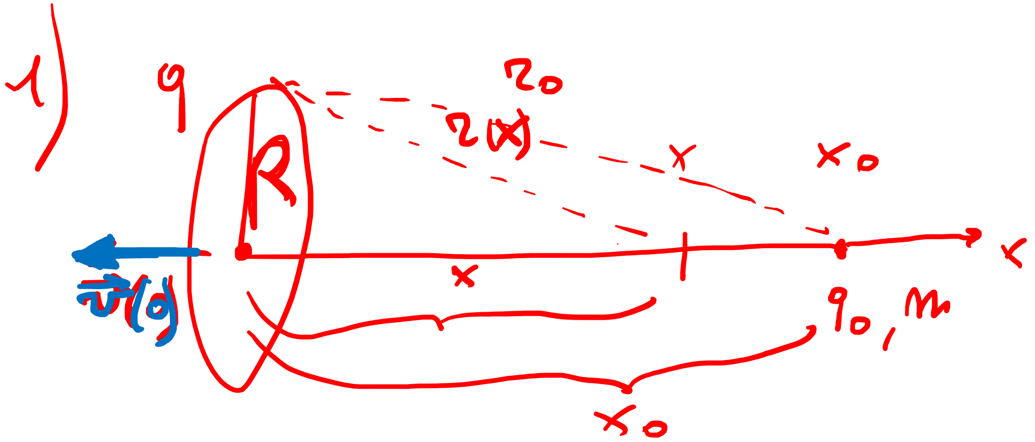
14/02/2023

1)



$$dq = \lambda \, dl$$

$$\lambda = \frac{q}{2\pi R}$$

$$V(x) = \int_0^{2\pi R} \frac{\lambda \, dl}{4\pi\varepsilon_0 \sqrt{R^2+x^2}}$$

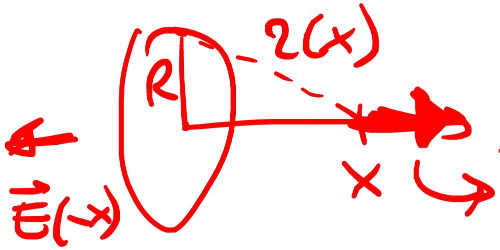
$$r(x) = \sqrt{R^2+x^2}$$

$$V(x) = \frac{q}{4\pi\varepsilon_0 \sqrt{R^2+x^2}} \qquad V(0) = \frac{q}{4\pi\varepsilon R} \qquad V(x_0) = \frac{q}{4\pi\varepsilon \sqrt{R^2+x_0^2}}$$

$$E_k(0) - \underbrace{E_k(x_0)}_{=0} = -q_0\left[V(0) - V(x_0)\right] = -\frac{\overbrace{q_0 q}^{<0}}{4\pi\varepsilon}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+x_0^2}}\right] > 0$$

$$v(0) = \sqrt{\frac{2E_k(0)}{m}} = \sqrt{\frac{-2q_0 q}{m 4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+x_0^2}}\right]}$$

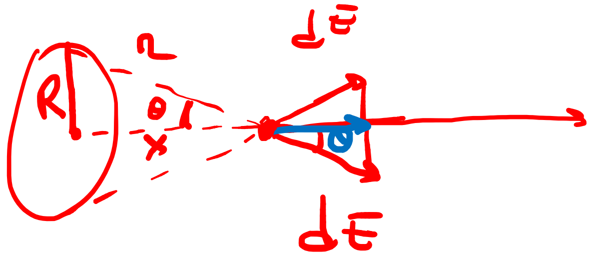
P.2



$r(x) = \sqrt{R^2 + x^2}$ $\qquad r^2(x) = R^2 + x^2$

$\hookrightarrow \vec{E}(x) = \hat{x}\, E_x(x) \longrightarrow$ facciamo i conti con $x \geq 0$

$E(-x) = -E(x)$



$$dE_x(x) = dE\cos\theta = dE \frac{x}{\sqrt{R^2+x^2}}$$

$$dE = \frac{dq}{4\pi\varepsilon\, r^2} = \frac{\lambda\, dl\; x}{4\pi\varepsilon(R^2+x^2)}$$

(per $x \geq 0$)

$$\vec{E}(x) = \hat{x}\frac{q\, x}{4\pi\varepsilon(R^2+x^2)^{3/2}}$$

$$\underline{\vec{F}(x)} = q_0\vec{E}(x) = \hat{x}\frac{\overbrace{q_0}^{<0} q\, x}{4\pi\varepsilon(R^2+x^2)^{3/2}}$$

(DIREZIONE $-\hat{x}$)

$$|F_{MAX}| \rightarrow \frac{\partial F(x)}{\partial x} = 0 \quad \ldots \quad \cancel{\frac{q_0 q}{4\pi\varepsilon}} \frac{\partial}{\partial x}\left[\frac{x}{(R^2+x^2)^{3/2}}\right] = 0$$

(con $x \geq 0$)

$$|F_{MAX}| \rightarrow \frac{dF(x)}{dx} = 0 \quad \ldots \quad \cancel{\frac{qQ}{4\pi\varepsilon}} \frac{d}{dx}\left[\frac{x}{(R^2+x^2)^{3/2}}\right] = 0$$

$$\frac{1}{(R^2+x^2)^{3/2}} + x\left(-\frac{3}{2}\right)(R^2+x^2)^{-5/2}(2x) = 0$$

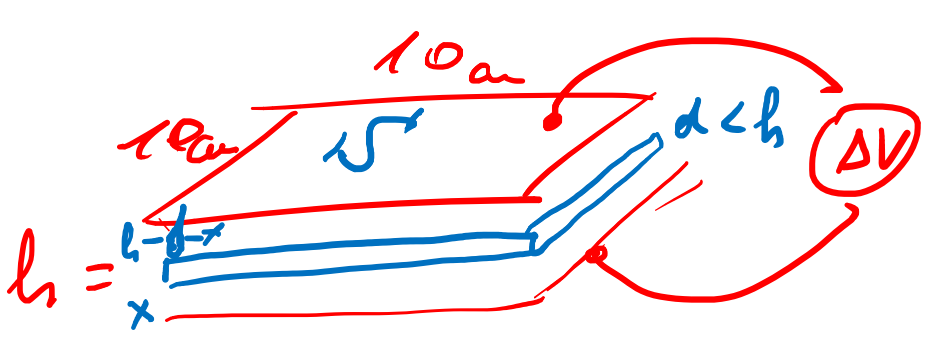
$$\frac{+3x^2}{(R^2+x^2)^{\cancel{5/2}\,2}} = \frac{1}{\cancel{(R^2+x^2)^{3/2}}} \rightarrow \frac{3x^2}{(R^2+x^2)} = 1$$

$$3x^2 = R^2 + x^2 \rightarrow 2x^2 = R^2 \rightarrow x^2 = \frac{R^2}{2} \rightarrow x = +\frac{R}{\sqrt{2}}$$

e il simmetrico

$$x = -\frac{R}{\sqrt{2}}$$

2)



$\Delta V = 24\,V$

$$C_x = \varepsilon \frac{S}{x} \qquad C_{h-d-x} = \varepsilon \frac{S}{h-d-x}$$

$$C_0 = \varepsilon \frac{S}{h} \qquad C = \frac{1}{\frac{1}{C_x} + \frac{1}{C_{h-d-x}}} = \frac{1}{\frac{x}{\varepsilon_0 S} + \frac{h-d-x}{\varepsilon S}} =$$

$$= \frac{1}{\frac{\cancel{x} + h - d - \cancel{x}}{\varepsilon S}} = \frac{\varepsilon_0 S}{h-d}$$

$$U_0 = \frac{1}{2} C_0 \Delta V^2 = \frac{1}{2} \frac{\varepsilon S}{h} \Delta V^2 \qquad U = \frac{1}{2} C \Delta V^2 = \frac{1}{2} \frac{\varepsilon S}{h-d} \Delta V^2$$

P.5

$$C_0 = \frac{q_0}{\Delta V} \qquad C = \frac{q}{\Delta V} \rightarrow q_0 = C_0 \Delta V \qquad q = C\Delta V$$

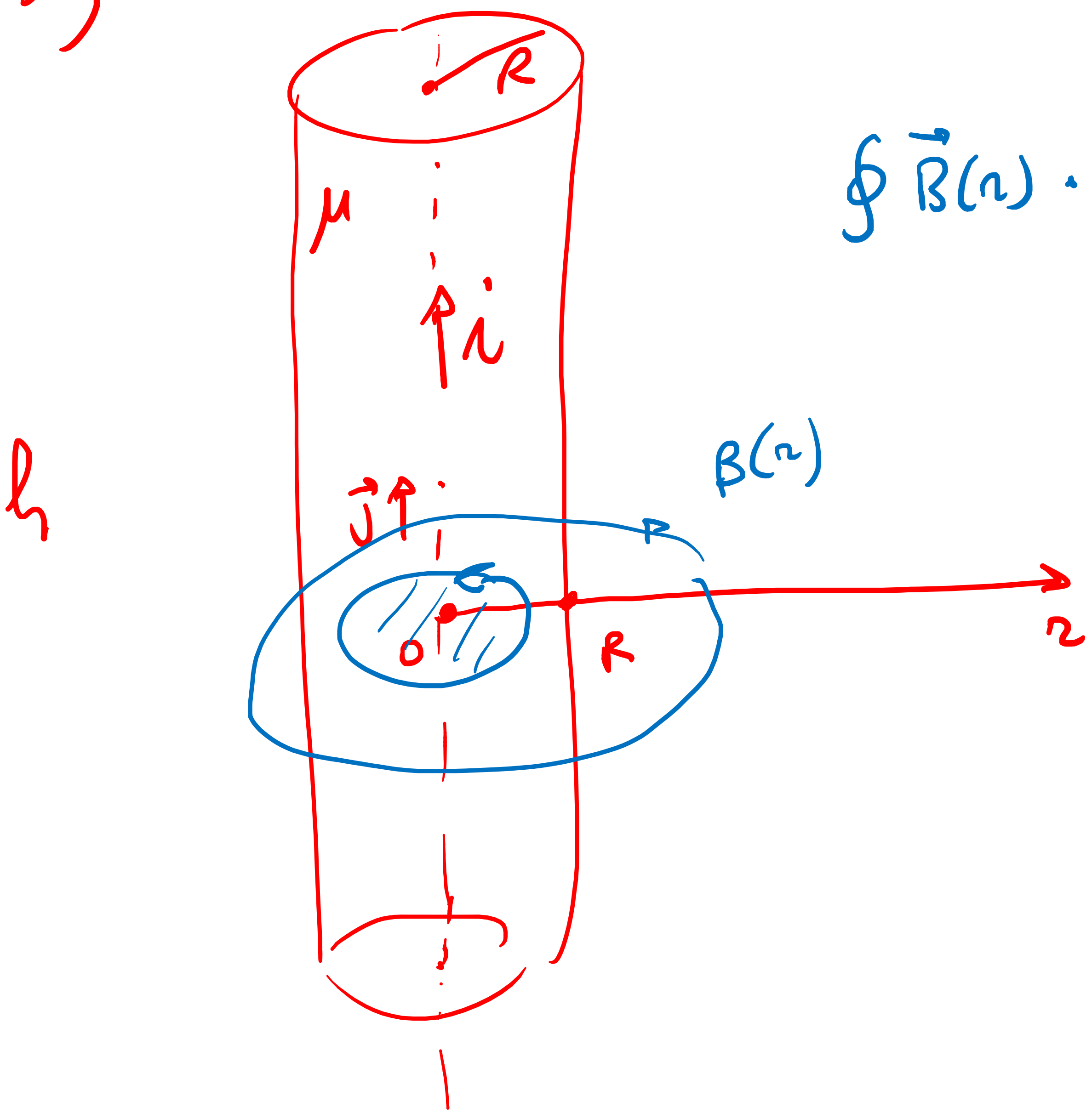
$$\mathcal{L}_{gen} = \Delta q \cdot \Delta V = (q - q_0)\Delta V = (C\Delta V - C_0\Delta V)\Delta V =$$

$$= (C - C_0)\Delta V^2 = \left(\frac{\varepsilon S}{h-d} - \frac{\varepsilon S}{h}\right)\Delta V^2$$

$$\mathcal{L}_{gen} = 2 \cdot \Delta U$$

3)

$h \gg R$



$$j = \frac{i}{\pi R^2}$$

$$\oint \vec{B}(r) \cdot d\vec{\ell} = B(r)\, 2\pi r = \mu(r)\, i_{conc}$$

$\downarrow$

$0 < r < R$

$$B(r)\, 2\pi \cancel{r} = \mu \frac{i}{\cancel{\pi} R^2} \cancel{\pi} r^{\cancel{2}}$$

$$B(r) = \frac{\mu i}{2\pi R^2}\, r$$

$r > R$

$$B(r)\, 2\pi r = \mu_0 i$$

$$B(r) = \frac{\mu_0 i}{2\pi r}$$

Nel conduttore $0 < r < R$ $\qquad B(r) = \dfrac{\mu i}{2\pi R^2}\, r$

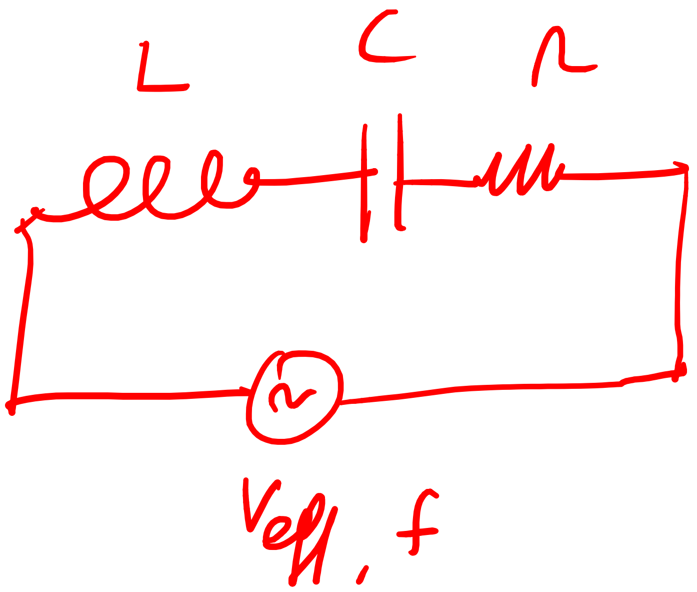
$$u_m(r) = \frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2\cancel{\mu}}\frac{\mu^{\cancel{2}} i^2}{4\pi^2 R^4}\, r^2 = \frac{\mu i^2}{8\pi^2 R^4}\, r^2$$

$$U_m = \int_{vol} u_m \, d\tau = h \int_0^R \int_0^{2\pi} \frac{\mu i^2}{8\pi^2 R^2}\, r^2 \, r \, d\theta \, dr =$$

$$= h \frac{\mu i^2 \, 2\pi}{8\pi^2 R^2} \int_0^R r^3 \, dr = h \frac{\mu i^2 \cancel{2\pi}}{\cancel{32}\,16\, \cancel{\pi^2}\, \cancel{R^2}}\, R^{\cancel{4}\,2} = h \frac{\mu i^2 R^2}{16\pi}$$

$$U_{m_h} = U_m / h = \frac{\mu i^2 R^2}{16\pi}$$

4)



$$\boxed{\omega = 2\pi f}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\cos\phi = \frac{R}{Z}$$ P_m pour $\cos\phi = 1$

$$\cos\phi = 1 \rightarrow R = Z \rightarrow \left(\omega L - \frac{1}{\omega C}\right) = 0 \rightarrow \omega = \frac{1}{\sqrt{LC}}$$

d'où $C = \dfrac{1}{\omega^2 L}$ & $C' = C/2$ $\quad Z' = \sqrt{R^2 + \left(\omega L - \dfrac{2}{\omega C}\right)^2}$

$\cos\phi' = \dfrac{R}{Z'}$ $\quad i'_{eff} = \dfrac{V_{eff}}{Z'}$ $\quad V'_{eff_R} = i'_{eff} \cdot R$ $\quad V'_{eff_L} = \omega L\, i'_{eff}$ $\quad V'_{eff_C} = \dfrac{i'_{eff}}{\omega C'}$