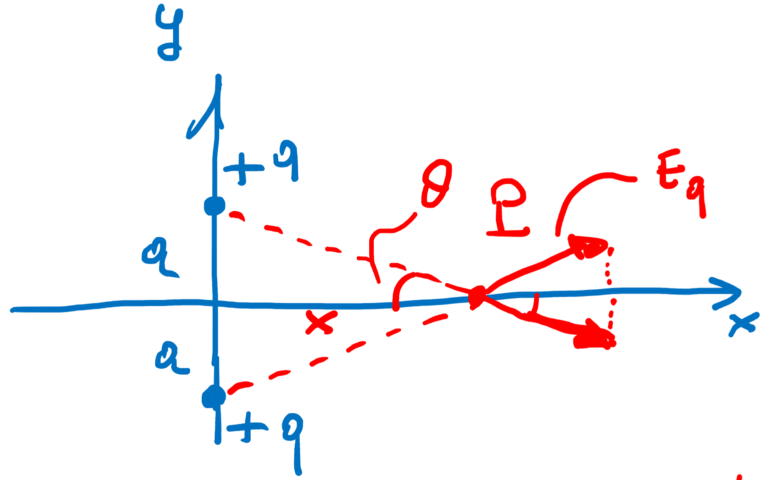
24 · 03 · 2022



per $x>0$

$$\vec{E}_T(x;0) = \hat{x}\cdot 2\cdot\cos\theta\cdot E_q(x)$$

con $E_q(x) = \dfrac{q}{4\pi\varepsilon}\dfrac{1}{(a^2+x^2)}$

per simmetria $\vec{E}_T(x;0)$ è diretto $\pm\hat{x}$

per $x>0$ $\vec{E}_T =$ diretto $+\hat{x}$ (se $q>0$)

per $x<0$ $\boxed{\vec{E}_T(x;0) = -\vec{E}_T(-x;0)}$ è diretto $-\hat{x}$ (se $q>0$)

mi concentro su $x>0$ perché per $x<0$ è simmetrico (per $x=0$ $\vec{E}_T(0;0)=0$)

$$\vec{E}_T(x;0) = \hat{x}\; 2\cdot\frac{x}{\sqrt{a^2+x^2}}\cdot\frac{q}{4\pi\varepsilon(a^2+x^2)} = \hat{x}\,\frac{\cancel{2}qx}{\cancel{2}4\pi\varepsilon}(x^2+a^2)^{-3/2}$$

P. 2

quindi: $\vec{E}_T(x;0) = \hat{x} \dfrac{q\,x}{2\pi\varepsilon_0} (x^2+a^2)^{-3/2}$

VERIFICO CHE SCRITTO COSI' RISULTA DIRETTO VERSO $+\hat{x}$ per $x>0$, VERSO $-\hat{x}$ per $x<0$ ED È $=0$ per $x=0$. OK!

per trovare il MAX del MODULO:

$$\frac{\partial E_T(x;0)}{\partial x} = \frac{\partial}{\partial x}\left\{\frac{q}{2\pi\varepsilon_0}\left[x\cdot(x^2+a^2)^{-3/2}\right]\right\} = 0$$

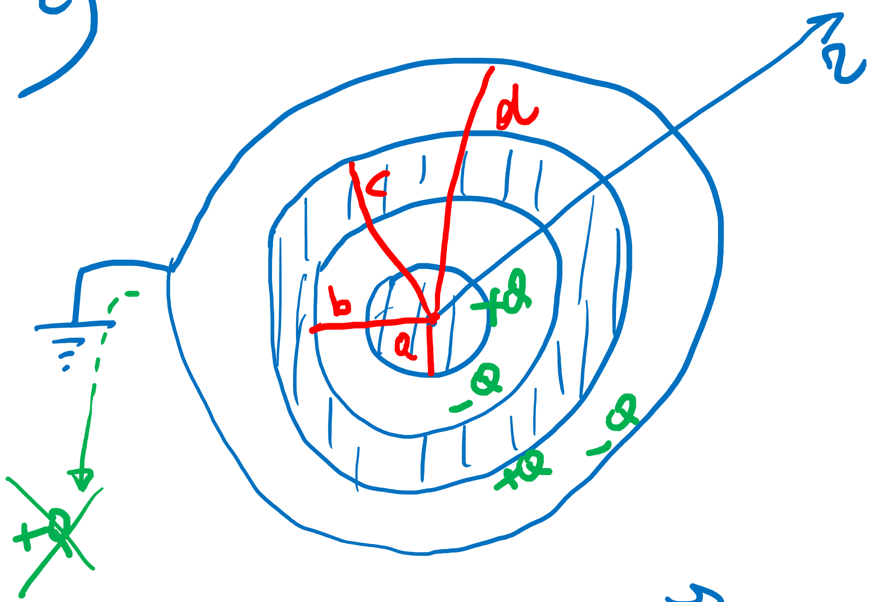
da cui $\dfrac{\partial}{\partial x}\left[x\cdot(x^2+a^2)^{-3/2}\right]=0 \rightarrow (x^2+a^2)^{-3/2} + x\cdot\left(-\dfrac{3}{\cancel{2}}\right)\cdot(x^2+a^2)^{-5/2}\cdot(\cancel{2}x)=0$

$\boxed{x^2+a^2\neq 0\ \forall x \in \mathbb{R}}$

$\rightarrow (x^2+a^2)^{-3/2} - 3x^2\cdot(x^2+a^2)^{-5/2} = 0 \rightarrow \dfrac{1}{\cancel{(x^2+a^2)^{3/2}}} = \dfrac{3x^2}{(x^2+a^2)^{\cancel{5/2}\ 2/2}} \rightarrow$

$\rightarrow 1 = \dfrac{3x^2}{(x^2+a^2)} \rightarrow (x^2+a^2)=3x^2 \rightarrow x^2+a^2=3x^2 \rightarrow 2x^2=a^2 \rightarrow \boxed{x=\pm\dfrac{a}{\sqrt{2}}}$

2)



METTERE UNA CARICO +Q IN $r=a^+$ significa indurre una carica $-Q$ in $r=b^-$ e poiché il conduttore (tra b e c) era NEUTRO, vuol dire che si forma una carica $+Q$ in $r=c^+$ che induce $-Q$ in $r=d^-$, a sua volta rimane una carica +Q che sparisce a TERRA (cioè per noi è lontana e non interviene).

TROVO IL CAMPO $\vec{E}$ CON GAUSS SU SFERE DI RAGGIO r

COME DIREZIONE $\vec{E}=+\hat{r}\,E(r)$ per $Q>0$

$$\phi(\vec{E}) = E(r)\cdot 4\pi r^2 = \frac{1}{\varepsilon_0} q_{INT}(r) \rightarrow E(r) = \frac{q_{INT}(r)}{4\pi\varepsilon_0 r^2}$$

$\rightarrow$ DIPENDE DALLE ZONE

per $r<a$ $\quad q_{INT}(r)=0$ $\quad$ per cui $E(r)=0$

per $a<r<b$ $\quad q_{INT}^{(r)}=Q$ $\quad$ per cui $E(r)=\dfrac{Q}{4\pi\varepsilon_0 r^2}$

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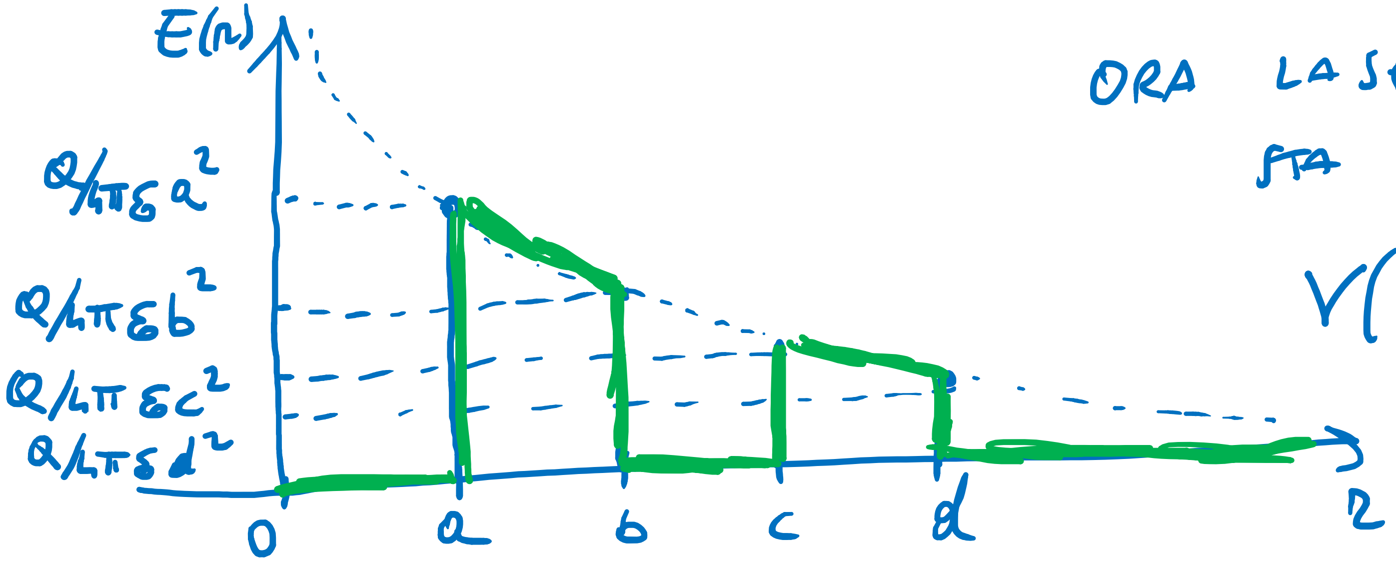
per $b < r < c$ $\quad q_{INT}(r) = 0$ per cui $E(r) = 0$

per $c < r < d$ $\quad q_{INT}(r) = Q$ per cui $E(r) = \dfrac{Q}{4\pi\varepsilon r^2}$

per $r > d$ $\quad q_{INT}(r) = 0$ per cui $E(r) = 0$



ORA LA SFERA AL CENTRO (EQUIPOTENZIALE) STA AL POT. V RISPETTO A MASSA:

$$V(a) - V(+\infty) = V(a) - V(d) =$$

$$= V - \phi =$$

$$= V = -\int_{+\infty}^{a} \vec{E}(r)\, d\vec{r} =$$

$$= -\int_{+d}^{a} E(r)\, dr =$$

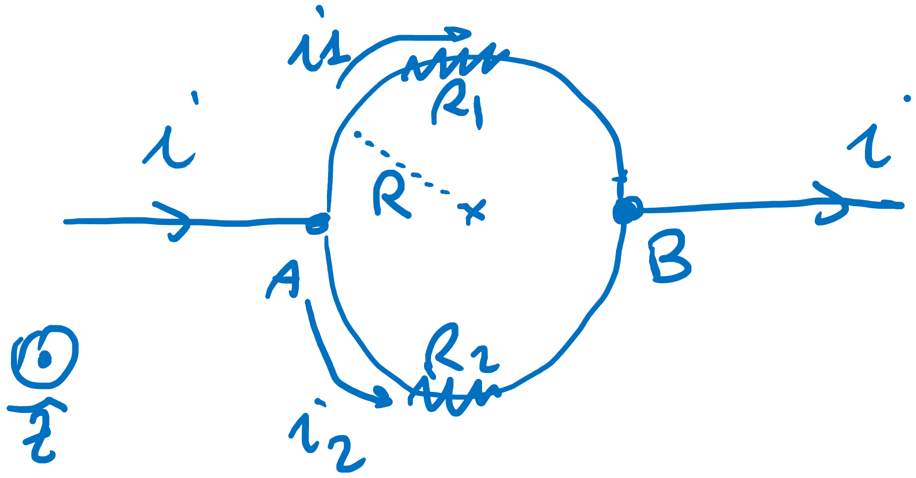
$\boxed{P.5}$

$$\rightarrow V = -\int_{+\infty}^{d} \cancel{E(r)}\,dr - \int_{d}^{c} E(r)\,dr - \int_{c}^{b} \cancel{E(r)}\,dr - \int_{b}^{e} E(r)\,dr =$$

$$\rightarrow V = -\int_{d}^{c} \frac{Q}{4\pi\varepsilon}\frac{dr}{r^2} - \int_{b}^{e} \frac{Q}{4\pi\varepsilon}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\Big|_{d}^{c} + \frac{1}{r}\Big|_{b}^{e}\right] =$$

$$\rightarrow V = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{c} - \frac{1}{d}\right) + \left(\frac{1}{e} - \frac{1}{b}\right)\Big] \rightarrow Q = \frac{4\pi\varepsilon V}{\left[\left(\frac{1}{c} - \frac{1}{d}\right) + \left(\frac{1}{e} - \frac{1}{b}\right)\right]}$$

3)



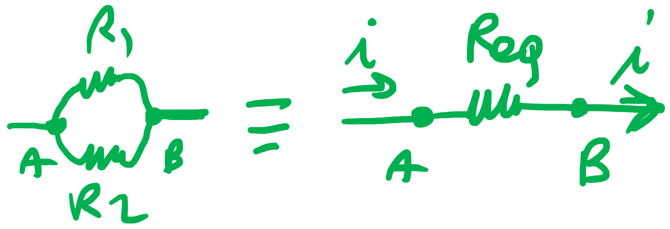
$$R = \frac{\overline{AB}}{2}$$

$$\vec{B}_1 = (-\hat{z})\frac{\mu_0 i_1}{4\cancel{\pi}}\frac{\cancel{R}\cancel{\pi}}{R^{\cancel{2}}} = \frac{\mu_0 i_1}{4R}(-\hat{z})$$

$$\vec{B}_2 = (+\hat{z})\frac{\mu_0 i_2}{4R}$$

$$\vec{B}_{TOT} = \vec{B}_1 + \vec{B}_2 = \hat{z}\left[\frac{\mu_0 i_2}{4R} - \frac{\mu_0 i_1}{4R}\right] = \hat{z}\,B$$

$$B = \frac{\mu_0}{4R}(i_2 - i_1)$$



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\Delta V_{AB} = R_{eq}\, i = \frac{R_1 R_2}{R_1 + R_2}\, i$$

$$i_1 = \frac{\Delta V_{AB}}{R_1} = \frac{\cancel{R_1} R_2\, i}{(R_1+R_2)\cancel{R_1}} = \frac{R_2}{R_1+R_2}\, i \qquad i_2 = \frac{R_1}{R_1+R_2}\, i$$

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$$B = \frac{\mu_0}{4R}\left(i_2 - i_1\right) = \frac{\mu_0}{4R}\left[\frac{R_1}{R_1+R_2}\, i - \frac{R_2}{R_1+R_2}\, i\right]$$

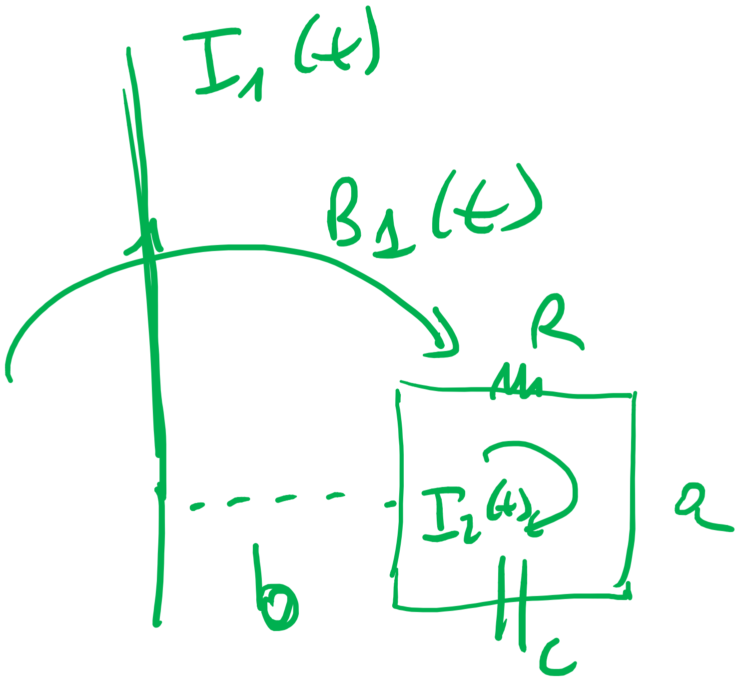
$$B = \frac{\mu_0}{4R}\, i \,\frac{1}{R_1+R_2}\left[R_1 - R_2\right] = \frac{\mu_0}{2\cancel{4}\cdot\frac{\overline{AB}}{\cancel{2}}}\,\frac{i}{(R_1+R_2)}\left(R_1 - R_2\right)$$

$$(R_1+R_2)\,2\,\overline{AB}\cdot B = \mu_0 i\,(R_1 - R_2) \rightarrow R_1\, 2\overline{AB}\cdot B + R_2\cdot 2\overline{AB}\cdot B = \mu_0 i R_1 - \mu_0 i R_2$$

$$\rightarrow R_2\left[2\cdot\overline{AB}\cdot B + \mu_0 i\right] = \left[\mu_0 i - 2\overline{AB}\cdot B\right] R_1 \rightarrow$$

$$\rightarrow R_2 = R_1\,\frac{\mu_0 i - 2\overline{AB}\cdot B}{\mu_0 i + 2\overline{AB}\cdot B}$$

4)



$$B_1(t) = \frac{\mu_0 I_1(t)}{2\pi r}$$

$$\phi(B_1) = a \int_b^{b+a} \frac{\mu_0 I_1(t)}{2\pi} \frac{dr}{r} =$$

$$= \frac{\mu_0 a I_1(t)}{2\pi} \ln \frac{b+a}{b}$$

$$I_1(t) = I_{1max} \cos(\omega t)$$

$$fem_{\text{sulla spira}}(t) = \mathcal{E}(t) = -\frac{d}{dt} \phi(B_1) =$$

$$= -\frac{d}{dt} \left[\frac{\mu_0 a}{2\pi} \ln\left(\frac{b+a}{b}\right) I_{1max} \cos(\omega t) \right]$$

$$\mathcal{E}(t) = \frac{\omega \mu_0 a}{2\pi} \ln\left(\frac{b+a}{b}\right) I_{1max} \ \text{sen}(\omega t) = \mathcal{E}_{max} \ \text{sen}(\omega t)$$

con $\mathcal{E}_{max} = \frac{\omega \mu_0 a}{2\pi} \ln\left(\frac{b+a}{b}\right) I_{1max}$

$I_2(t) = I_{2max} \ \text{sen}(\omega t + \varphi)$ con $I_{2max} = \frac{\mathcal{E}_{max}}{\sqrt{R^2 + (1/\omega C)^2}}$

$\varphi = -\text{arctg} \frac{1}{\omega R C} = -\varphi_z$

$\tilde{Z} = R + \frac{1}{j\omega C} = R - j\frac{1}{\omega C}$

$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

$\varphi_z = \text{arctg} \frac{1/\omega C}{R} = \text{arctg} \frac{1}{\omega R C}$