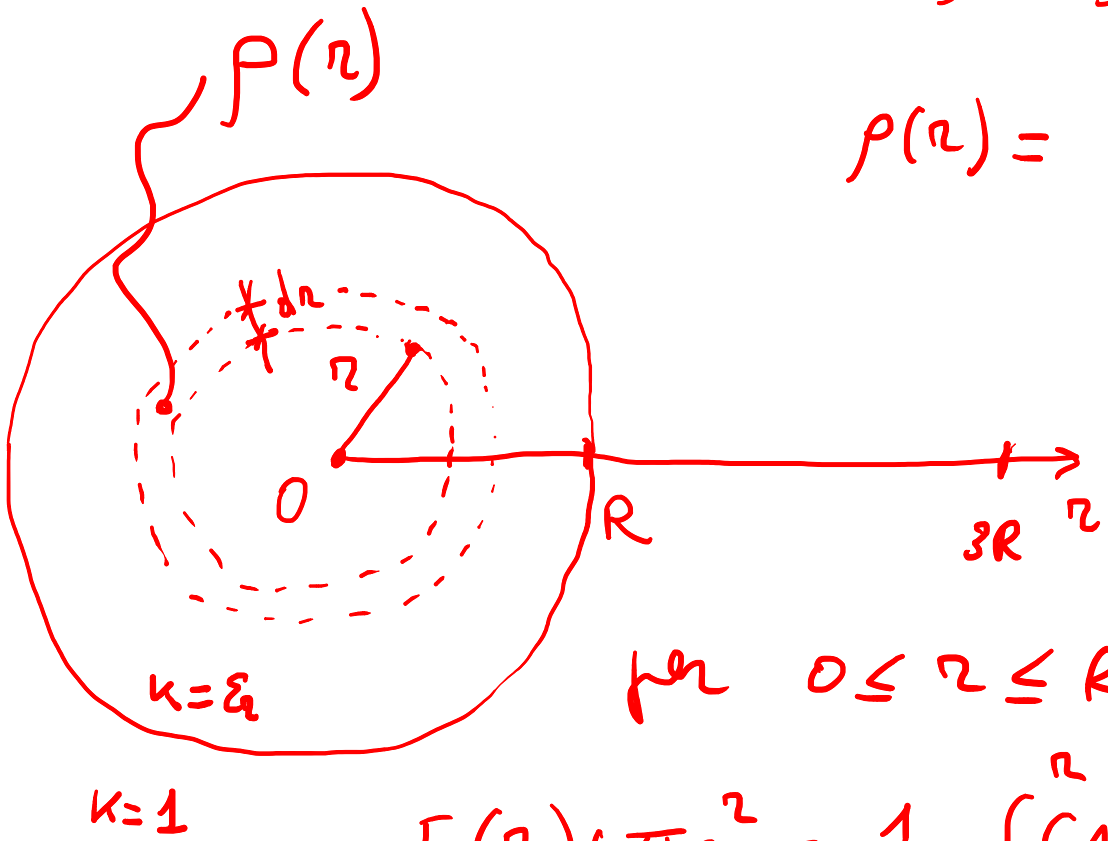
13-09-2022

P.1

1)



$$\rho(r) = \begin{cases} Ar + Br^2 & \text{per } r \le R \\ \emptyset & \text{per } r > R \end{cases}$$

USO GAUSS

$$\phi(\vec{E}(r)) = \frac{q^{INT}(r)}{\varepsilon_0 k(r)}$$

$$\downarrow$$

per $0 \le r \le R$

$$E(r)4\pi r^2 = \frac{1}{\varepsilon_0 k}\int_0^r \rho(r')4\pi r'^2 dr'$$

$$E(r)4\pi r^2 = \frac{1}{\varepsilon_0 k}\int_0^r (Ar' + Br'^2)4\pi r'^2 dr' =$$

$$= \frac{1}{\varepsilon_0 k}4\pi\left[\int_0^r Ar'^3 dr' + \int_0^r Br'^4 dr'\right] =$$

$$= \frac{4\pi}{\varepsilon_0 k}\left[A\frac{r^4}{4} + B\frac{r^5}{5}\right] \rightarrow E(r) = \frac{Ar^2}{4\varepsilon_0 k} + \frac{Br^3}{5\varepsilon_0 k}$$

IL CAMPO È RADIALE PER SIMMETRIA

$\vec{E}(r) = E(r)\hat{r}$ per $\forall r$

per $r > R$ $\qquad E(r)4\pi r^2 = \dfrac{q^{INT}(R)}{\varepsilon_0}$

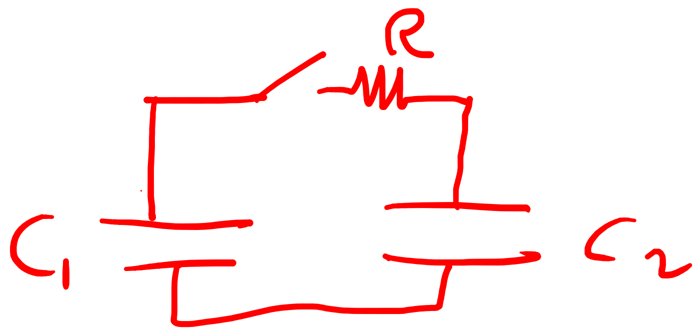
$$E(r)\cancel{4\pi} r^2 = \frac{\cancel{4\pi}}{\varepsilon_0}\left[\int_0^R A r'^3 dr' + \int_0^R B r'^4 dr'\right]$$

$$= \frac{\cancel{4\pi}}{\varepsilon_0}\left[\frac{AR^4}{4} + \frac{BR^5}{5}\right]$$

$$\rightarrow E(r) = \frac{1}{\cancel{4\pi}\varepsilon_0}\left[\frac{AR^4}{4r^2} + \frac{BR^5}{5r^2}\right] = \left[\frac{AR^4}{4} + \frac{BR^5}{5}\right]\frac{1}{\varepsilon_0 r^2}$$

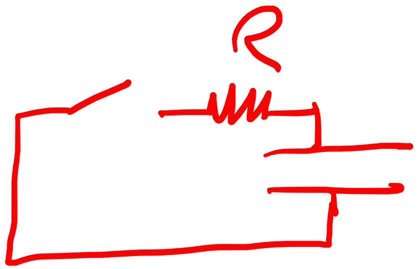
$$E(3R) = \left[\frac{AR^4}{4} + \frac{BR^5}{5}\right]\frac{1}{\varepsilon_0 9R^2} = \frac{AR^2}{36\varepsilon_0} + \frac{BR^3}{45\varepsilon_0}$$

2)



CARICO CON CARICA Q   SCARICO

$\equiv$

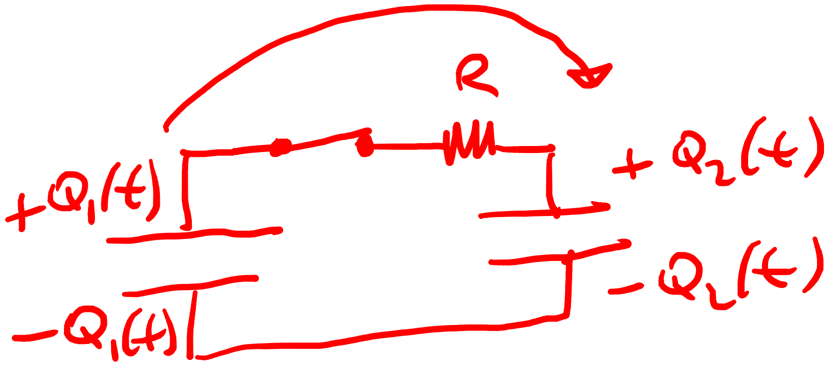


$$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{C_1 C_2}{C_1+C_2}$$

SERIE

SICCOME $C_1 = C_2$ $\quad C_{eq} = \frac{C^{\cancel{2}}}{2\cancel{C}} = \frac{C}{2}$ $\quad \tau = R\, C_{eq} = \frac{RC}{2}$

↑ NON SERVE ↑



$$\begin{cases} Q_1(t) + Q_2(t) = Q = Q_{INIZIALE} \\ Q_1(t=0) = Q \qquad Q_2(t=0) = \emptyset \\ Q_1(t=t^*) = \frac{3}{4}Q \qquad Q_2(t=t^*) = \frac{1}{4}Q \end{cases}$$

EQ. DIFF. $\quad V_{C_1}(t) - V_{C_2}(t) = R\, i(t) \quad$ CON $\quad i(t) = -\frac{dQ_1(t)}{dt}$

$$\frac{Q_1(t)}{C} - \frac{Q_2(t)}{C} = R\left(-\frac{dQ_1(t)}{dt}\right)$$

M1 $Q_2(t) = Q - Q_1(t)$

$$\frac{Q_1(t)}{C} - \frac{Q - Q_1(t)}{C} = -R\frac{dQ_1(t)}{dt}$$

$$\frac{Q_1(t)}{\cancel{C}} + \frac{Q_1(t)}{\cancel{C}} - \frac{Q}{\cancel{C}} = -RC\frac{dQ_1(t)}{dt}$$

$$Q_1(t) - \frac{Q}{2} = -\frac{RC}{2}\frac{dQ_1(t)}{dt}$$

$$Q^*(t) = Q_1(t) - Q/2$$

$$dQ^*(t) = dQ_1(t)$$

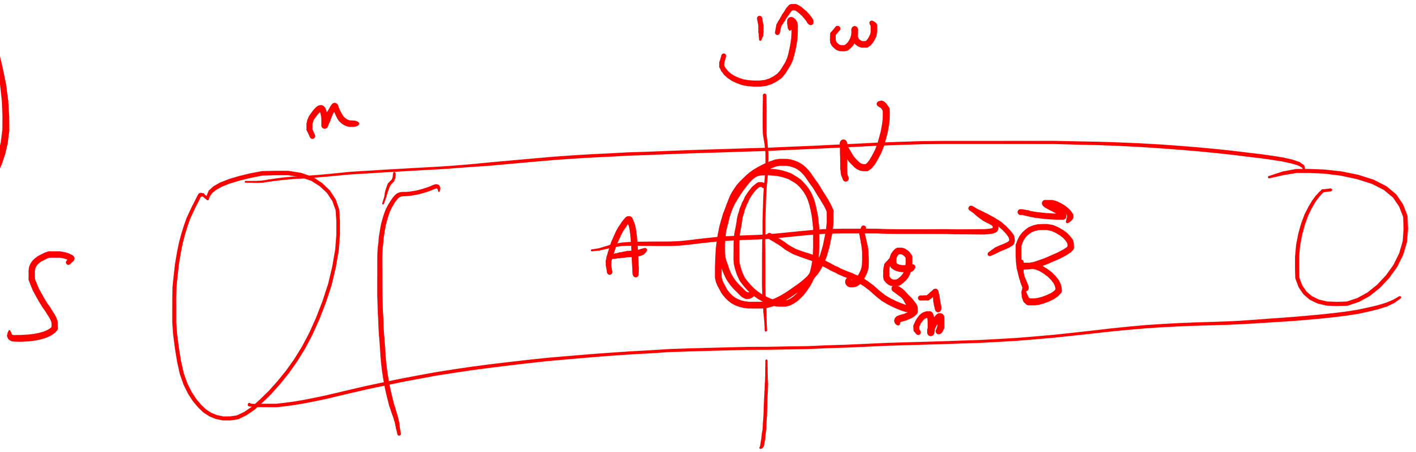
$$-\frac{2}{RC}\int_0^{t^*} dt = \int_{Q_1=Q}^{Q_1=\frac{3}{4}Q} \frac{dQ_1(t)}{Q_1(t) - Q/2} = \int \frac{dQ^*(t)}{Q^*(t)} = \ln\frac{Q^*_{FIN}}{Q^*_{IN}} = \ln\frac{Q_1(FIN) - Q/2}{Q_1(IN) - Q/2}$$

$$-\frac{2}{RC}t^* = \ln\left(\frac{\frac{3}{4}Q - Q/2}{Q - Q/2}\right) = \ln\left(\frac{\frac{3}{4}Q - \frac{2}{4}Q}{\frac{2}{2}Q - \frac{1}{2}Q}\right) = \ln\frac{\frac{1}{4}}{\frac{1}{2}} = \ln\frac{1}{2}$$

$$-\frac{2}{RC}t^* = \ln\frac{1}{2} = \ln 2^{-1} = -\ln 2$$

$$t^* = \frac{RC}{2}\ln 2$$

3)



$\theta(t) = \omega t$

Se suppongo che sul solenoide scorra $i \Rightarrow B = \mu_0 n i$

$$\phi(\vec{B})_{\text{su N spire}} = N B A \cos\theta(t) = N B A \cos(\omega t)$$

$$M = \frac{\phi(\vec{B})_{\text{bobina}}}{i} = \frac{NBA\cos(\omega t)}{i} = \frac{N\mu_0 n \cancel{i} A \cos\omega t}{\cancel{i}} = N\mu_0 n A \cos(\omega t) = M(t)$$

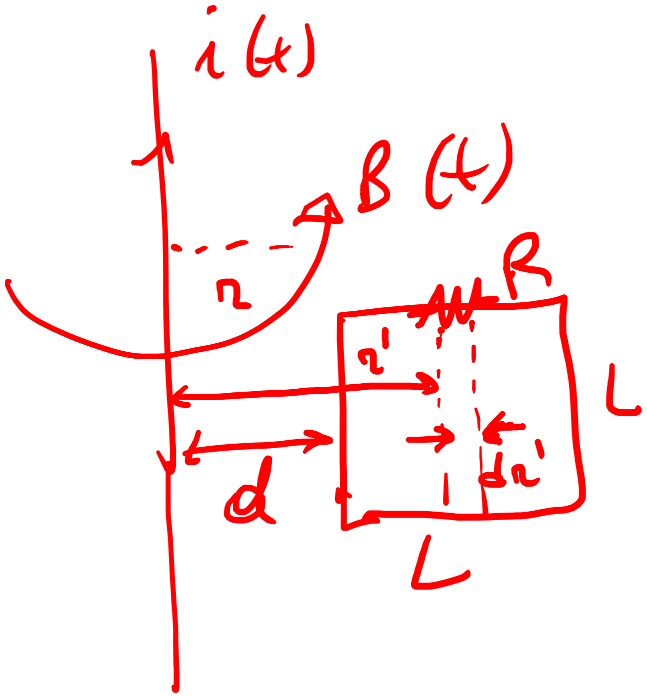
$$M = \frac{\phi_{\text{bobina}}}{i} = \frac{\phi_{\text{solenoide}}}{I} = N\mu_0 A n \cos(\omega t) \rightarrow$$

P.7

$$\phi_{solenoide} = M(t) I = \mu_0 N A n I \cos(\omega t)$$

$$\text{fem solenoide} = -\frac{d}{dt}\phi_{solenoide} = +\mu_0 N A n I \omega \,\text{sen}(\omega t)$$

4)



$$B(t) = \frac{\mu_0 i(t)}{2\pi r} = \frac{\mu_0 (i_0 + \alpha t)}{2\pi r}$$

P.8

$$d\phi(B) = L\, B(t)\, dr = \frac{\mu_0 L (i_0 + \alpha t)}{2\pi r} dr$$

$$\phi(B) = \int_d^{d+L} d\phi(B) = \frac{\mu_0 L (i_0 + \alpha t)}{2\pi} \int_d^{d+L} \frac{dr}{r} =$$

$$\phi(\vec{B}) = \frac{\mu_0 L (i_0 + \alpha t)}{2\pi} \ln\left(\frac{d+L}{d}\right)$$

$$f_{em} = -\frac{d}{dt}\phi(B) = -\frac{\mu_0 L}{2\pi} \ln\left(\frac{d+L}{d}\right) \alpha$$

$$i_{INDOTTO} = \frac{f_{em}}{R} = -\frac{\mu_0 L \alpha}{2\pi R} \ln\left(\frac{d+L}{d}\right)$$