$V(h) = \int_0^L dV(x) = \int_0^L \frac{\lambda(x)\,dx}{4\pi\varepsilon\sqrt{x^2+h^2}} =$$

$$= \int_0^L \frac{a\,x\,dx}{4\pi\varepsilon\sqrt{x^2+h^2}} = \frac{a}{4\pi\varepsilon}\,\frac{1}{2}\int_0^L \frac{dx^2}{\sqrt{x^2+h^2}}$$

$$\left(\int \frac{dx^2}{\sqrt{x^2+h^2}} = \int \frac{dt}{\sqrt{t+h^2}} = \int \frac{dt'}{\sqrt{t'}} = \int t'^{-\frac{1}{2}}\,dt' = \right.$$

$$\left. = \frac{t'^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{t'^{\frac{1}{2}}}{\frac{1}{2}} = \frac{2\sqrt{t'}}{1} = 2\sqrt{t+h^2} = 2\sqrt{x^2+h^2}\right)$$

$$V(h) = \frac{a}{4\pi\varepsilon}\,\frac{1}{\cancel{2}}\left[\cancel{2}\sqrt{L^2+h^2} - \cancel{2}\sqrt{h^2}\right] = \frac{a}{4\pi\varepsilon}\left(\sqrt{L^2+h^2} - h\right)$$

P. 2

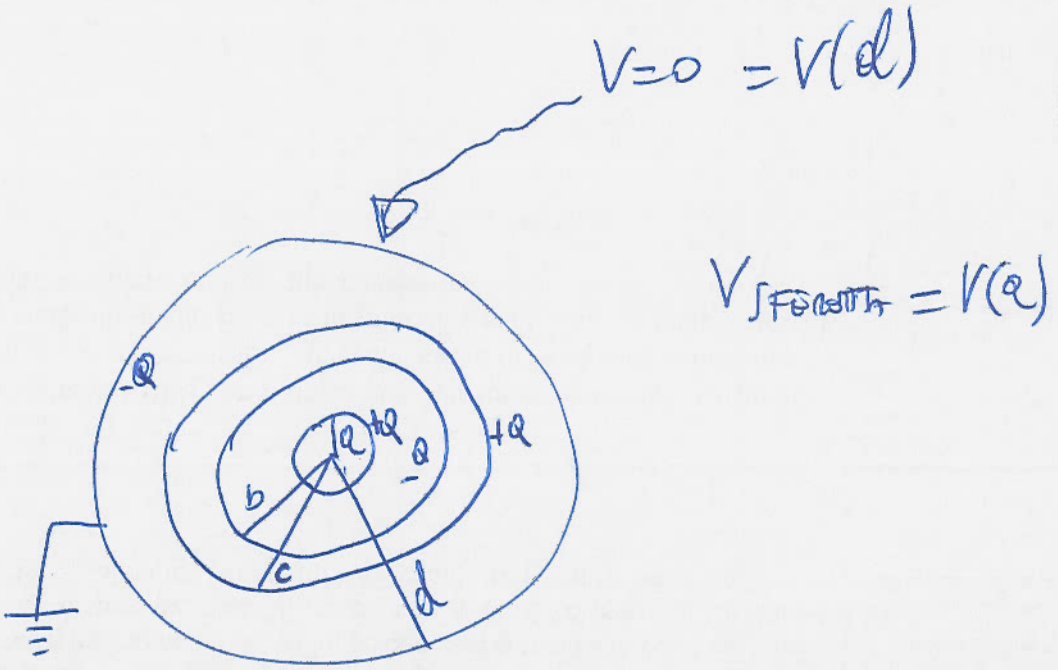
$$V(A) = V\left(\frac{L}{3}\right) = \frac{e}{4\pi\varepsilon}\left(\sqrt{L^2+\frac{L^2}{9}} - \frac{L}{3}\right)$$

$$V(B) = V\left(\frac{L}{2}\right) = \frac{e}{4\pi\varepsilon}\left(\sqrt{L^2+\frac{L^2}{4}} - \frac{L}{2}\right)$$

$$V(A) - V(B) = \frac{e}{4\pi\varepsilon}\left(\sqrt{\frac{9L^2+L^2}{9}} - \frac{L}{3} - \sqrt{\frac{4L^2+L^2}{4}} + \frac{L}{2}\right) =$$

$$= \frac{eL}{4\pi\varepsilon}\left(\sqrt{\frac{10}{9}} - \frac{1}{3} - \sqrt{\frac{5}{4}} + \frac{1}{2}\right)$$

2)



$$V(a) - V(d) = -\int_d^a E(r)\,dr$$

per $a < r < b$ $\quad \Phi(\vec{E}) = E(r)\,4\pi r^2 = \frac{Q}{\varepsilon}$

$$E(r) = \frac{Q}{4\pi\varepsilon}\frac{1}{r^2}$$

per $b < r < c$ $\quad \Phi(\vec{E}) = 0$

$$E(r) = 0$$

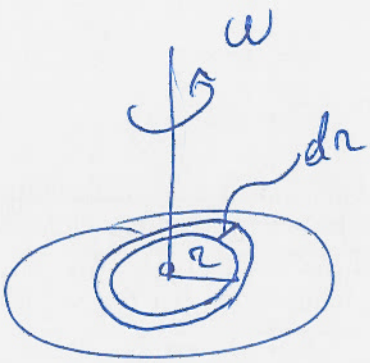
per $c < r < d$ $\quad \Phi(E) = E(r)\,4\pi r^2 = \frac{Q - Q + Q}{\varepsilon} = \frac{Q}{\varepsilon}$

$$E(r) = \frac{Q}{4\pi\varepsilon\, r^2}$$

$$\overset{\Delta V}{\overset{\shortparallel}{V(a) - V(d)}} = V(a) = -\int_d^c \frac{Q}{4\pi\varepsilon}\frac{dr}{r^2} - \int_b^a \frac{Q}{4\pi\varepsilon}\frac{dr}{r^2}$$

$$= \frac{Q}{4\pi\varepsilon}\left[\frac{1}{c} - \frac{1}{d} + \frac{1}{a} - \frac{1}{b}\right] \quad U = \frac{1}{2}Q\Delta V$$

3)



disco

$dq =$ carica su $2\pi r\,dr$

$$dq(r) = \sigma(r)\, 2\pi r\, dr = a\, r^2\, 2\pi r\, dr =$$

$$dq(r) = a\, 2\pi\, r^3\, dr$$

$$di = \frac{dq}{dt} = \frac{dq}{T} \qquad \text{con } T = \frac{2\pi}{\omega}$$

$$di = \omega \frac{a\, 2\pi\, r^3\, dr}{2\pi} = \omega a r^3 dr$$

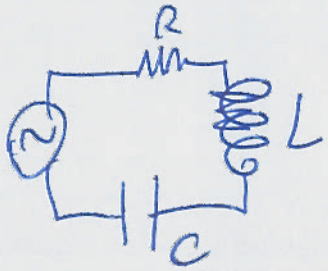
$$dB_{disco}(r) = \frac{\mu_0\, di}{2r} = \frac{\mu_0 \omega a\, r^{\cancel{3}\,2}\, dr}{2\cancel{r}} = \frac{\mu_0 \omega a\, r^2 dr}{2}$$

$$B_{disco} = \int_0^R dB_{disco} = \frac{\mu_0 \omega a}{2} \int_0^R r^2 dr = \frac{\mu_0 \omega a}{2} \cdot \frac{R^3}{3}$$

$$B_{solenoide} = -B_{disco} = \mu_0 n i$$

$$i = -\frac{B_{disco}}{\mu_0 n} = -\frac{\cancel{\mu_0} \omega a}{2\cancel{\mu_0} n} \cdot \frac{R^3}{3} = -\frac{\omega a}{2n} \frac{R^3}{3}$$

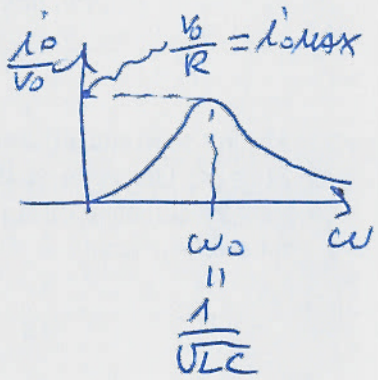
4)



$$V(t) = V_0 \cos(\omega t)$$

$$V_0 = Z i_0 \qquad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$i_0(\omega) = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



$$P_m = \frac{1}{2} V_0 i_0 \cos\phi =$$

$$= \frac{1}{2} V_0 \underbrace{i_0 \frac{R}{Z}}_{= i_0} = \frac{1}{2} i_0^2 R$$

$$P_m(\omega) = \frac{1}{2} R\, i_0^2(\omega)$$

$$P_{m\,MAX} = \frac{1}{2} R\, i_{0MAX}^2$$

i_{0MAX} tale che

$$\omega L - \frac{1}{\omega C} = 0$$

$$\Downarrow$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}} = \omega_0$$

$$\boxed{i_{0MAX} = \frac{V_0}{R}}$$

$$\frac{1}{2}P_{m\,max} = P_m(\omega^*) = \frac{1}{2}\,\frac{1}{2}R\,i_{0\,max}^2 =$$

$$= \frac{1}{2}\,\frac{1}{2}\,\cancel{R}\,\frac{V_0^2}{R^{\cancel{2}}} = \frac{V_0^2}{4R}$$

$$P_m(\omega^*) = \frac{V_0^2}{4R} = \frac{1}{2}R\,\underbrace{\frac{V_0^2}{\left[R^2+\left(\omega^* L - \frac{1}{\omega^* C}\right)^2\right]}}_{=i_0^2(\omega^*)}$$

$$\Downarrow$$

$$\left[R^2 + \left(\omega^* L - \frac{1}{\omega^* C}\right)^2\right] = 2R^2$$

$$\left(\omega^* L - \frac{1}{\omega^* C}\right)^2 = R^2 \quad ; \quad \omega^* L - \frac{1}{\omega^* C} = \pm R$$

$$\omega^{*2} LC \mp \omega^* RC - 1 = 0$$

$$\omega^* = \frac{\pm RC \cancel{\pm} \sqrt{R^2C^2 + 4LC}}{2LC} > 0$$

$$\omega_1 = \frac{+RC + \sqrt{R^2C^2+4LC}}{2LC} \qquad \omega_2 = \frac{-RC + \sqrt{R^2C^2+4LC}}{2LC}$$

$$\Delta\omega = \omega_1 - \omega_2 = \cancel{2}\,\frac{R\cancel{C}}{\cancel{2}L\cancel{C}} = \frac{R}{L}$$