1) per $r \geq R$ $\quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \Rightarrow \boxed{V(r \geq R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$

per $r \leq R$ Troviamo $|\vec{E}_i|$ tramite Gauss: $4\pi r^2 E = \frac{\int_0^r \rho \, 4\pi r^2 dr}{\varepsilon_0}$

$\rho = \alpha r$

$$\Rightarrow \cancel{4\pi} r^2 E_i = \frac{\cancel{4\pi}}{\varepsilon_0}\alpha \int_0^r r^3 dr = \frac{\alpha}{\varepsilon_0}\frac{r^4}{4} \Rightarrow E_i = \frac{\alpha}{4\varepsilon_0} r^2$$

$$Q = \int_0^R \alpha r \, 4\pi r^2 dr = 4\pi\alpha \frac{R^4}{4} \Rightarrow \alpha = \frac{Q}{\pi R^4} \Rightarrow \vec{E}_i = \frac{1}{4\pi\varepsilon_0} Q \frac{r^2}{R^4}\hat{r}$$

$$V(r \leq R) - V(R) = \int_r^R E_i \, dr$$

$$V(r \leq R) - \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0} Q \left(\frac{1}{3R} - \frac{r^3}{3R^4}\right)$$

$$\Rightarrow \boxed{V(r \leq R) = \frac{1}{4\pi\varepsilon_0} Q \left[\frac{4}{3R} - \frac{r^3}{3R^4}\right]}$$

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2) Campo $\vec{E}$ tra le due armature $\quad \vec{E} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{r^2}\hat{r}$

ΔV tra le due armature

$$V_2 - V_1 = -\int_{r_1}^{r_2} \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2} dr = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

La capacità quindi è $C = \frac{Q}{V_2 - V_1} = 4\pi\varepsilon_r\varepsilon_0 \frac{r_1 r_2}{r_2 - r_1} \Rightarrow$

$$\boxed{\varepsilon_r = C\frac{r_2 - r_1}{4\pi\varepsilon_0 r_1 r_2} = \frac{(4{,}45 \cdot 10^{-12}\,F)\,(10^{-2}\,m)}{4\pi\left(8{,}85 \cdot 10^{-12}\,\frac{C^2}{Nm^2}\right)\left(2 \cdot 10^{-4}\,m^2\right)} = 2}$$

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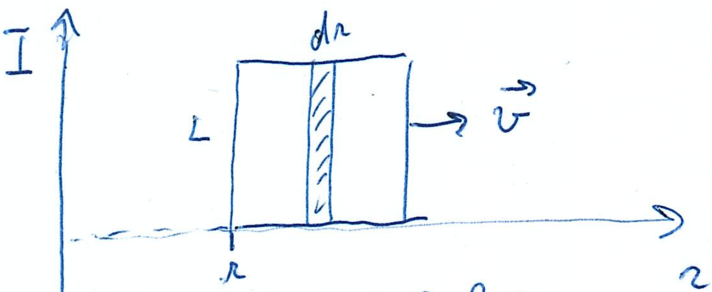
3)



B creato dal filo

$$B = \frac{\mu_0 I}{2\pi r}$$; flusso attraverso la spira $$\phi = \int_r^{r+L} \frac{\mu_0 I}{2\pi r} L\, dr$$

$$\phi = \frac{\mu_0 I}{2\pi} L \ln \frac{r+L}{r} \, ; \quad f.e.m. = -\frac{d\phi}{dt}$$

$$f.e.m. = -\frac{\mu_0 I L}{2\pi} \frac{1}{\frac{r+L}{r}} \left(\frac{r v - (r+L) v}{r^2} \right) = -\frac{\mu_0 I L}{2\pi} \frac{r}{r+L} \left(-\frac{L v}{r^2} \right)$$

$$= \frac{\mu_0 I L^2}{2\pi} \frac{v}{r(r+L)} \Rightarrow \boxed{I_{sp} = \frac{\mu_0 I L^2}{R\, 2\pi} \frac{v}{r(r+L)}}$$

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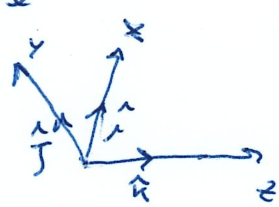
4) Data la frequenza $$\lambda = \frac{c}{\nu} = \frac{3 \cdot 10^8 \, m/s}{5 \cdot 10^5 \, Hz} = 600 \, m$$

$$\Rightarrow k = \frac{2\pi}{\lambda} = 10^{-2} \, m^{-1}$$

Dato che l'onda è polarizzata linearmente

$$\vec{E} = E_0 \sin(kz - \omega t)\, \hat{i}$$

$$\vec{B} = \frac{E_0}{c} \sin(kz - \omega t)\, \hat{j}$$



Il valore medio della potenza trasmessa per unità di superficie è la media del modulo del vettore di Poynting

$$P = \frac{1}{\mu_0} \overline{\vec{E} \times \vec{B}} = \frac{1}{\mu_0} \frac{1}{2} E_0 \frac{E_0}{c} = \frac{1}{2\mu_0} \frac{E_0^2}{c}$$

$$\Rightarrow \boxed{E_0 = \sqrt{2 \mu_0 c P} = \sqrt{2 (4\pi \cdot 10^{-7} \, H/m)(3\, 10^8 \, m/s)(30 \, W/m^2)} = 150{,}4 \, V/m}$$