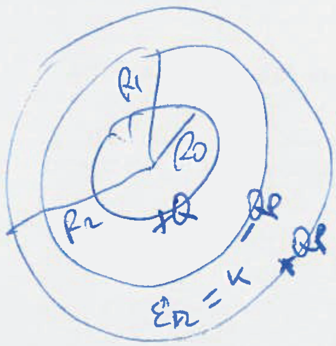
1)



Calcolo il campo $\vec{E}(r)$ con Gauss:

ho vari modi equivalenti

A) penso che il dielettrico sia sostituito dal vuoto ($\varepsilon = \varepsilon_0$) con opportune cariche di polarizzazione

$$Q_P = Q\left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right) = Q\left(\frac{k-1}{k}\right)$$

per $0 < r < R_0$ $\quad \phi(\vec{E}) = 0 \rightarrow \vec{E}(r) = 0$

per $R_0 < r < R_1$ $\quad \phi(\vec{E}) = E(r) 4\pi r^2 = \frac{Q}{\varepsilon_0}$

$$E'(r) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

per $R_1 < r < R_2$ $\quad \phi(\vec{E}) = E(r) 4\pi r^2 = \frac{Q - Q_P}{\varepsilon_0} = \frac{Q - Q\left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)}{\varepsilon_0}$

$$E''(r) = \frac{\cancel{Q} - \cancel{Q} + Q/\varepsilon_r}{4\pi\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}$$

per $r > R_2$ $\quad \phi(\vec{E}) = E(r) 4\pi r^2 = \frac{Q - \cancel{Q_P} + \cancel{Q_P}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$

$$E'''(r) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

oppure

B) uso il vettore $\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon_0 k \vec{E}$

con Gauss $\phi(\vec{D}) = q_{int}$ $\hookrightarrow \vec{E} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r} = \frac{\vec{D}}{\varepsilon_0 k}$

per $0 < r < R_0$ $\phi(\vec{D}) = 0 \rightarrow \vec{D} = 0 \rightarrow \vec{E} = 0$

per $R_0 < r < R_1$ $\phi(\vec{D}) = D(r) 4\pi r^2 = Q$

$$D(r) = \frac{Q}{4\pi r^2} \rightarrow E' = \frac{D}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

per $R_1 < r < R_2$ $\phi(\vec{D}) = D(r) 4\pi r^2 = Q$

$$D(r) = \frac{Q}{4\pi r^2} \rightarrow E'' = \frac{D}{\varepsilon_0 \varepsilon_r} \rightarrow E'' = \frac{Q}{4\pi\varepsilon_0 \varepsilon_r r^2}$$

per $r > R_2$ $\phi(\vec{D}) = D(r) 4\pi r^2 = Q$

$$D(r) = \frac{Q}{4\pi r^2} \rightarrow E''' = \frac{D}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$V_{CONDUTT} = V_{COND} - V_{\infty} = \Delta V = -\int_{\infty}^{R_0} \vec{E} \cdot d\vec{r} =$$

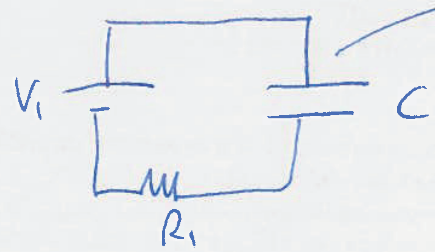
$$= -\int_{+\infty}^{R_2} \frac{Q}{4\pi\varepsilon_0} \frac{dr}{r^2} - \int_{R_2}^{R_1} \frac{Q}{4\pi\varepsilon_0 \varepsilon_r} \frac{dr}{r^2} - \int_{R_1}^{R_0} \frac{Q}{4\pi\varepsilon_0} \frac{dr}{r^2} =$$

$$= \frac{Q}{4\pi\varepsilon_0} \left[\left(\frac{1}{R_2} - 0 \right) + \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \left(\frac{1}{R_0} - \frac{1}{R_1} \right) \right] = \Delta V$$

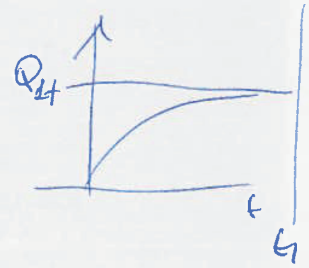
$$U_e = \frac{1}{2} Q \Delta V^2$$

2)

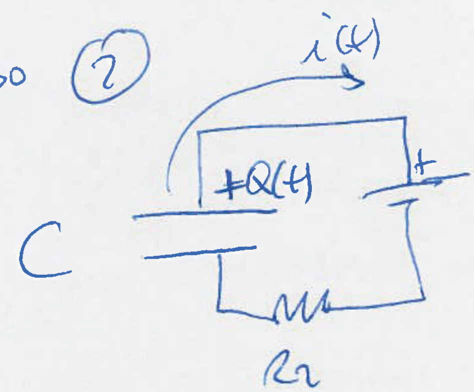
Caso (1)



$$Q_{1f} = CV_1$$
$$\shortparallel$$
$$Q(t=t_1)$$



Caso (2)



$$V_2 = \frac{V_1}{4}$$

$$\boxed{Q(t_1) = Q_{1f} = CV_1 \quad t_1 = 0}$$

maglie
$$\begin{cases} V_C(t) - V_2 = R_2\, i(t) \\ i(t) = -\dfrac{dQ(t)}{dt} \end{cases}$$

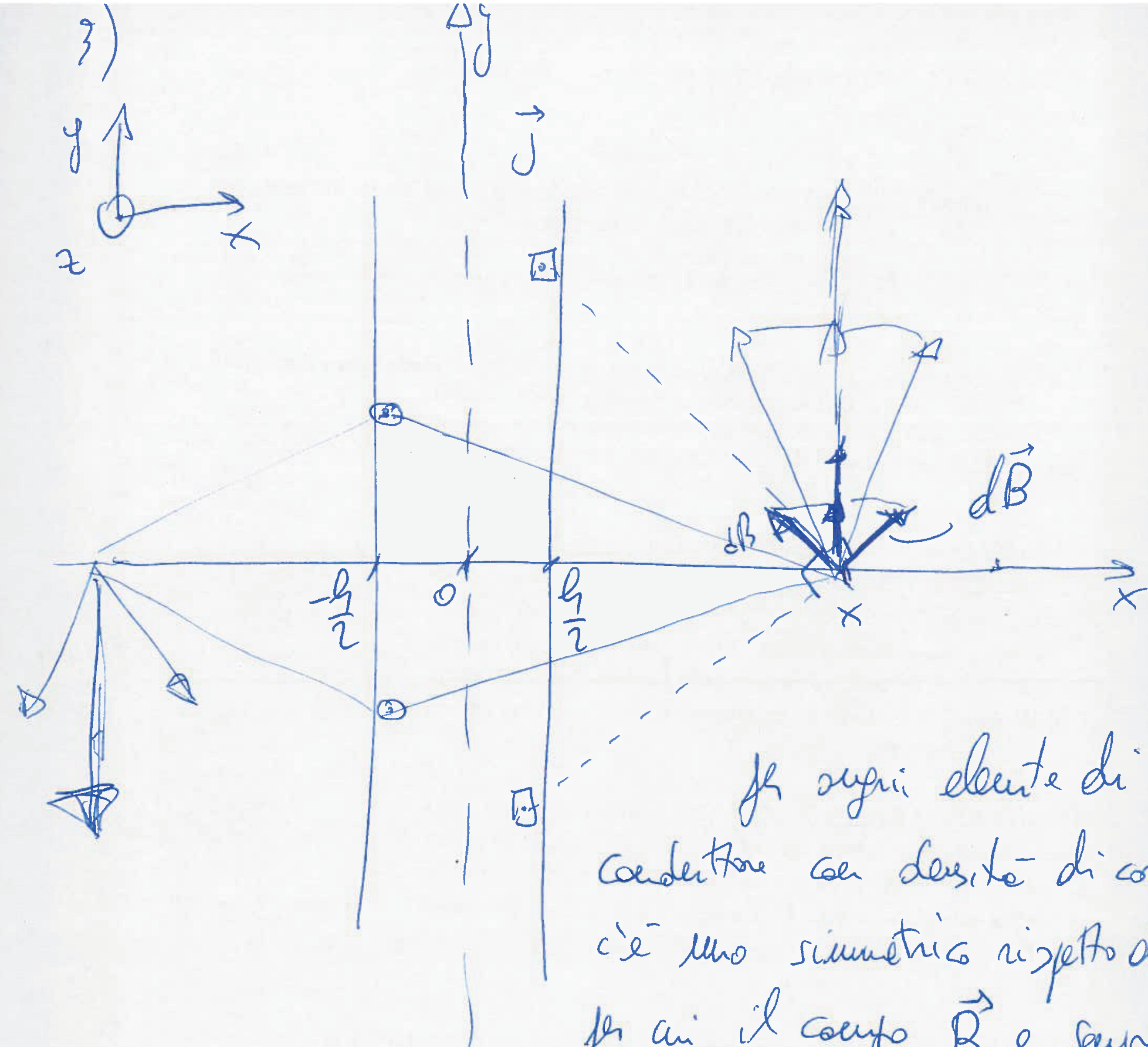
$$\frac{Q(t)}{C} - V_2 = -R_2 \frac{dQ(t)}{dt} \; ; \quad Q(t) - V_2 C = -R_2 C \frac{dQ(t)}{dt} \; ;$$

$$-\frac{1}{R_2 C}\int_0^{t^*} dt = \int_{Q_{1f}}^{Q_{1f}/2} \frac{dQ(t)}{Q(t) - V_2 C} \; ; \quad -\frac{t^*}{R_2 C} = \ln \frac{\frac{CV_1}{2} - V_2 C}{CV_1 - V_2 C} \; ;$$

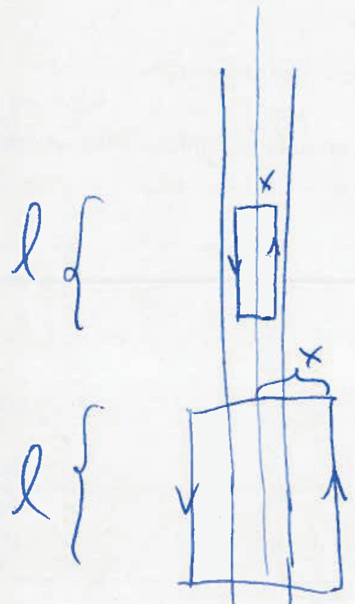
$$Q_{1f} = CV_1 = C 4 V_2$$

$$; \; t^* = -R_2 C \ln \frac{2CV_2 - V_2 C}{4V_2 - V_2 C} = -R_2 C \ln \frac{1}{3} = R_2 C \ln 3$$

3)



per ogni elemento di conduttore con densità di corrente $\vec{J}$ c'è uno simmetrico rispetto all'asse x per cui il campo $\vec{B}$ è sempre in direzione $+\hat{y}$ (per $x>0$) o $-\hat{y}$ (per $x<0$)



per cui facendo la circuitazione di $\vec{B}$ su un rettangolo di lati $2x$ e l si ha:

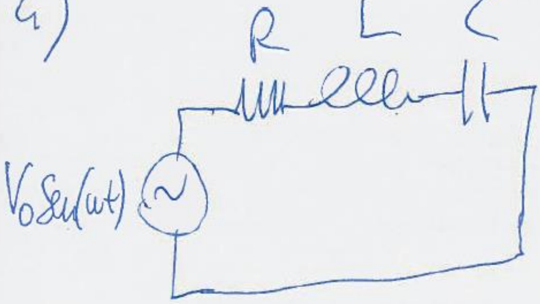
per $0<x<\frac{h}{2}$

$$\oint \vec{B}\cdot d\vec{l} = 2B(x)\cancel{l} = \mu_0 j \cancel{l}\, 2x \to B(x)=\mu_0 j x$$

per $x>\frac{h}{2}$ $\oint \vec{B}\cdot d\vec{l} = 2B(x)\cancel{l} = \mu_0 j \cancel{l}\, h$

$$\to B(x) = \mu_0 j \frac{h}{2} = cost$$

4)



$$\omega = \frac{1}{\sqrt{LC}}$$: RISONANZA

Z = IMPEDENZA CIRCUITO =

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = R$$ ALLA RISONANZA

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{Z} = \frac{V_0}{R}$$

$$V_R = I_0 R = \frac{V_0}{Z} R = V_0$$

POT DISSIPATA $= \frac{1}{2} V_0 I_0 \cos\phi = \frac{1}{2} V_0 I_0 \frac{R}{Z}$

$$= \frac{1}{2} V_0 I_0 \frac{\cancel{R}}{\cancel{R}} = \frac{1}{2} V_0 I_0 = \frac{1}{2} \frac{V_0^2}{R}$$

Se $R' = R/2$ $\qquad Z' = \sqrt{\frac{R^2}{4} + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{R}{2}$

$$I_0' = \frac{V_0}{Z'} = \frac{2V_0}{R} = 2I_0 \qquad V_R' = I_0' \frac{R}{2} = \frac{2 V_0}{\cancel{R}} \frac{\cancel{R}}{\cancel{2}} = V_0$$

POT DISSIP' $= \frac{1}{2} V_0' I_0' \overset{1}{\cancel{\cos(\phi)}} = \frac{1}{2} V_0' I_0' = \frac{1}{2} V_0 \frac{2V_0}{R} = \frac{V_0^2}{R}$