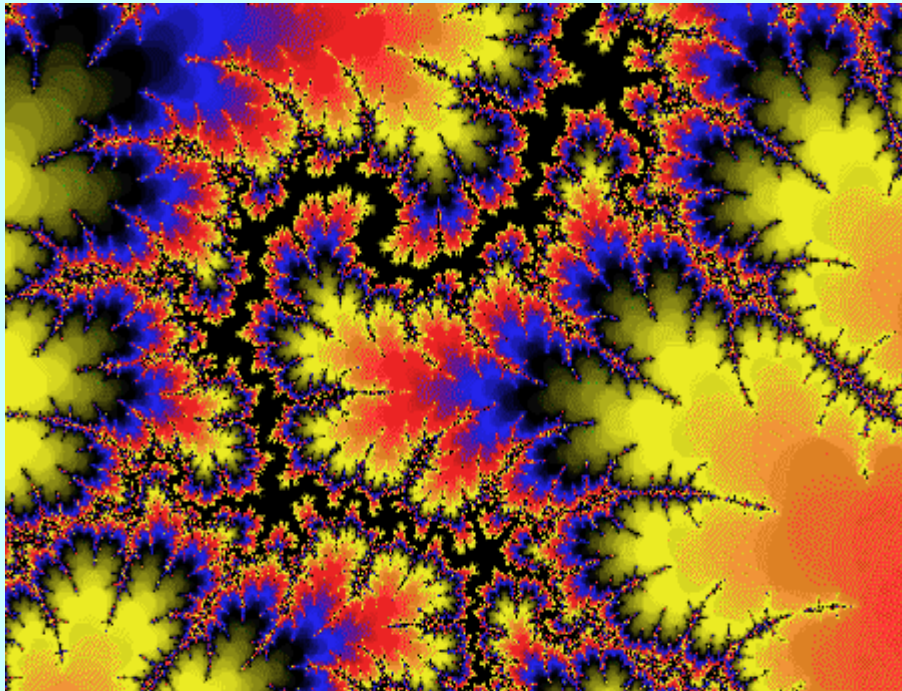


SAPIENZA UNIVERSITA' DI ROMA

Venerdì 23 aprile 2021



MATEMATICA E MUSICA

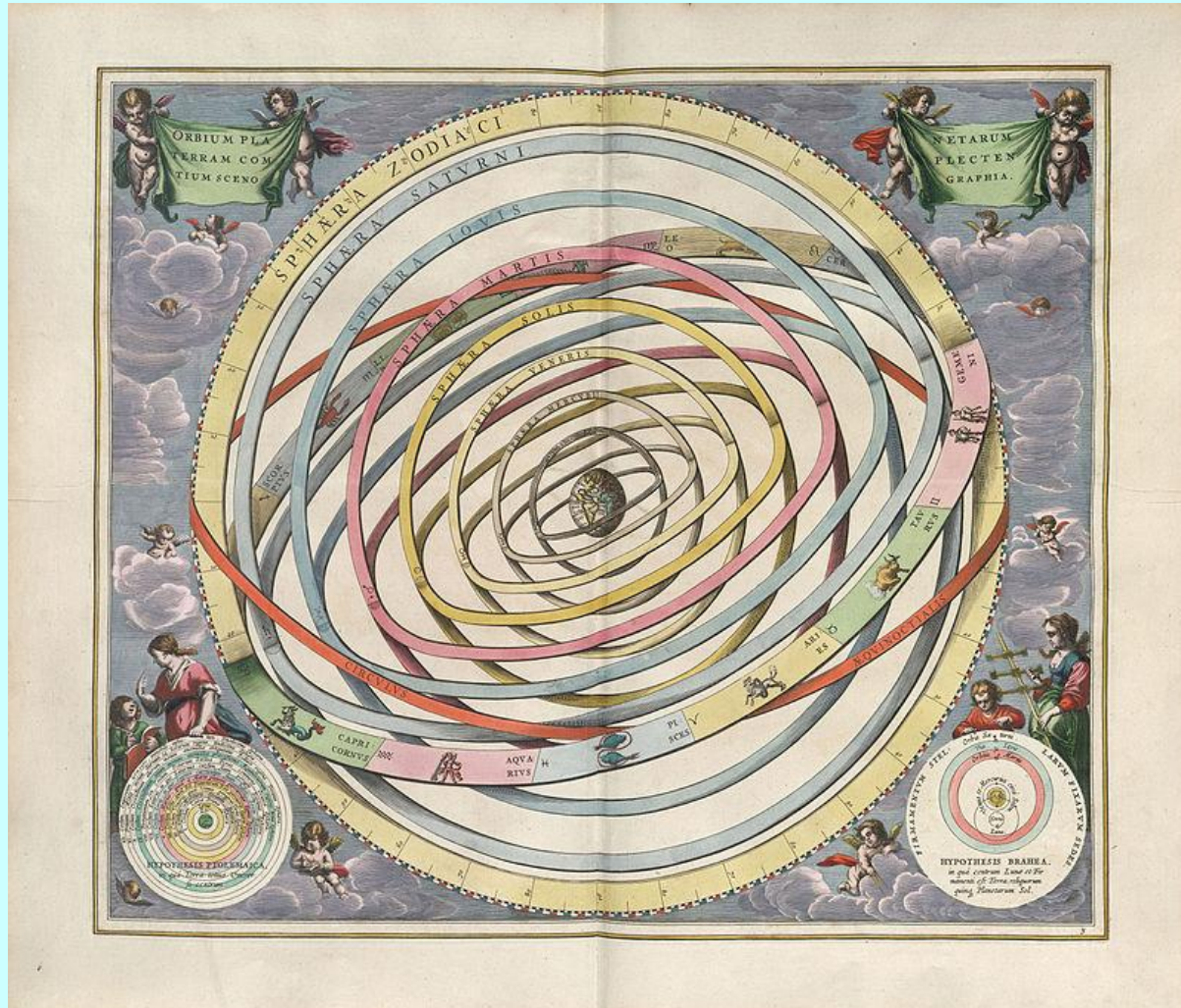
Bruno Cifra

SAPIENZA UNIVERSITA' DI ROMA

“L'armonia delle folle”

***Rappresentazioni musicali
di modelli matematici***

"L'armonia delle sfere"



Ioannis Keplleri
**HARMONICES
MUNDI**

LIBRI V. QVORVM

Primus GEOMETRICVS, De Figurarum Regularium, quæ Proportiones Harmonicas constituunt, ortu & demonstrationibus.
Secundus ARCHITECTONICVS, seu ex GEOMETRIA FIGVRATA, De Figurarum Regularium Congruentia in plano vel solido:
Tertius propriè HARMONICVS, De Proportionum Harmonicarum ortu ex Figuris; deque Natura & Differentiis rerum ad cantum pertinentium, contra Veteres:
Quartus METAPHYSICVS, PSYCHOLOGICVS & ASTROLOGICVS, De Harmoniarum mentali Essentiâ earumque generibus in Mundo; præsertim de Harmonia radiorum, ex corporibus cælestibus in Terram descendentibus, eiusque effectu in Natura seu Anima sublunari & Humana:
Quintus ASTRONOMICVS & METAPHYSICVS, De Harmoniis absolutissimis motuum cælestium, ortuque Eccentricitatum ex proportionibus Harmonicis.
Appendix habet comparationem huius Operis cum Harmonices Cl. Ptolemæi libro II cumque Roberti de Fluctibus, dicti Flud. Medici Oxoniensis speculationibus Harmonicis, operi de Macrocosmo & Microcosmo insertis.



Cum S. C. M^o. Privilegio ad annos XV.

Lincii Austriae,

Sumptibus GODOFREDI TAMPACHII Bibl. Francof.
Excudebat IOANNES PLANCVS.

ANNO M. DC. XIX.

*1st Edition 1619
6 plates on 5*

Harmonices mundi

**Giovanni Keplero
1619**

Bruno Cifra

G. ZARLINO “*istituzioni harmoniche*” (1558)

-**Scala naturale**: introduzione del *senario* (1-2-3-4-5-6)
come ampliamento della *tetractys pitagorica*

$$1+2+3 = 1*2*3=6$$

6 furono i giorni necessari a Dio per la creazione

6 sono i pianeti: Luna, Mercurio, Marte, Venere, Giove, Saturno

6 sono le qualità naturali: misura, colore, forma, intervallo, stato, moto

6 sono le direzioni: giù, su, avanti, indietro, destra, sinistra

6 sono le superfici del cubo...

G. KEPLERO “*Harmonice mundi*” (1619)

- ordine armonico dei moti celesti nel rapporto delle velocità
angolari di rivoluzione

IL SISTEMA NATURALE

Sistema Zarliniano

La costruzione della **scala** avviene attraverso il metodo della “buona armonia” (*rapporti semplici*)

La base è l'*accordo perfetto maggiore*, costruito come segue:

$3/2$		
v	$5/4v$	$3/2v$
DO	MI	SOL
$5/4$		$6/5$

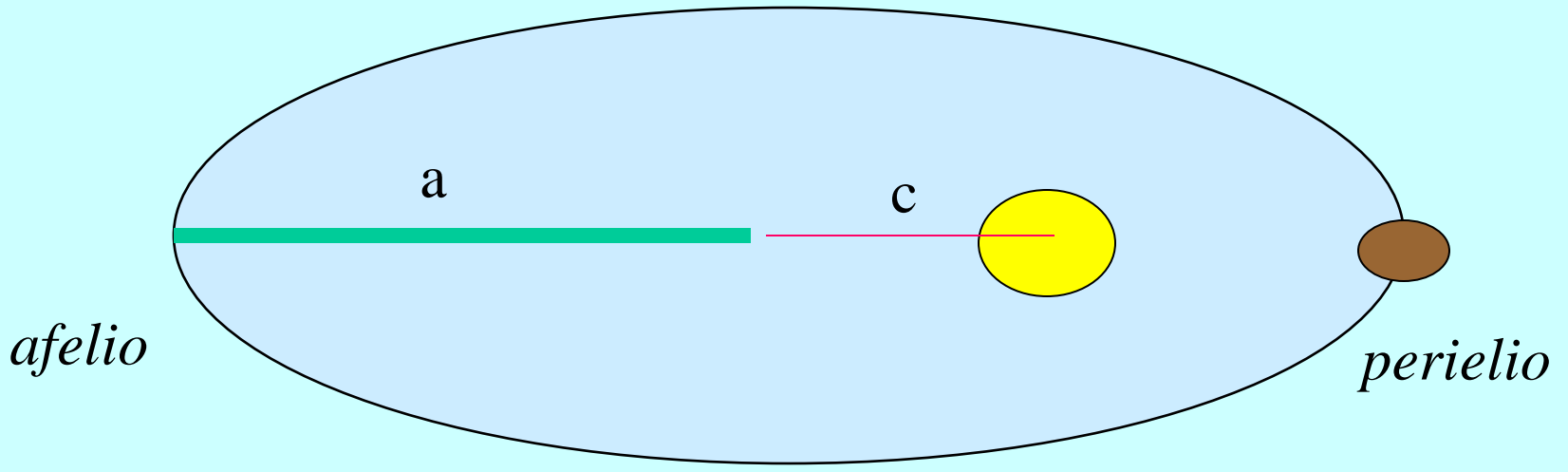
$3/2$		$3/2$			$3/2$	
$2/3v$	$5/6v$	v	$5/4v$	$3/2v$	$15/8v$	$9/4v$
FA	LA	DO	MI	SOL	SI	RE
$5/4$	$6/5$	$5/4$	$6/5$	$5/4$	$6/5$	

Moltiplicando per opportune potenze di 2 si può riordinare tutto all'interno dell' *ottava*:

$9/8$		$10/9$	$16/15$	$9/8$	$10/9$	$9/8$	$16/15$
DO	RE	MI	FA	SOL	LA	SI	DO
v	$9/8v$	$5/4v$	$4/3v$	$3/2v$	$5/3v$	$15/8v$	$2v$

Scala diatonica Zarliniana

Scala cromatica Zarliniana	
DO	1
Do#	25/24
Sem. diat. piccolo	16/15
Reb	27/25
RE	9/8
Re#	75/64
Mib	6/5
MI	5/4
Fab	32/35
Mi#	125/96
FA	4/3
Fa#	25/18
Sem. diat. piccolo	64/45
Solb	36/25
SOL	3/2
Sol #	25/16
Lab	8/5
LA	5/3
La#	125/72
Sem. diat. piccolo	16/9
Sib	9/5
SI	15/8
Dob	48/25
Si#	125/44
DO	2

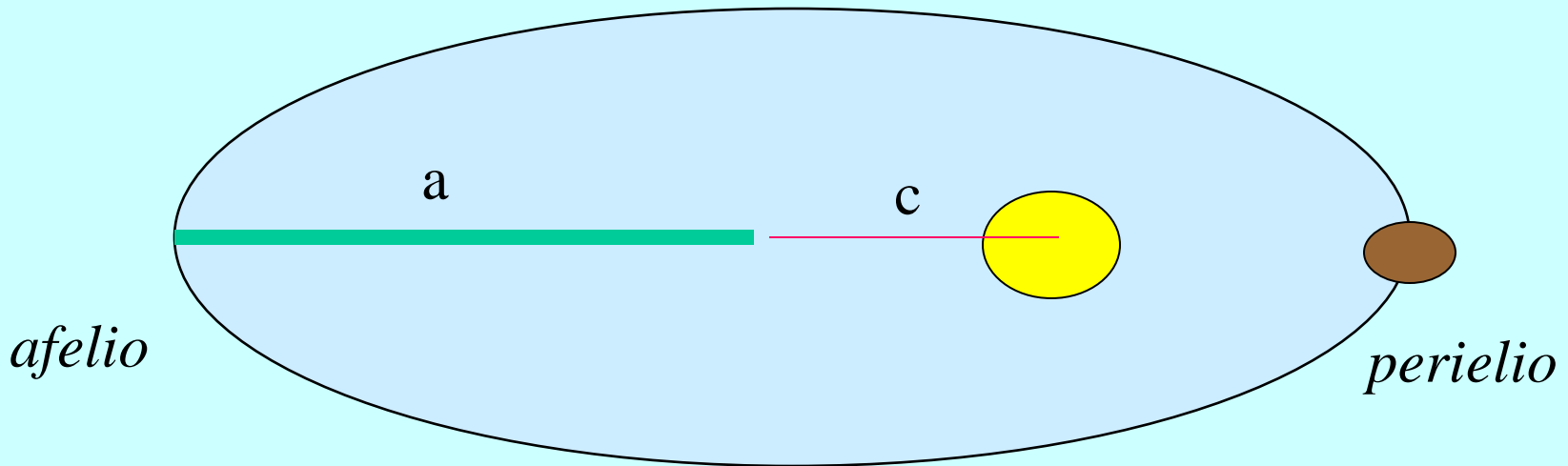


$$m\omega^2 R = GMm/R^2 \quad \Rightarrow \quad R^3\omega^2 = GM$$

$$(a-c)^3\omega_p^2 = GM \quad \text{nel perielio}$$

$$(a+c)^3\omega_a^2 = GM \quad \text{nell'afelio}$$

$$\omega_p^2/\omega_a^2 = (a+c)^3/(a-c)^3 = (1+e)^3/(1-e)^3 \quad e=c/a$$



	Moti apparenti diurni	valori ideali	rapporti	intervalli musicali			
SATURNO	afelio	1' 46"	1' 48"	5/4	terza maggiore	1,25	1,25
	perielio	2' 15"	2' 15"				
GIOVE	afelio	4' 30"	4' 35"	6/5	terza minore	1,2	1,2
	perielio	5' 30"	5' 30"				
MARTE	afelio	26' 14"	25' 21"	3/2	quinta	1,481169	1,5
	perielio	38' 1"	38' 1"				
TERRA	afelio	57' 3"	57' 28"	16/15	semitono	1,066705	1,066667
	perielio	61' 18"	61' 18"				
VENERE	afelio	94' 50"	94' 50"	25/24	<i>semitono cromatico</i> semitono minore	1,041652	1,041667
	perielio	97' 37"	98' 47"				
MERCURIO	afelio	164' 0"	164' 0"	12/5	ottava + terza minore	2,402439	2,4
	perielio	384' 0"	394' 0"				

		Moti apparenti diurni	rapporti	intervalli musicali		
SATURNO	afelio	1' 46"				
			3/1	ottava + quinta	3,113208	3
GIOVE	perielio	5' 30"				
SATURNO	perielio	2' 15"				
			2/1	ottava	2	2
GIOVE	afelio	4' 30"				
GIOVE	afelio	4' 30"				
			8/1	tre ottave	8,448148	8
MARTE	perielio	38' 1'				
GIOVE	perielio	5' 30"				
			24/5	due ottave + terza minore	4,769697	4,8
MARTE	afelio	26' 14"				
MARTE	afelio	26' 14"				
			12/5	ottava + terza minore	2,336722	2,4
TERRA	perielio	61' 18"				
MARTE	perielio	38' 1"				
			3/2	quinta	1,500658	1,5
TERRA	afelio	57' 3"				
TERRA	afelio	57' 3"				
			5/3	sesta maggiore	1,711072	1,666667
VENERE	perielio	97' 37"				
TERRA	perielio	61' 18"				
			8/5	sesta minore	1,547036	1,6
VENERE	afelio	94' 50"				
VENERE	afelio	94' 50"				
			4/1	due ottave	4,049209	4
MERCURIO	perielio	384' 0"				
VENERE	perielio	97' 37"				
			5/3	sesta maggiore	1,680041	1,666667
MERCURIO	afelio	164' 0"				

ARMONIA DELLE SFERE

Mercurio

Sesta maggiore $5/3$

Venere

Sesta minore $8/5$

Terra

Quinta $3/2$

Marte

Terza minore $6/5$

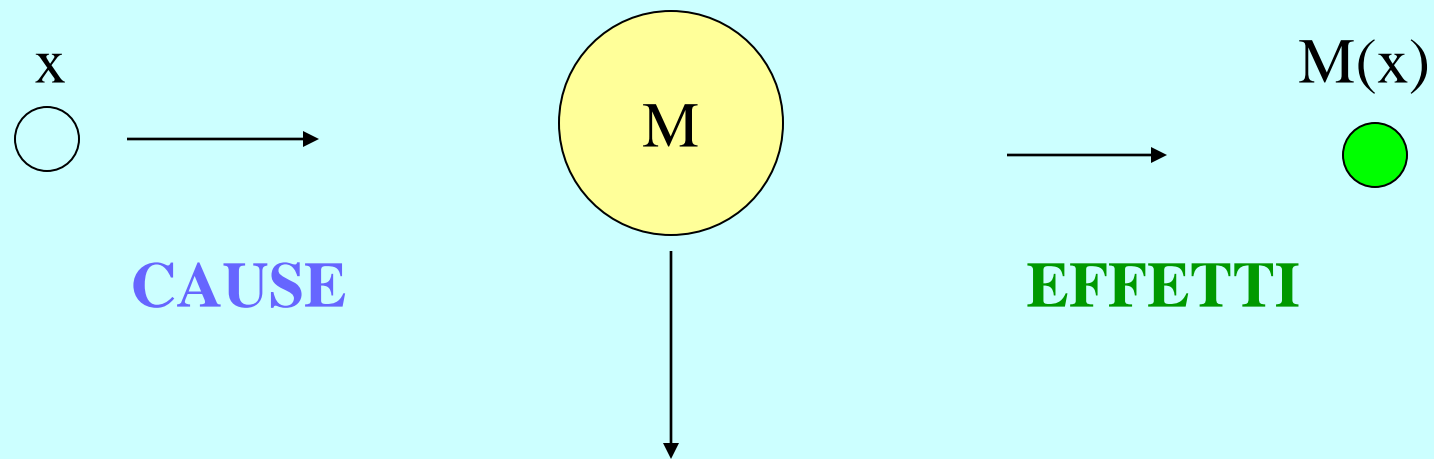
Giove

Ottava 2

Saturno

MODELLO MATEMATICO

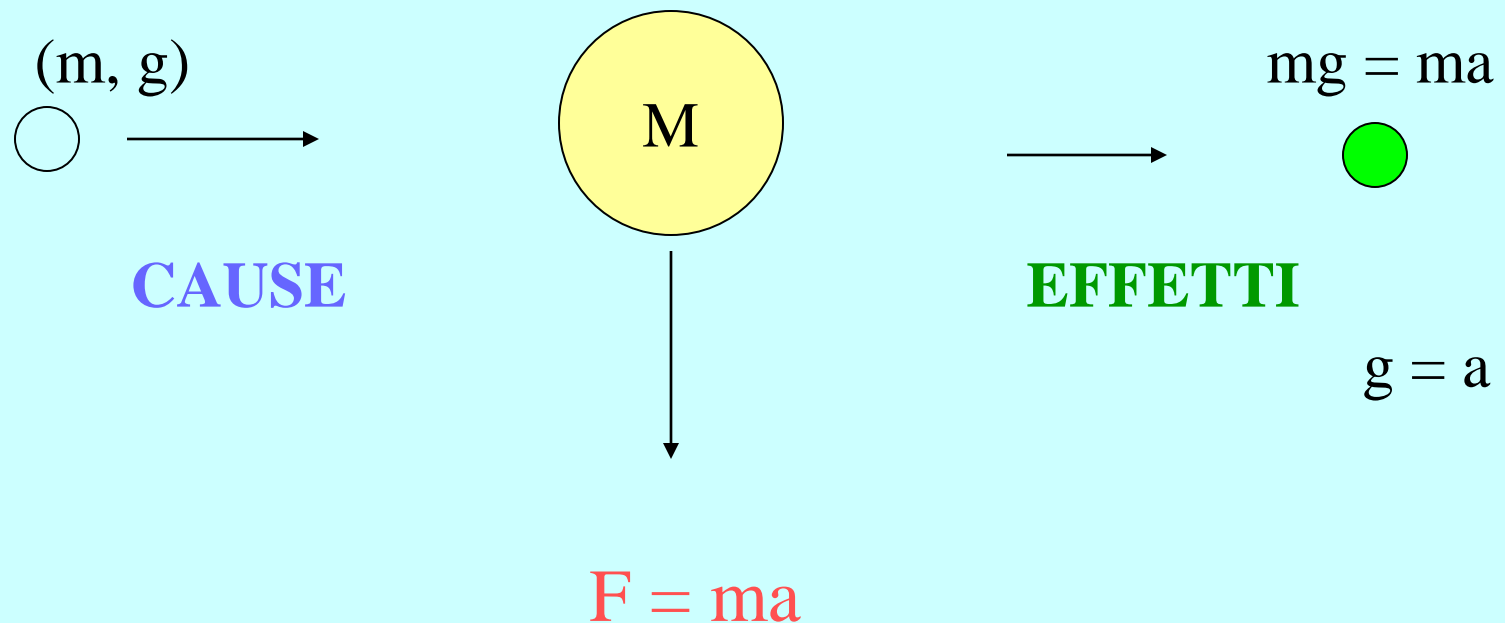
Rappresentazione simbolica degli enti caratteristici di un “fenomeno” e delle loro relazioni.



Macchina che riproduce la logica evolutiva

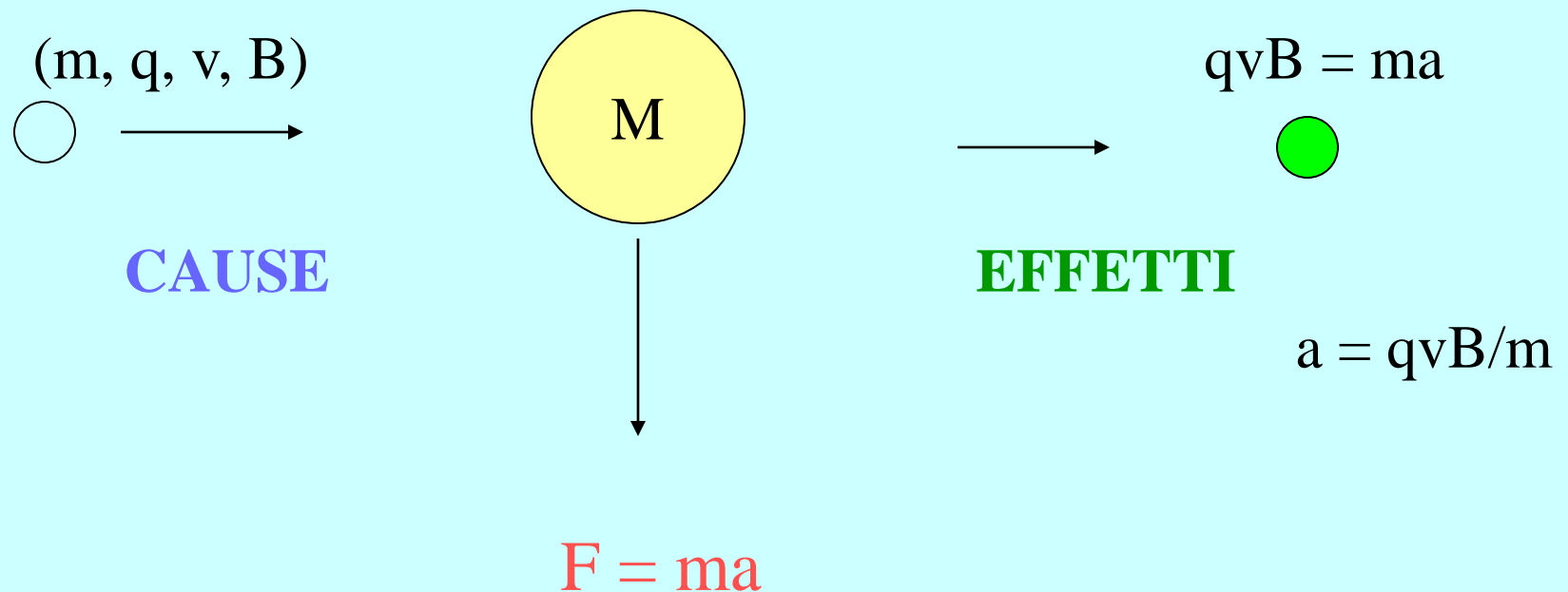
MODELLO MATEMATICO

Rappresentazione simbolica degli enti caratteristici di un “fenomeno” e delle loro relazioni.



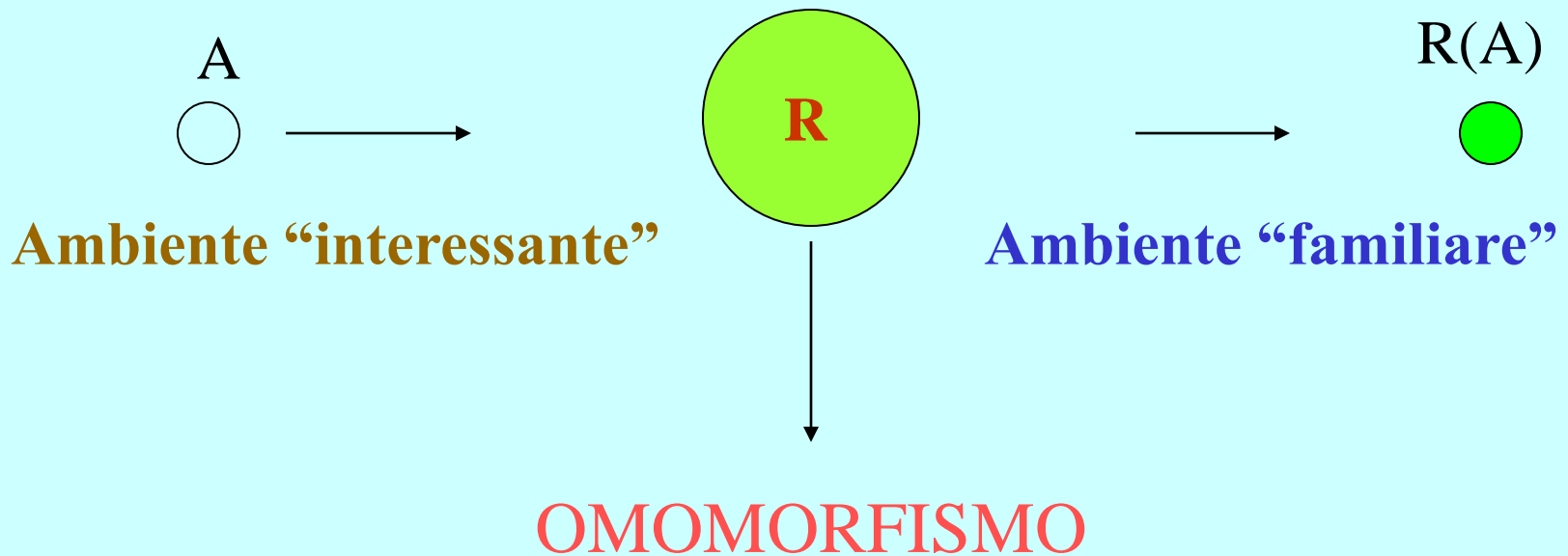
MODELLO MATEMATICO

Rappresentazione simbolica degli enti caratteristici di un “fenomeno” e delle loro relazioni.



RAPPRESENTAZIONE

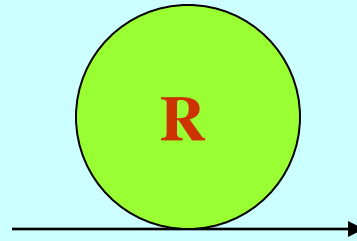
Traduzione (simbolica) di un ambiente A in un ambiente $R(A)$ che *rispetta* la struttura




RAPPRESENTAZIONE

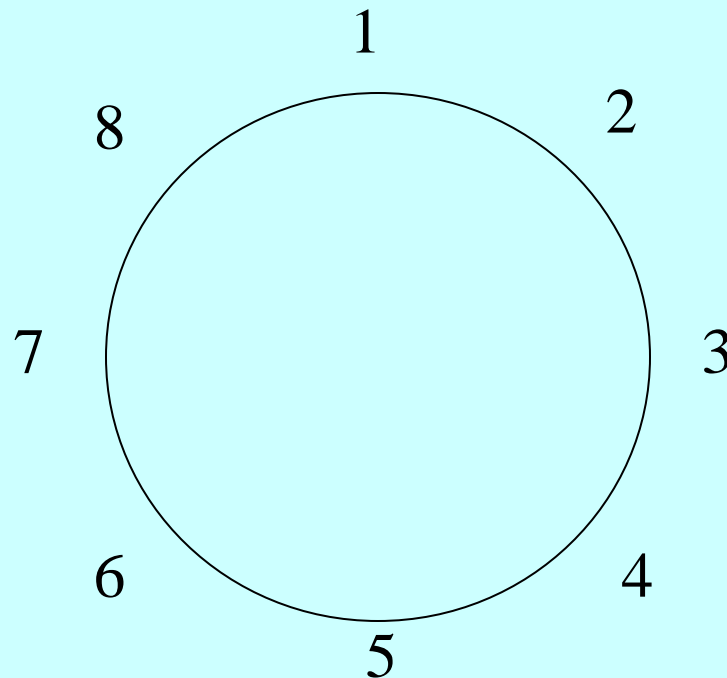
1	6	3
4		8
7	2	5

A

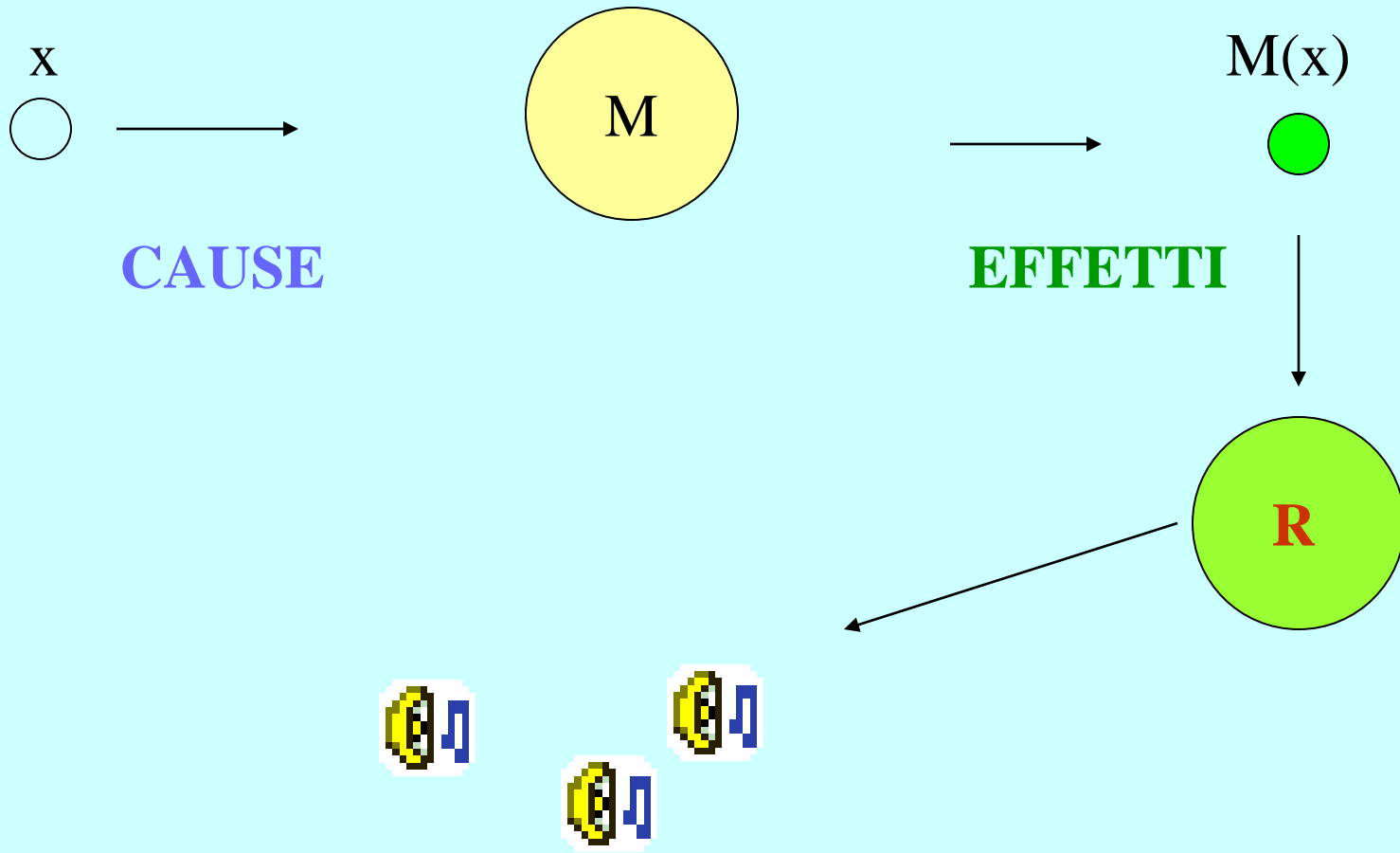


R(A)

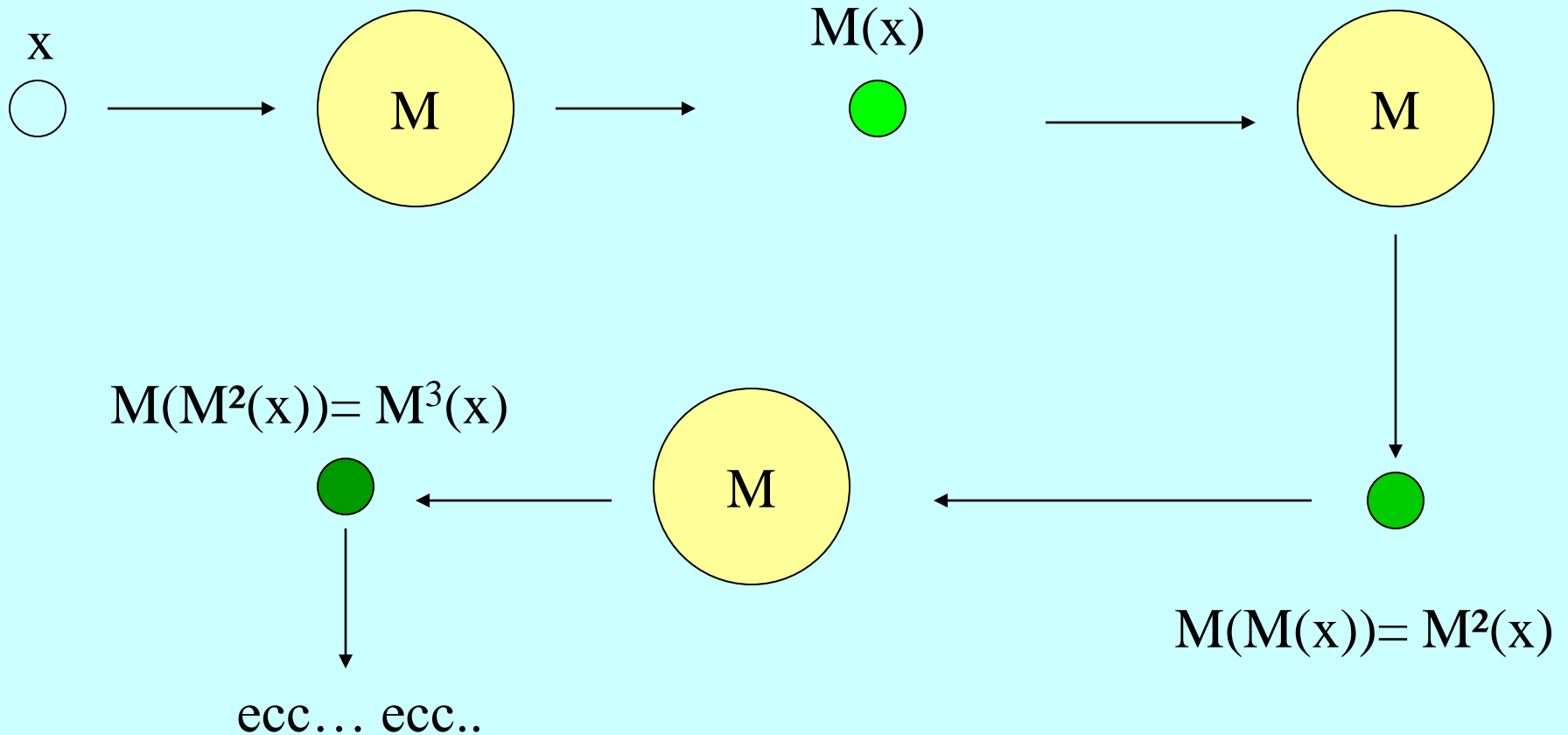


Rappresentazione musicale su modelli matematici



Processi iterativi

Manipolazione ricorsiva

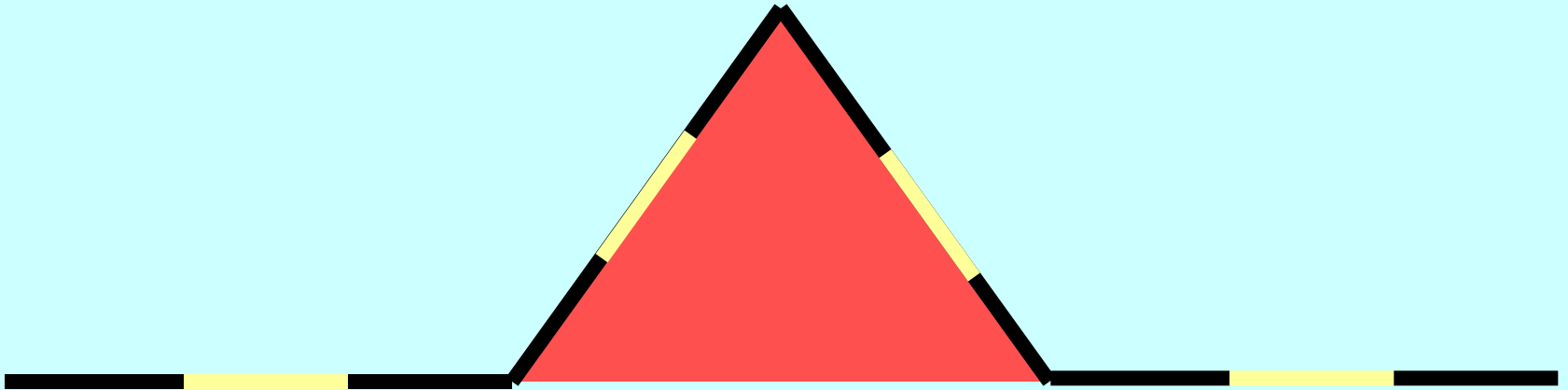


Processi iterativi



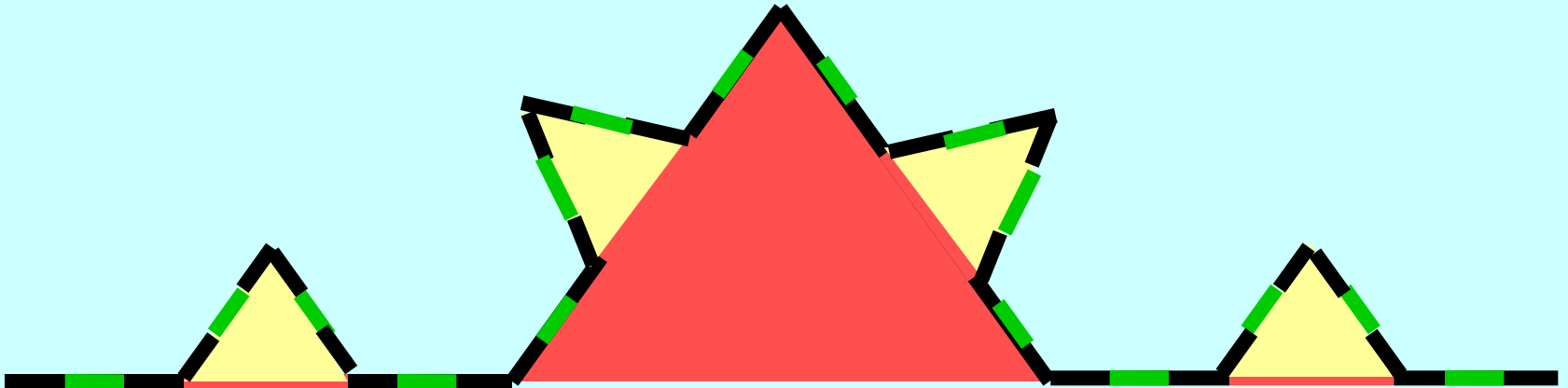
Livello 0

Processi iterativi



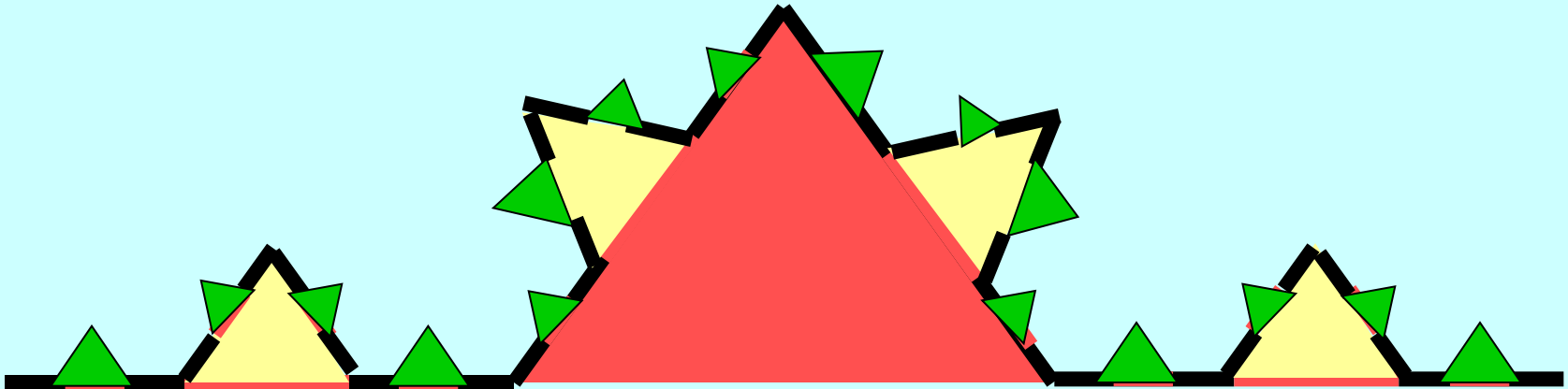
Livello 1

Processi iterativi



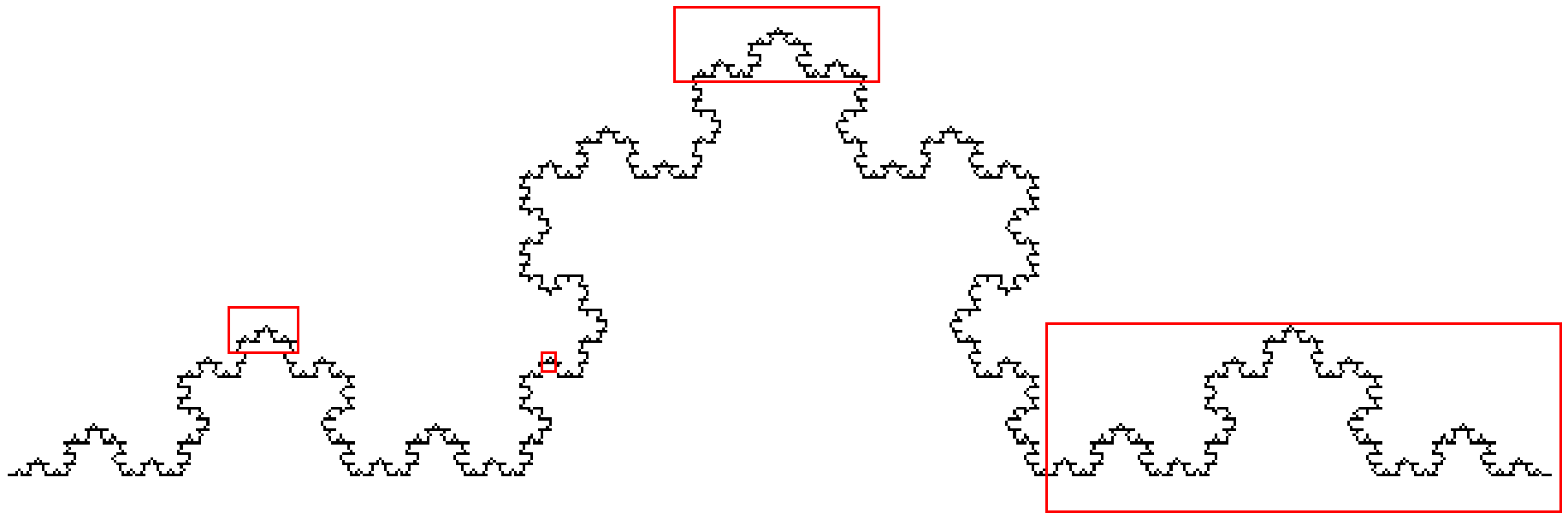
Livello 2

Processi iterativi

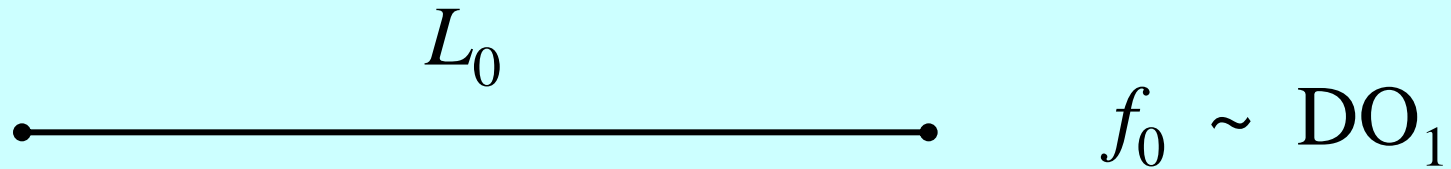


Livello 3

Processi iterativi



PITAGORA

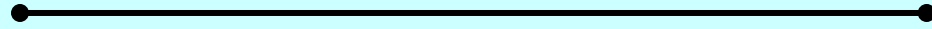


INTERVALLO DI OTTAVA

Rapporto $2=2/1$

PITAGORA

L_0



$f_0 \sim \text{DO}_1$

$L_2 = 2/3 L_0$



$f_2 = 3/2 f_0 \sim \text{SOL}$

INTERVALLO DI QUINTA

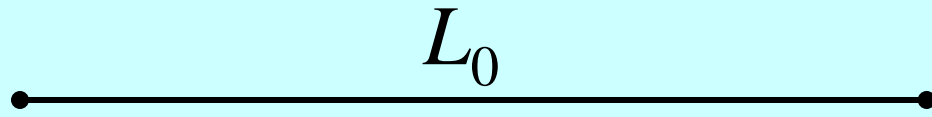
Rapporto $3/2$

$L_1 = L_0/2$



$f_1 = 2f_0 \sim \text{DO}_2$

PITAGORA



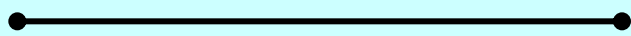
$$f_0 \sim \text{DO}_1$$

$$L_3 = 2L = 8/9L_0$$



$$f_3 = 9/8f_0 \sim \text{RE}$$

$$L_2 = 2/3L_0$$



$$f_2 = 3/2f_0 \sim \text{SOL}$$

$$L = 2/3L_2 = 4/9L_0$$



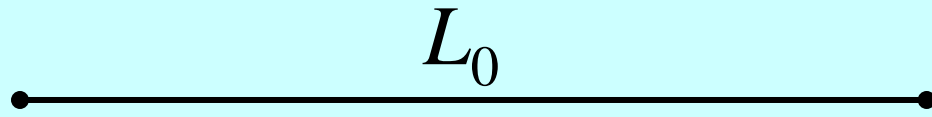
$$f = 9/4f_0 \sim \text{RE}_2$$

$$L_1 = L_0/2$$



$$f_1 = 2f_0 \sim \text{DO}_2$$

PITAGORA



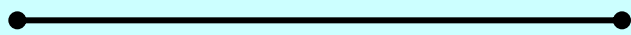
$$f_0 \sim \text{DO}_1$$

$$L_3 = 2L = 8/9L_0$$



$$f_3 = 9/8f_0 \sim \text{RE}$$

$$L_2 = 2/3L_0$$



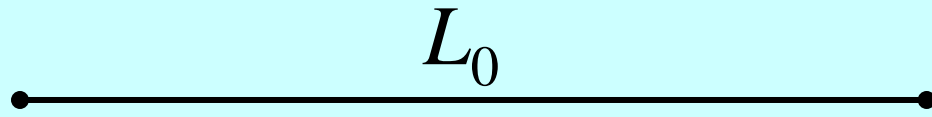
$$f_2 = 3/2f_0 \sim \text{SOL}$$

$$L_1 = L_0/2$$



$$f_1 = 2f_0 \sim \text{DO}_2$$

PITAGORA



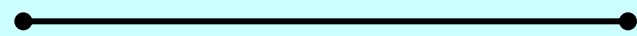
$$f_0 \sim \text{DO}_1$$

$$L_3 = 8/9 L_0$$



$$f_3 = 9/8 f_0 \sim \text{RE}$$

$$L_2 = 2/3 L_0$$



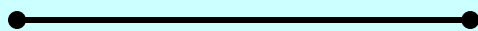
$$f_2 = 3/2 f_0 \sim \text{SOL}$$

$$L_4 = 2/3 L_3 = 16/27 L_0$$



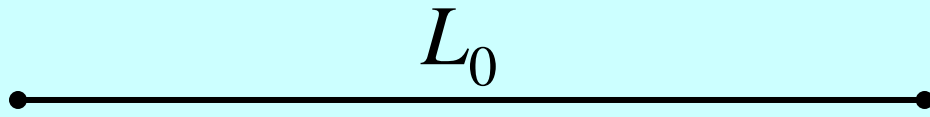
$$f_4 = 27/16 f_0 \sim \text{LA}$$

$$L_1 = L_0/2$$



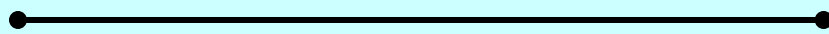
$$f_1 = 2 f_0 \sim \text{DO}_2$$

PITAGORA



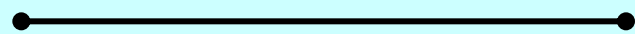
$$f_0 \sim \text{DO}_1$$

$$L_3 = 8/9 L_0$$



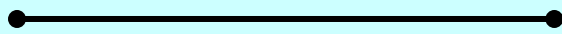
$$f_3 = 9/8 f_0 \sim \text{RE}$$

$$L_2 = 2/3 L_0$$



$$f_2 = 3/2 f_0 \sim \text{SOL}$$

$$L_4 = 16/27 L_0$$



$$f_4 = 27/16 f_0 \sim \text{LA}$$

$$L = 2/3 L_4 = 32/81 L_0$$



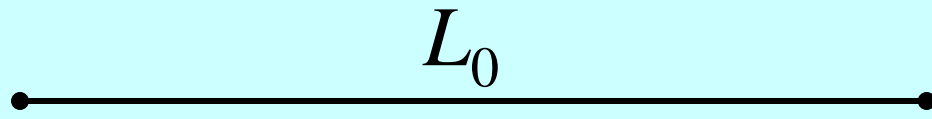
$$f = 81/32 f_0 \sim \text{MI}_2$$

$$L_1 = L_0/2$$



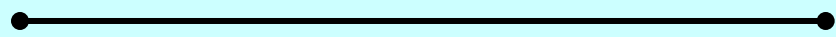
$$f_1 = 2 f_0 \sim \text{DO}_2$$

PITAGORA



$$f_0 \sim \text{DO}_1$$

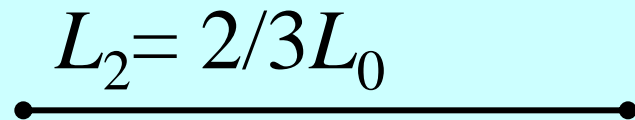
$$L_3 = 8/9 L_0$$



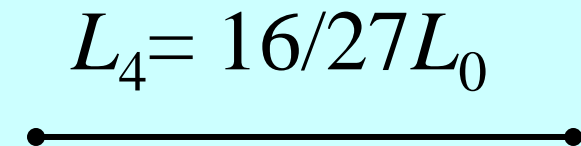
$$f_3 = 9/8 f_0 \sim \text{RE}$$



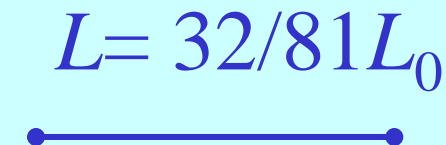
$$f_5 = 81/64 f_0 \sim \text{MI}$$



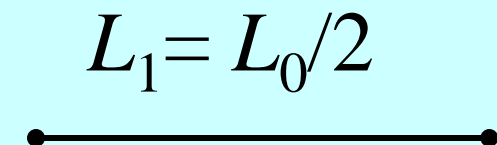
$$f_2 = 3/2 f_0 \sim \text{SOL}$$



$$f_4 = 27/16 f_0 \sim \text{LA}$$

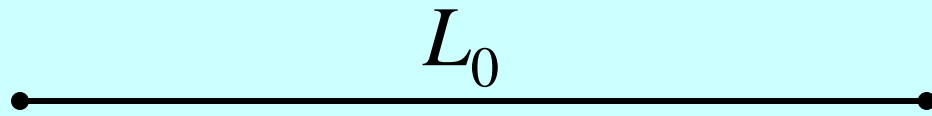


$$f = 81/32 f_0 \sim \text{MI}_2$$



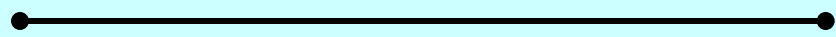
$$f_1 = 2 f_0 \sim \text{DO}_2$$

PITAGORA



$$f_0 \sim \text{DO}_1$$

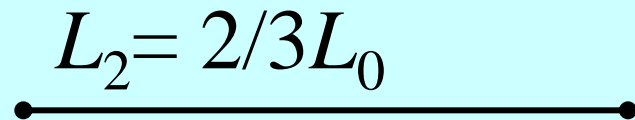
$$L_3 = 8/9 L_0$$



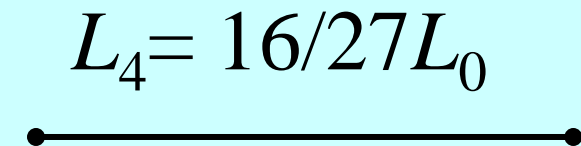
$$f_3 = 9/8 f_0 \sim \text{RE}$$



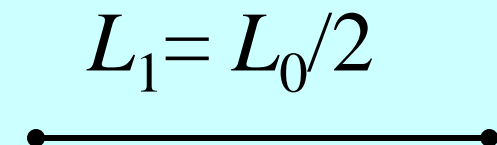
$$f_5 = 81/64 f_0 \sim \text{MI}$$



$$f_2 = 3/2 f_0 \sim \text{SOL}$$

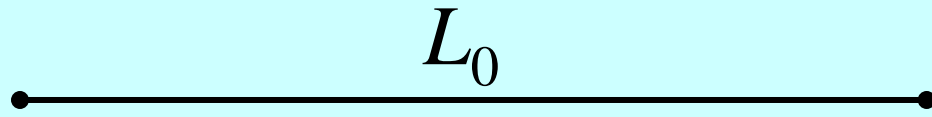


$$f_4 = 27/16 f_0 \sim \text{LA}$$

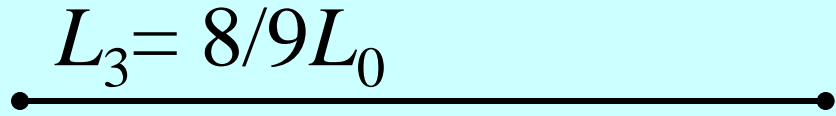


$$f_1 = 2 f_0 \sim \text{DO}_2$$

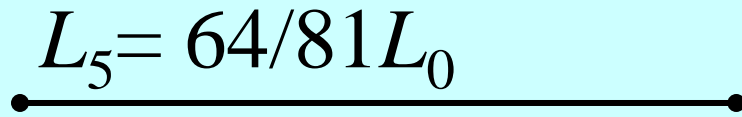
PITAGORA



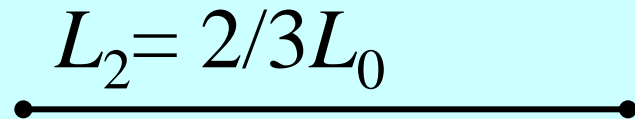
$$f_0 \sim \text{DO}_1$$



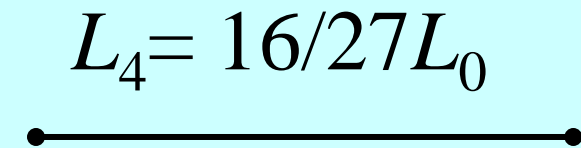
$$f_3 = 9/8 f_0 \sim \text{RE}$$



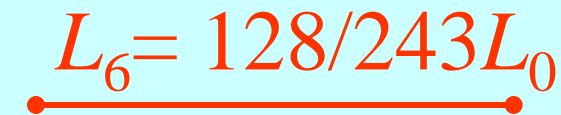
$$f_5 = 81/64 f_0 \sim \text{MI}$$



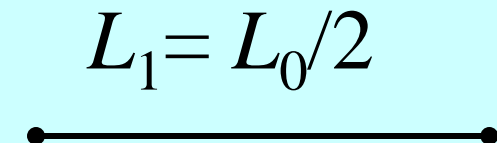
$$f_2 = 3/2 f_0 \sim \text{SOL}$$



$$f_4 = 27/16 f_0 \sim \text{LA}$$



$$f_6 = 243/128 f_0 \sim \text{SI}$$



$$f_1 = 2 f_0 \sim \text{DO}_2$$

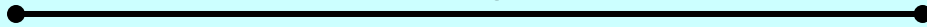
PITAGORA

$$L = 3/2L_0$$



$$f = 2/3f_0 \sim \text{FA}_0$$

$$L_0$$



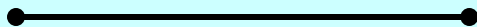
$$f_0 \sim \text{DO}_1$$

$$L_2 = 2/3L_0$$



$$f_2 = 3/2f_0 \sim \text{SOL}$$

$$L_1 = L_0/2$$



$$f_1 = 2f_0 \sim \text{DO}_2$$

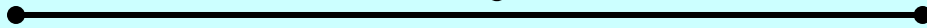
PITAGORA

$$L = 3/2L_0$$



$$f = 2/3f_0 \sim \text{FA}_0$$

$$L_0$$



$$f_0 \sim \text{DO}_1$$

$$L_7 = 3/4L_0$$



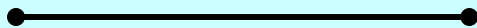
$$f_7 = 4/3f_0 \sim \text{FA}$$

$$L_2 = 2/3L_0$$



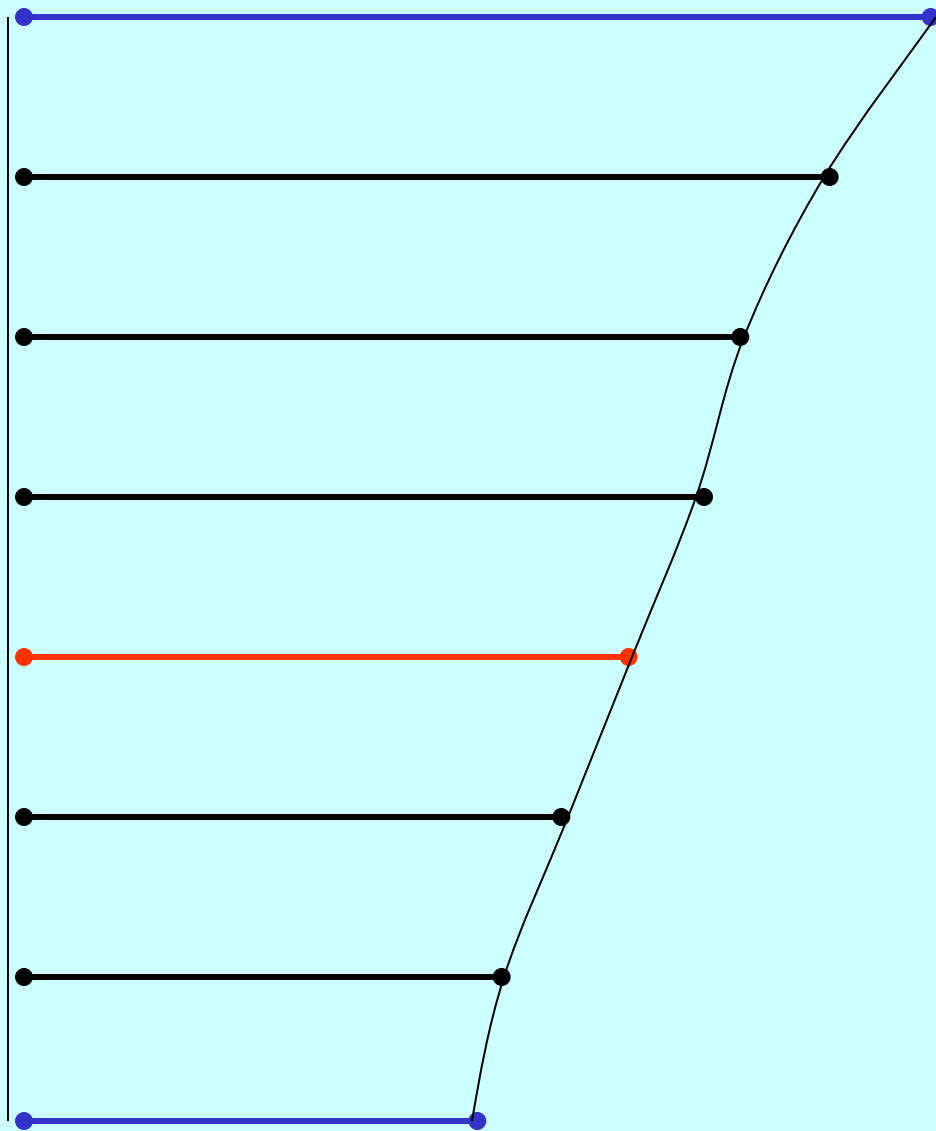
$$f_2 = 3/2f_0 \sim \text{SOL}$$

$$L_1 = L_0/2$$



$$f_1 = 2f_0 \sim \text{DO}_2$$

SCALA PITAGORICA



$$f_0 \sim DO_1$$

$$f_3 = 9/8f_0 \sim RE$$

$$f_5 = 81/64f_0 \sim MI$$

$$f_7 = 4/3f_0 \sim FA$$

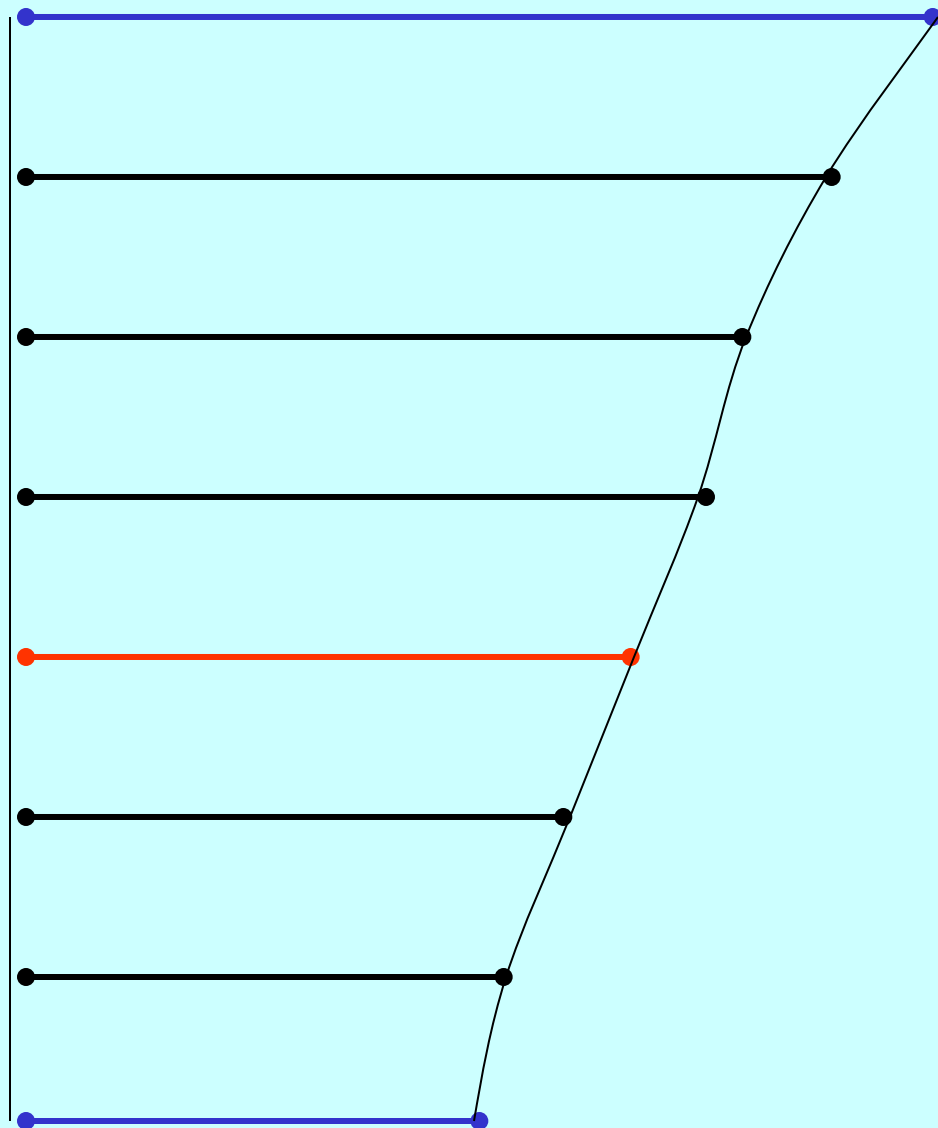
$$f_2 = 3/2f_0 \sim SOL$$

$$f_4 = 27/16f_0 \sim LA$$

$$f_6 = 243/128f_0 \sim SI$$

$$f_1 = 2f_0 \sim DO_2$$

SCALA PITAGORICA



Strumenti antichi: Parpa



POPOLAZIONI

Popolazione = *Insieme di individui*

Carattere fondamentale: *età degli individui*

Modalità del carattere età (**fasce**): *giovane, adulto, vecchio...ecc..*

POPOLAZIONI

Evoluzione popolazione nel tempo:

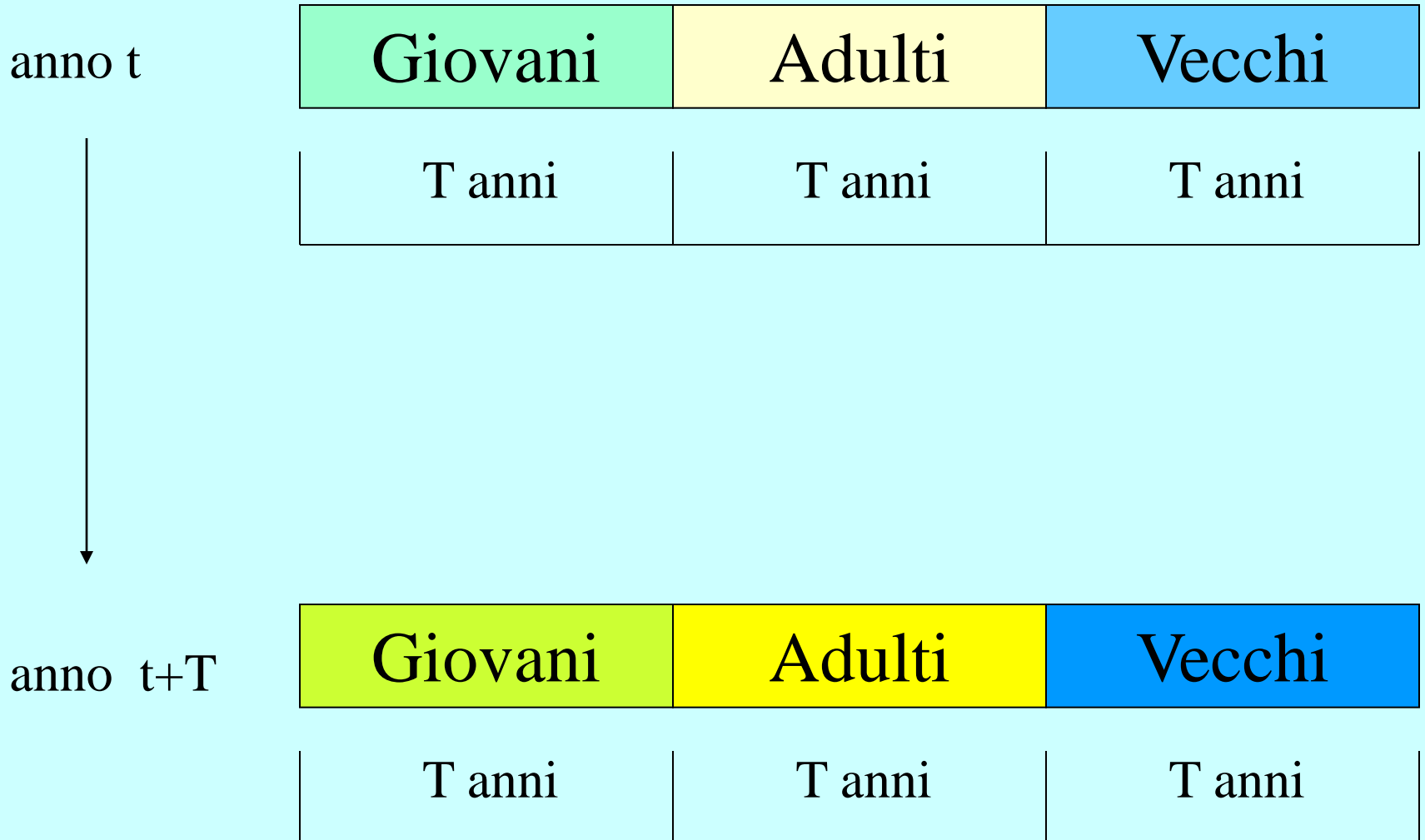
Generazione prima fascia: *fertilità*

Metamorfosi di fascia :

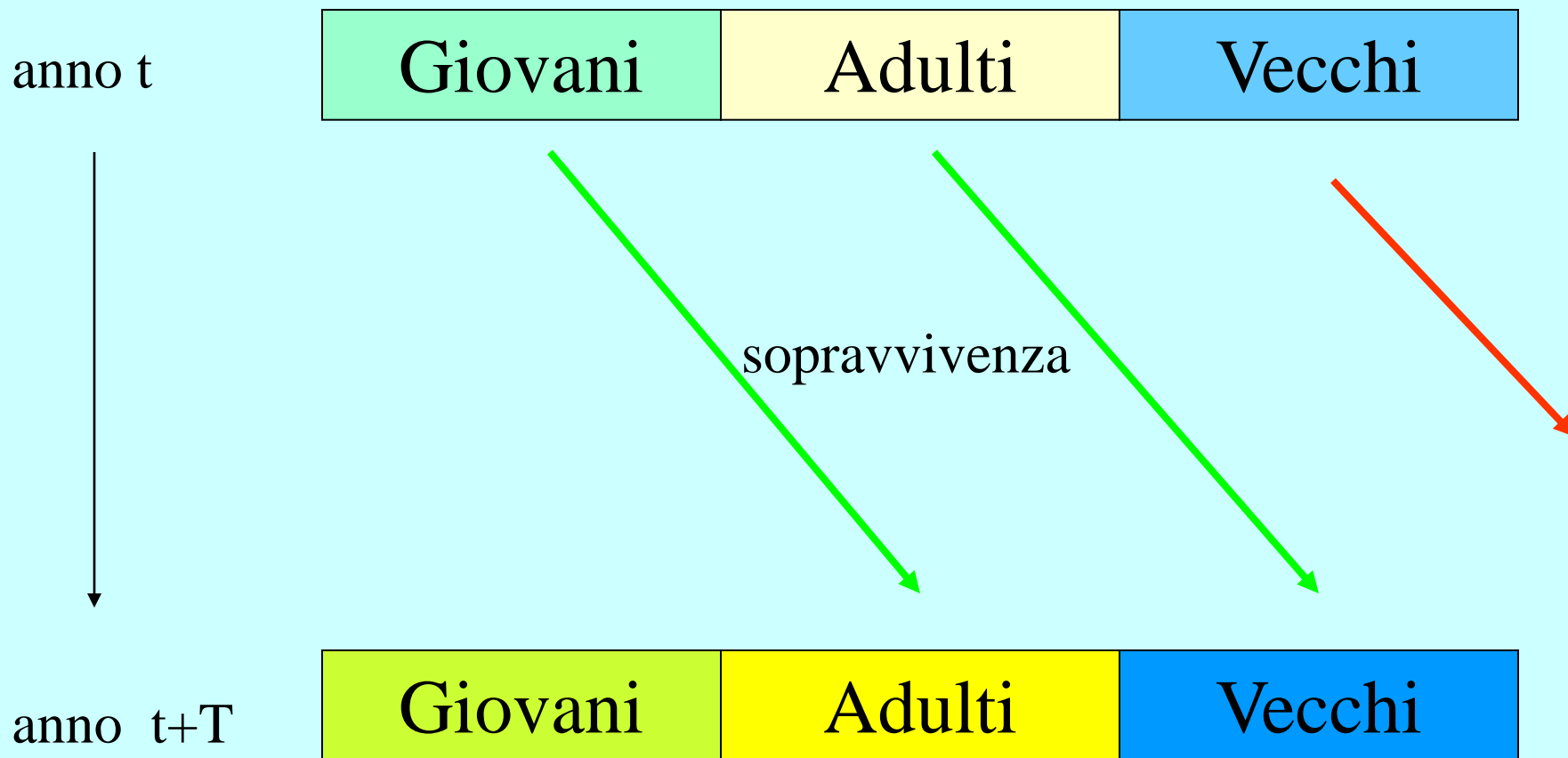
probabilità di sopravvivenza

migrazioni

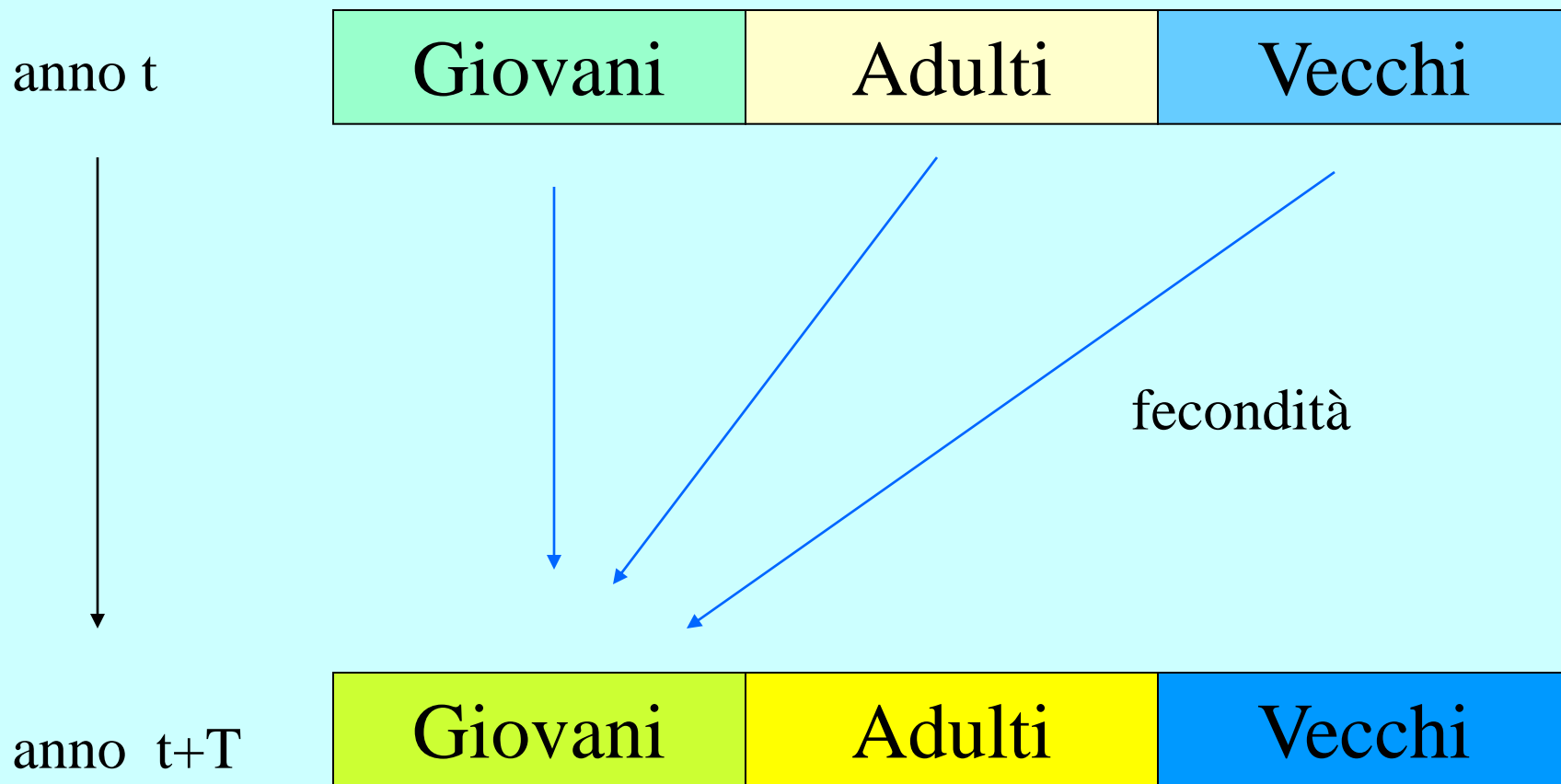
POPOLAZIONI - MODELLO DI LESLIE



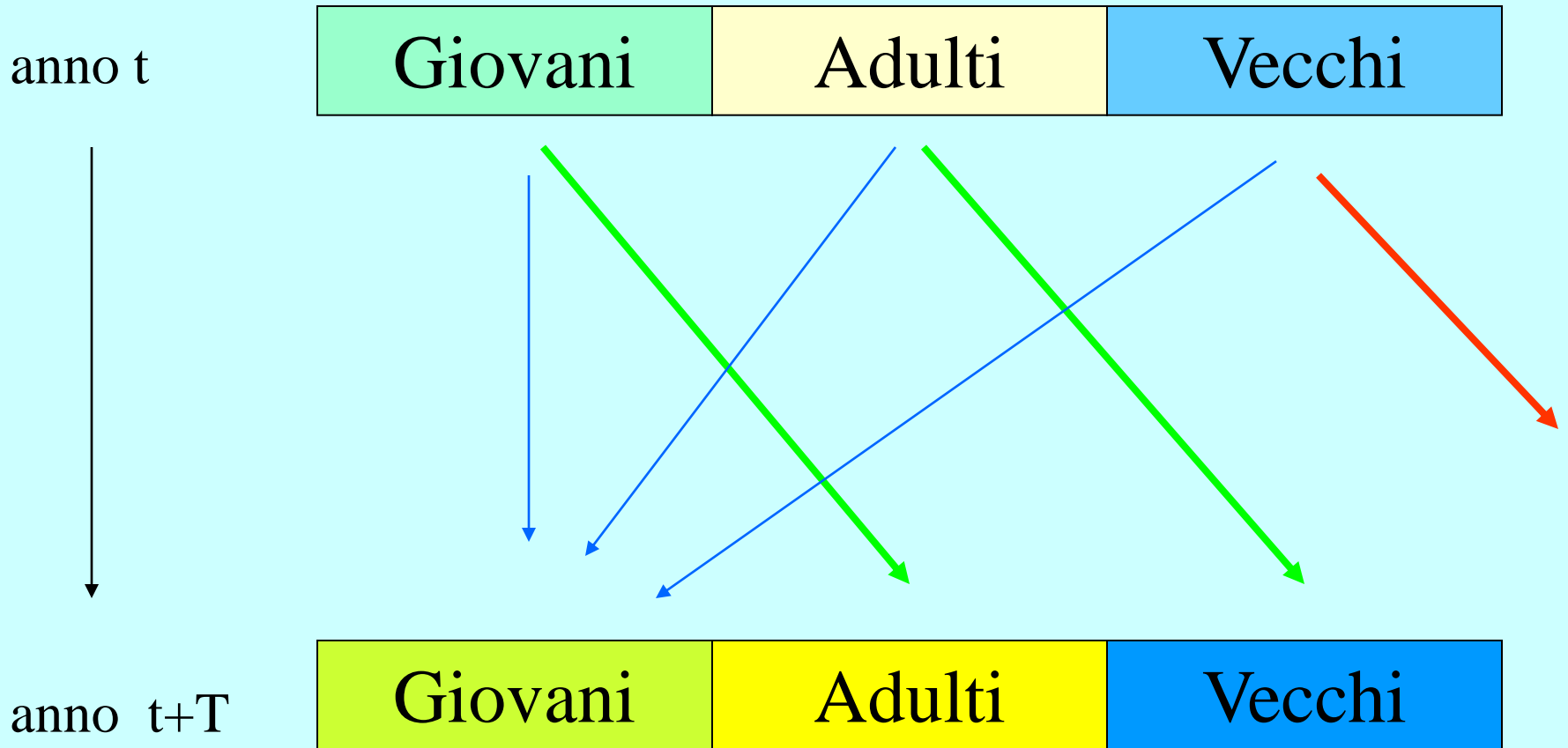
POPOLAZIONI - MODELLO DI LESLIE



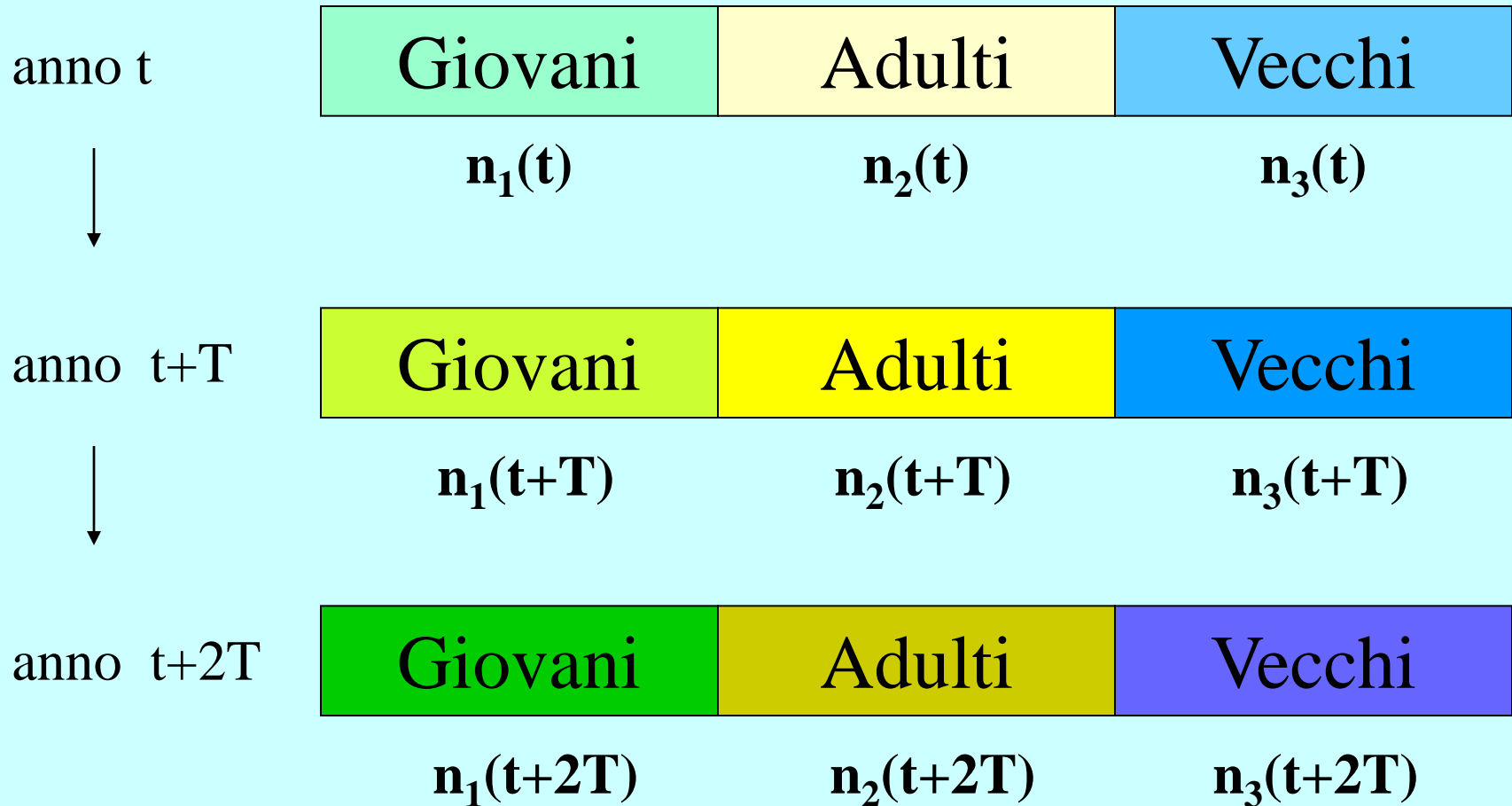
POPOLAZIONI - MODELLO DI LESLIE



POPOLAZIONI - MODELLO DI LESLIE



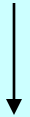
POPOLAZIONI - MODELLO DI LESLIE



POPOLAZIONI - MODELLO DI LESLIE

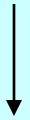
anno t

$$\mathbf{N}(t) = \left(n_1(t) ; n_2(t) ; n_3(t) \right)$$



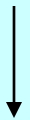
anno $t+T$

$$\mathbf{N}(t+T) = \left(n_1(t+T) ; n_2(t+T) ; n_3(t+T) \right)$$



anno $t+2T$

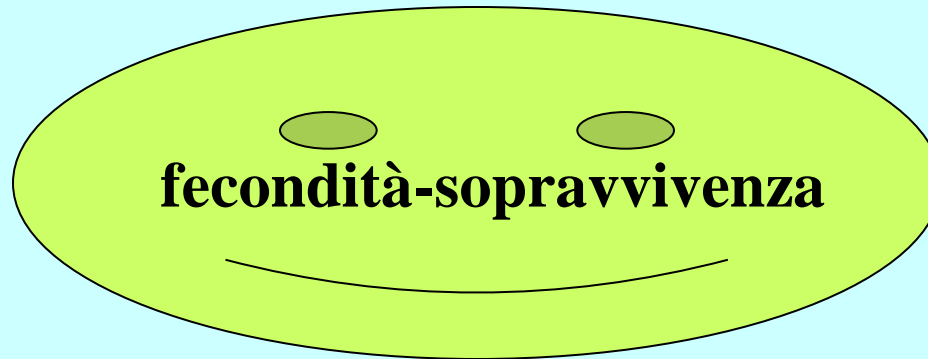
$$\mathbf{N}(t+2T) = \left(n_1(t+2T) ; n_2(t+2T) ; n_3(t+2T) \right)$$



POPOLAZIONI - MODELLO DI LESLIE

anno t

$$\mathbf{N}(t) = \left(n_1(t) ; n_2(t) ; n_3(t) \right)$$



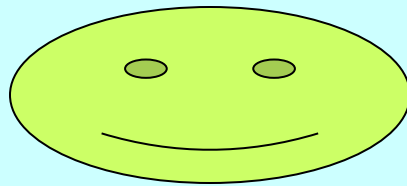
anno t+T

$$\mathbf{N}(t+T) = \left(n_1(t+T) ; n_2(t+T) ; n_3(t+T) \right)$$

POPOLAZIONI - MODELLO DI LESLIE

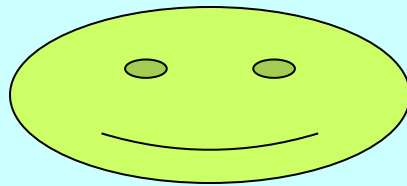
anno t

$$\mathbf{N}(t) = \left(n_1(t) ; n_2(t) ; n_3(t) \right)$$



anno t+T

$$\mathbf{N}(t+T) = \left(n_1(t+T) ; n_2(t+T) ; n_3(t+T) \right)$$



anno t+2T

$$\mathbf{N}(t+2T) = \left(n_1(t+2T) ; n_2(t+2T) ; n_3(t+2T) \right)$$

POPOLAZIONI - MODELLO DI LESLIE

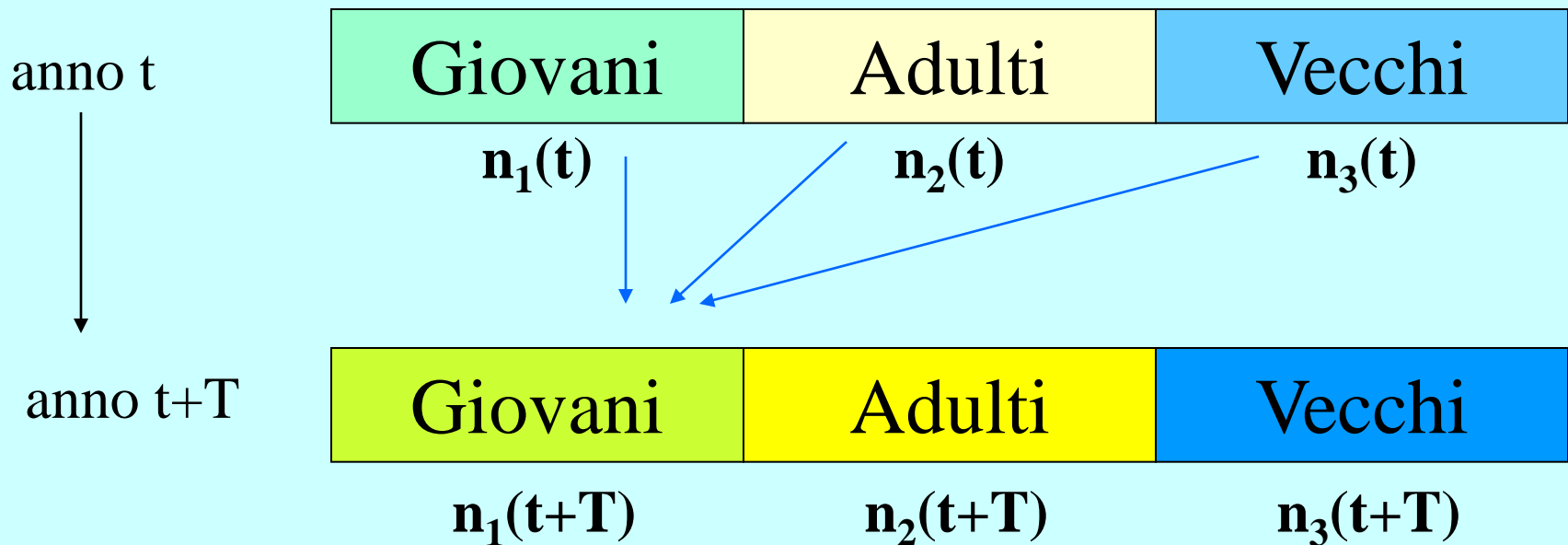
$$\begin{cases} n_1(t+T) = f_1 n_1(t) + f_2 n_2(t) + f_3 n_3(t) \\ n_2(t+T) = p_1 n_1(t) \\ n_3(t+T) = p_2 n_2(t) \end{cases}$$

POPOLAZIONI - MODELLO DI LESLIE

anno t	Giovani $n_1(t)$	Adulti $n_2(t)$	Vecchi $n_3(t)$
anno t+T	Giovani $n_1(t+T)$	Adulti $n_2(t+T)$	Vecchi $n_3(t+T)$

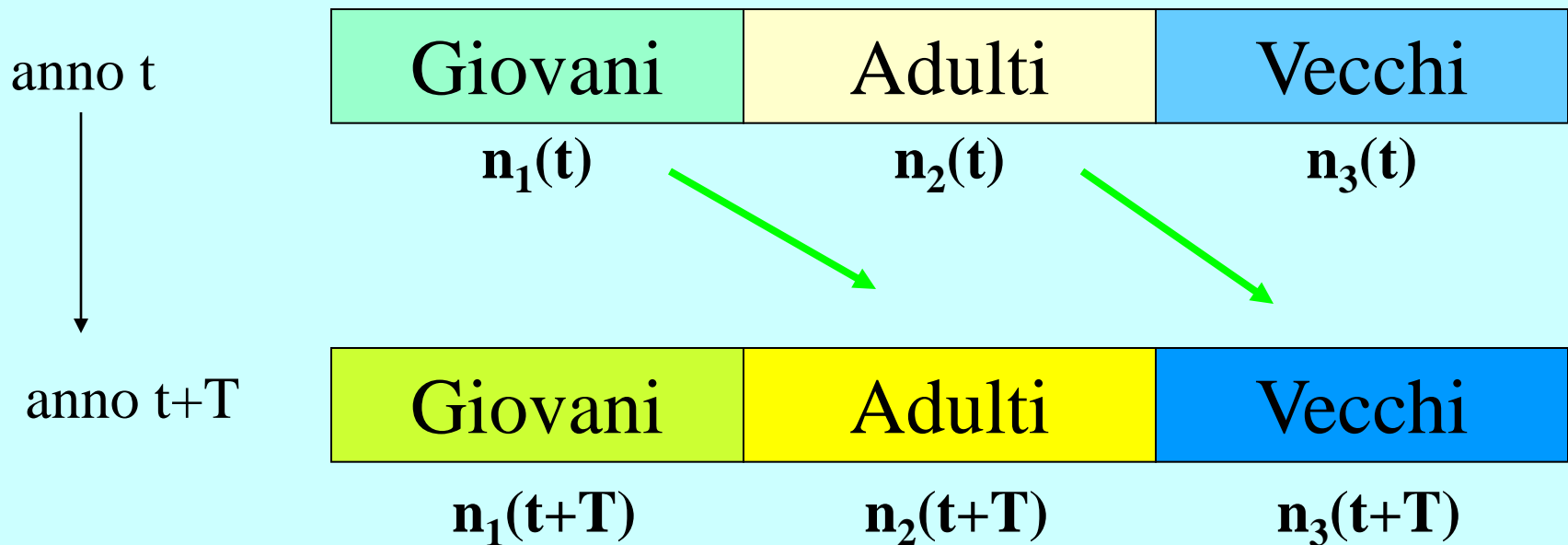
$$\begin{cases} n_1(t+T) = f_1 n_1(t) + f_2 n_2(t) + f_3 n_3(t) \\ n_2(t+T) = p_1 n_1(t) \\ n_3(t+T) = p_2 n_2(t) \end{cases}$$

POPOLAZIONI - MODELLO DI LESLIE



$$\begin{cases} n_1(t+T) = f_1 n_1(t) + f_2 n_2(t) + f_3 n_3(t) \\ n_2(t+T) = p_1 n_1(t) \\ n_3(t+T) = p_2 n_2(t) \end{cases}$$

POPOLAZIONI - MODELLO DI LESLIE



$$\begin{cases} n_1(t+T) = f_1 n_1(t) + f_2 n_2(t) + f_3 n_3(t) \\ n_2(t+T) = p_1 n_1(t) \\ n_3(t+T) = p_2 n_2(t) \end{cases} \quad (p_1 < 1, \quad p_2 < 1)$$

POPOLAZIONI - MODELLO DI LESLIE

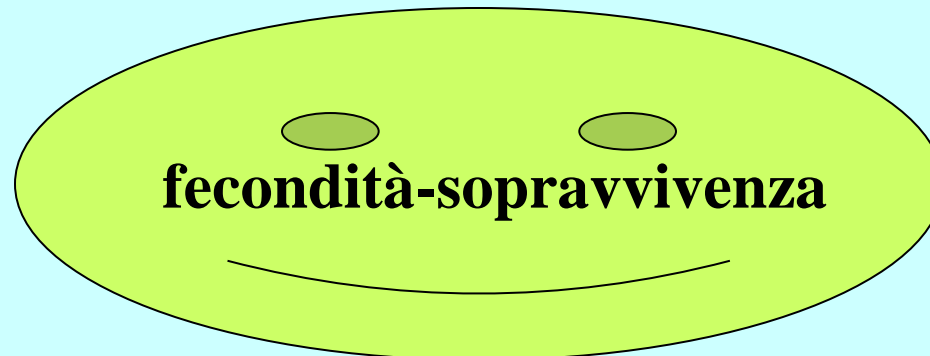
$$\begin{cases} n_1(t+T) = f_1 n_1(t) + f_2 n_2(t) + f_3 n_3(t) \\ n_2(t+T) = p_1 n_1(t) \\ n_3(t+T) = p_2 n_2(t) \end{cases}$$

$$\begin{pmatrix} n_1(t+T) \\ n_2(t+T) \\ n_3(t+T) \end{pmatrix} = \begin{pmatrix} f_1 & f_2 & f_3 \\ p_1 & 0 & 0 \\ 0 & p_2 & 0 \end{pmatrix} \begin{pmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \end{pmatrix}$$

POPOLAZIONI - MODELLO DI LESLIE

$$A = \begin{pmatrix} f_1 & f_2 & f_3 \\ p_1 & 0 & 0 \\ 0 & p_2 & 0 \end{pmatrix}$$

MATRICE DI LESLIE



POPOLAZIONI - MODELLO DI LESLIE

$$\begin{pmatrix} n_1(t+T) \\ n_2(t+T) \\ n_3(t+T) \end{pmatrix} = \begin{pmatrix} f_1 & f_2 & f_3 \\ p_1 & 0 & 0 \\ 0 & p_2 & 0 \end{pmatrix} \begin{pmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \end{pmatrix}$$

$$\mathbf{N}(t+T) = \mathbf{A}\mathbf{N}(t)$$

POPOLAZIONI - MODELLO DI LESLIE

$$\mathbf{N}(t + T) = A\mathbf{N}(t)$$

$$\mathbf{N}(t + 2T) = A\mathbf{N}(t+T) = AA\mathbf{N}(t) = A^2\mathbf{N}(t)$$

$$\mathbf{N}(t + 3T) = A\mathbf{N}(t+2T) = AAA\mathbf{N}(t) = A^3\mathbf{N}(t)$$

$$\mathbf{N}(t + 4T) = A\mathbf{N}(t+3T) = AAAA\mathbf{N}(t) = A^4\mathbf{N}(t)$$

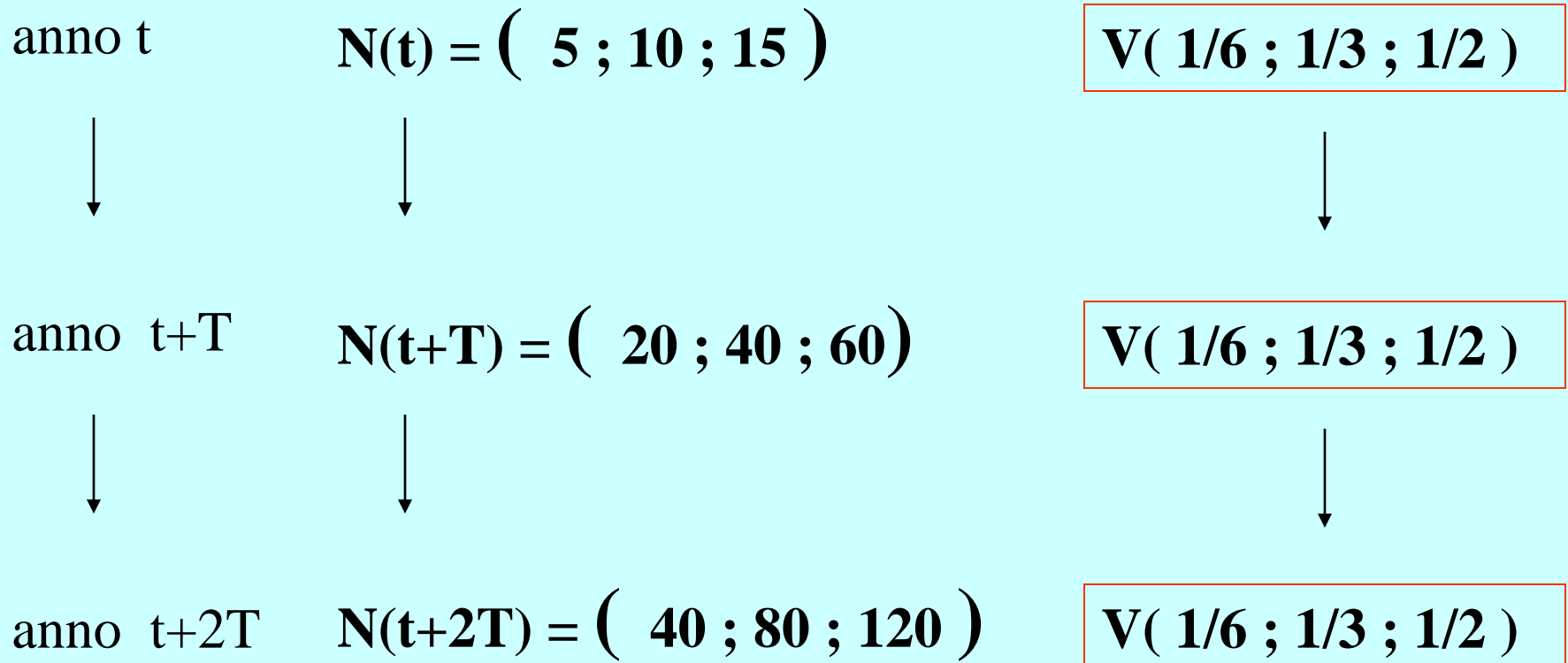
$$\mathbf{N}(t + kT) = A^k\mathbf{N}(t)$$

POPOLAZIONI - MODELLO DI LESLIE

$$\mathbf{N}(t + kT) = A^k \mathbf{N}(t)$$

$$\mathbf{N}(kT) = A^k \mathbf{N}(0)$$

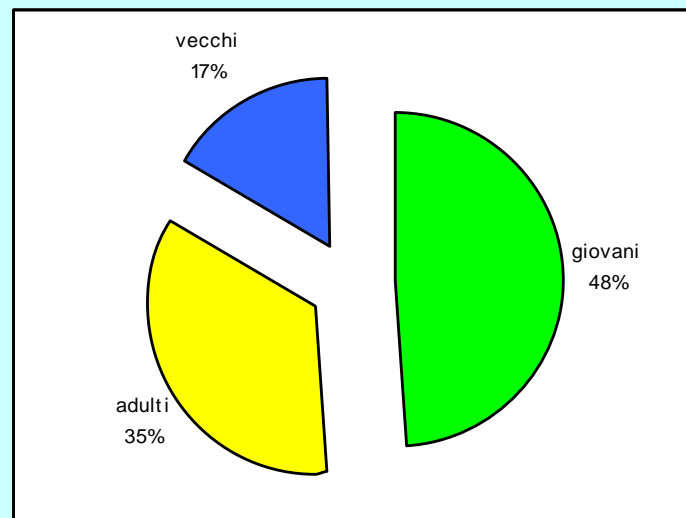
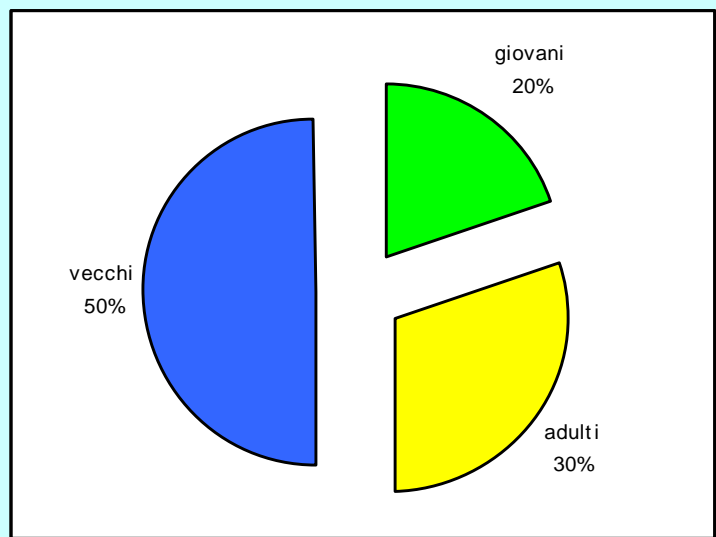
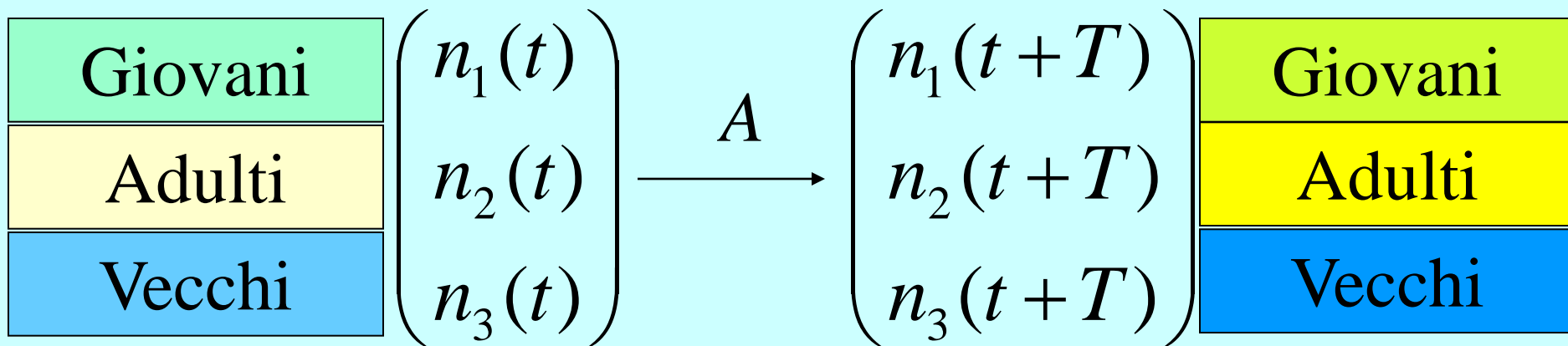
POPOLAZIONI - MODELLO DI LESLIE



POPOLAZIONI - MODELLO DI LESLIE

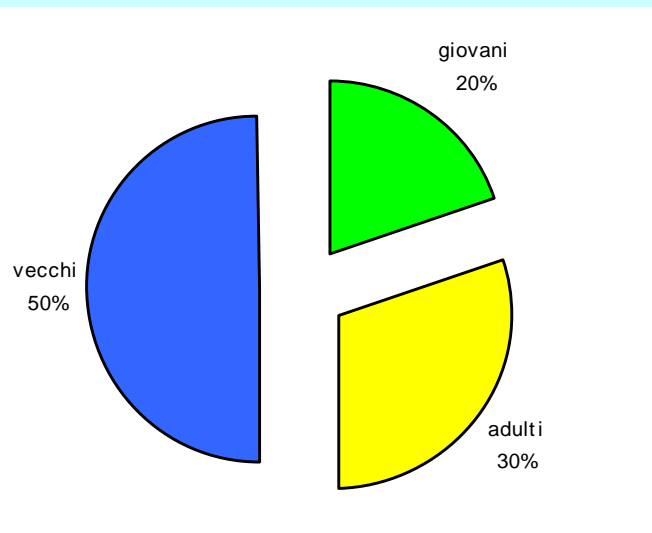
anno t

anno t+T



POPOLAZIONI - MODELLO DI LESLIE

VETTORE PROFILO



$$\begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix} = \begin{pmatrix} \frac{n_1}{n_1 + n_2 + n_3} \\ \frac{n_2}{n_1 + n_2 + n_3} \\ \frac{n_3}{n_1 + n_2 + n_3} \end{pmatrix}$$

POPOLAZIONI - MODELLO DI LESLIE

f_1	f_2	p_1			
1	2	0,2			
t	$n_1(t)$	$n_2(t)$	$n(t)$	$v_1(t)$	$v_2(t)$
0	1	0	1	1	0
1	1	0,2	1,2	0,8333333333	0,166666667
2	1,4	0,2	1,6	0,875	0,125
3	1,8	0,28	2,08	0,865384615	0,134615385
4	2,36	0,36	2,72	0,867647059	0,132352941
5	3,08	0,472	3,552	0,867117117	0,132882883
6	4,024	0,616	4,64	0,867241379	0,132758621
7	5,256	0,8048	6,0608	0,867212249	0,132787751
8	6,8656	1,0512	7,9168	0,867219078	0,132780922
9	8,968	1,37312	10,34112	0,867217477	0,132782523
10	11,71424	1,7936	13,50784	0,867217853	0,132782147
11	15,30144	2,342848	17,644288	0,867217765	0,132782235
12	19,987136	3,060288	23,047424	0,867217785	0,132782215
13	26,107712	3,9974272	30,1051392	0,867217781	0,132782219
14	34,1025664	5,2215424	39,3241088	0,867217782	0,132782218
15	44,5456512	6,82051328	51,36616448	0,867217781	0,132782219
16	58,18667776	8,90913024	67,095808	0,867217781	0,132782219
17	76,00493824	11,63733555	87,64227379	0,867217781	0,132782219
18	99,27960934	15,20098765	114,480597	0,867217781	0,132782219
19	129,6815846	19,85592187	149,5375065	0,867217781	0,132782219
20	169,3934284	25,93631693	195,3297453	0,867217781	0,132782219

POPOLAZIONI - MODELLO DI LESLIE

f_1	f_2	p_1			
0,4	0,5	0,5			
t	$n_1(t)$	$n_2(t)$	$n(t)$	$v_1(t)$	$v_2(t)$
0	3	2	5	0,6	0,4
1	2,2	1,5	3,7	0,594594595	0,405405405
2	1,63	1,1	2,73	0,597069597	0,402930403
3	1,202	0,815	2,017	0,595934556	0,404065444
4	0,8883	0,601	1,4893	0,59645471	0,40354529
5	0,65582	0,44415	1,0997	0,59621626	0,40378374
6	0,484403	0,32791	0,812313	0,596325554	0,403674446
7	0,3577162	0,2422015	0,5999177	0,596275456	0,403724544
8	0,26418723	0,1788581	0,44304533	0,596298419	0,403701581
9	0,195103942	0,132093615	0,327197557	0,596287893	0,403712107
10	0,144088384	0,097551971	0,241640355	0,596292718	0,403707282
11	0,106411339	0,072044192	0,178455531	0,596290507	0,403709493
12	0,078586632	0,05320567	0,131792301	0,59629152	0,40370848
13	0,058037488	0,039293316	0,097330803	0,596291056	0,403708944
14	0,042861653	0,029018744	0,071880397	0,596291269	0,403708731
15	0,031654033	0,021430826	0,05308486	0,596291171	0,403708829
16	0,023377026	0,015827017	0,039204043	0,596291216	0,403708784
17	0,017264319	0,011688513	0,028952832	0,596291195	0,403708805
18	0,012749984	0,008632159	0,021382144	0,596291205	0,403708795
19	0,009416073	0,006374992	0,015791065	0,5962912	0,4037088
20	0,006953925	0,004708037	0,011661962	0,596291202	0,403708798

POPOLAZIONI - MODELLO DI LESLIE

f_1	f_2	p_1			
0	2	0,5			
t	$n_1(t)$	$n_2(t)$	$n(t)$	$v_1(t)$	$v_2(t)$
0	1	2	3	0,333333333	0,666666667
1	4	0,5	4,5	0,888888889	0,111111111
2	1	2	3	0,333333333	0,666666667
3	4	0,5	4,5	0,888888889	0,111111111
4	1	2	3	0,333333333	0,666666667
5	4	0,5	4,5	0,888888889	0,111111111
6	1	2	3	0,333333333	0,666666667
7	4	0,5	4,5	0,888888889	0,111111111
8	1	2	3	0,333333333	0,666666667
9	4	0,5	4,5	0,888888889	0,111111111
10	1	2	3	0,333333333	0,666666667
11	4	0,5	4,5	0,888888889	0,111111111
12	1	2	3	0,333333333	0,666666667
13	4	0,5	4,5	0,888888889	0,111111111
14	1	2	3	0,333333333	0,666666667
15	4	0,5	4,5	0,888888889	0,111111111
16	1	2	3	0,333333333	0,666666667
17	4	0,5	4,5	0,888888889	0,111111111
18	1	2	3	0,333333333	0,666666667
19	4	0,5	4,5	0,888888889	0,111111111
20	1	2	3	0,333333333	0,666666667

POPOLAZIONI - MODELLO DI LESLIE

f_1	f_2	p_1			
0	1	0,25			
t	$n_1(t)$	$n_2(t)$	$n(t)$	$v_1(t)$	$v_2(t)$
0	2	2	4	0,5	0,5
1	2	0,5	2,5	0,8	0,2
2	0,5	0,5	1	0,5	0,5
3	0,5	0,125	0,625	0,8	0,2
4	0,125	0,125	0,25	0,5	0,5
5	0,125	0,03125	0,15625	0,8	0,2
6	0,03125	0,03125	0,0625	0,5	0,5
7	0,03125	0,0078125	0,0390625	0,8	0,2
8	0,0078125	0,0078125	0,015625	0,5	0,5
9	0,0078125	0,001953125	0,009765625	0,8	0,2
10	0,001953125	0,001953125	0,00390625	0,5	0,5
11	0,001953125	0,000488281	0,002441406	0,8	0,2
12	0,000488281	0,000488281	0,000976563	0,5	0,5
13	0,000488281	0,00012207	0,000610352	0,8	0,2
14	0,00012207	0,00012207	0,000244141	0,5	0,5
15	0,00012207	3,05176E-05	0,000152588	0,8	0,2
16	3,05176E-05	3,05176E-05	6,10352E-05	0,5	0,5
17	3,05176E-05	7,62939E-06	3,8147E-05	0,8	0,2
18	7,62939E-06	7,62939E-06	1,52588E-05	0,5	0,5
19	7,62939E-06	1,90735E-06	9,53674E-06	0,8	0,2
20	1,90735E-06	1,90735E-06	3,8147E-06	0,5	0,5

POPOLAZIONI - MODELLO DI LESLIE

f_1	f_2	p_1			
0	3	0,5			
t	$n_1(t)$	$n_2(t)$	$n(t)$	$v_1(t)$	$v_2(t)$
0	1	2	3	0,333333333	0,666666667
1	6	0,5	6,5	0,923076923	0,076923077
2	1,5	3	4,5	0,333333333	0,666666667
3	9	0,75	9,75	0,923076923	0,076923077
4	2,25	4,5	6,75	0,333333333	0,666666667
5	13,5	1,125	14,625	0,923076923	0,076923077
6	3,375	6,75	10,125	0,333333333	0,666666667
7	20,25	1,6875	21,9375	0,923076923	0,076923077
8	5,0625	10,125	15,1875	0,333333333	0,666666667
9	30,375	2,53125	32,90625	0,923076923	0,076923077
10	7,59375	15,1875	22,78125	0,333333333	0,666666667
11	45,5625	3,796875	49,359375	0,923076923	0,076923077
12	11,390625	22,78125	34,171875	0,333333333	0,666666667
13	68,34375	5,6953125	74,0390625	0,923076923	0,076923077
14	17,0859375	34,171875	51,2578125	0,333333333	0,666666667
15	102,515625	8,54296875	111,0585938	0,923076923	0,076923077
16	25,62890625	51,2578125	76,88671875	0,333333333	0,666666667
17	153,7734375	12,81445313	166,5878906	0,923076923	0,076923077
18	38,44335938	76,88671875	115,3300781	0,333333333	0,666666667
19	230,6601563	19,22167969	249,8818359	0,923076923	0,076923077
20	57,66503906	115,3300781	172,9951172	0,333333333	0,666666667

POPOLAZIONI - MODELLO DI LESLIE

f_1	f_2	p_1			
0	2	0,5			
t	$n_1(t)$	$n_2(t)$	$n(t)$	$v_1(t)$	$v_2(t)$
0	2	1	3	0,666666667	0,333333333
1	2	1	3	0,666666667	0,333333333
2	2	1	3	0,666666667	0,333333333
3	2	1	3	0,666666667	0,333333333
4	2	1	3	0,666666667	0,333333333
5	2	1	3	0,666666667	0,333333333
6	2	1	3	0,666666667	0,333333333
7	2	1	3	0,666666667	0,333333333
8	2	1	3	0,666666667	0,333333333
9	2	1	3	0,666666667	0,333333333
10	2	1	3	0,666666667	0,333333333
11	2	1	3	0,666666667	0,333333333
12	2	1	3	0,666666667	0,333333333
13	2	1	3	0,666666667	0,333333333
14	2	1	3	0,666666667	0,333333333
15	2	1	3	0,666666667	0,333333333
16	2	1	3	0,666666667	0,333333333
17	2	1	3	0,666666667	0,333333333
18	2	1	3	0,666666667	0,333333333
19	2	1	3	0,666666667	0,333333333
20	2	1	3	0,666666667	0,333333333

La geometria di A

$$A\vec{u} = \lambda\vec{u}$$



Autovettore



Autovalore

Gli autovettori sono dei generatori

$$\vec{N} = \alpha_1 \vec{u}_1 + \alpha_2 \vec{u}_2 + \alpha_3 \vec{u}_3$$

$$A\vec{N} = \alpha_1 A\vec{u}_1 + \alpha_2 A\vec{u}_2 + \alpha_3 A\vec{u}_3 = \\ \alpha_1 \lambda_1 \vec{u}_1 + \alpha_2 \lambda_2 \vec{u}_2 + \alpha_3 \lambda_3 \vec{u}_3$$

$$A^n \vec{N} = \alpha_1 \lambda_1^n \vec{u}_1 + \alpha_2 \lambda_2^n \vec{u}_2 + \alpha_3 \lambda_3^n \vec{u}_3$$

TEOREMA

Esiste un solo
autovalore positivo

$$\lambda_1$$

ed è dominante

$$\lambda_1 \geq |\lambda_i|$$

Esempi

λ_1 strettamente dominante $\lambda_1 > |\lambda_i|$

$\lambda_1 > 1$ la popolazione esplode e si profila come \vec{u}_1

$$\begin{aligned} A^n \vec{N} &= \alpha_1 \lambda_1^n \vec{u}_1 + \alpha_2 \lambda_2^n \vec{u}_2 + \alpha_3 \lambda_3^n \vec{u}_3 \\ &= \lambda_1^n \left(\alpha_1 \vec{u}_1 + \alpha_2 \frac{\lambda_2^n}{\lambda_1^n} \vec{u}_2 + \alpha_3 \frac{\lambda_3^n}{\lambda_1^n} \vec{u}_3 \right) \\ &\cong \alpha_1 \lambda_1^n \vec{u}_1 \end{aligned}$$

$\lambda_1 < 1$ la popolazione si profila come \vec{u}_1 ma si estingue

Esempi

λ_1 non strettamente dominante

per esempio $\lambda_1 = -\lambda_2$ in una popolazione a due fasce

$$\begin{aligned} A^n \vec{N} &= \lambda_1^n \left(\alpha_1 \vec{u}_1 + \alpha_2 \frac{\lambda_2^n}{\lambda_1^n} \vec{u}_2 \right) \\ &= \lambda_1^n (\alpha_1 \vec{u}_1 + \alpha_2 (-1)^n \vec{u}_2) \end{aligned}$$

il profilo oscilla

la popolazione esplode se $\lambda_1 > 1$

la popolazione si estingue se $\lambda_1 < 1$

la popolazione oscilla se $\lambda_1 = 1$

POPOLAZIONI - MODELLO DI LESLIE

Popolazione crescente - profilo convergente

Popolazione decrescente - profilo convergente

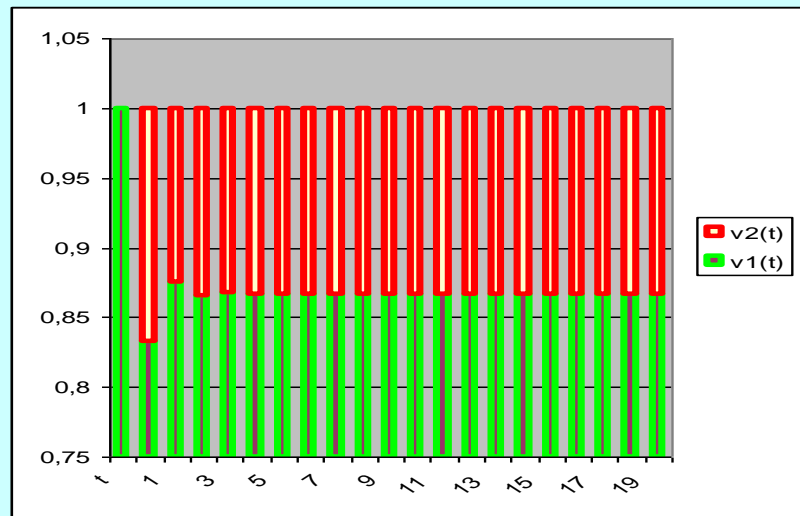
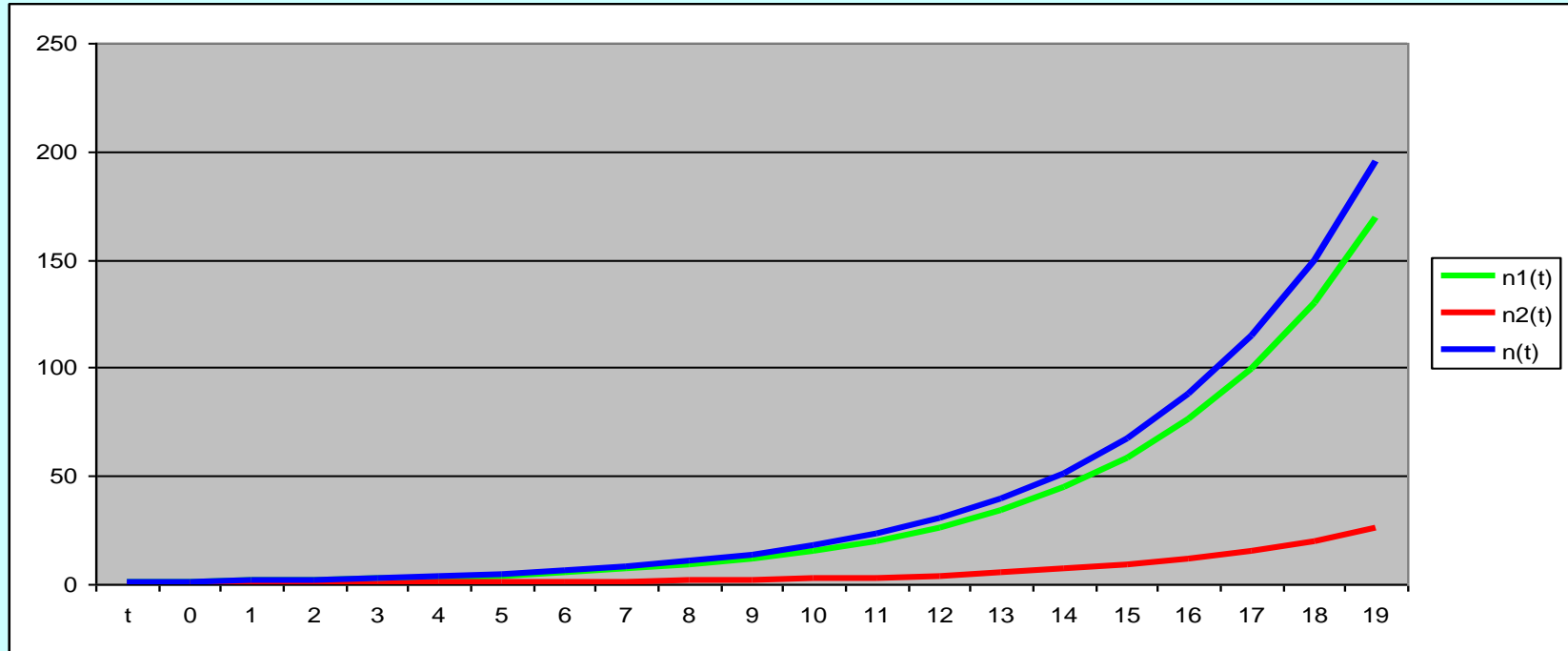
Popolazione pseudocrescente - profilo oscillante

Popolazione pseudodecrescente - profilo oscillante

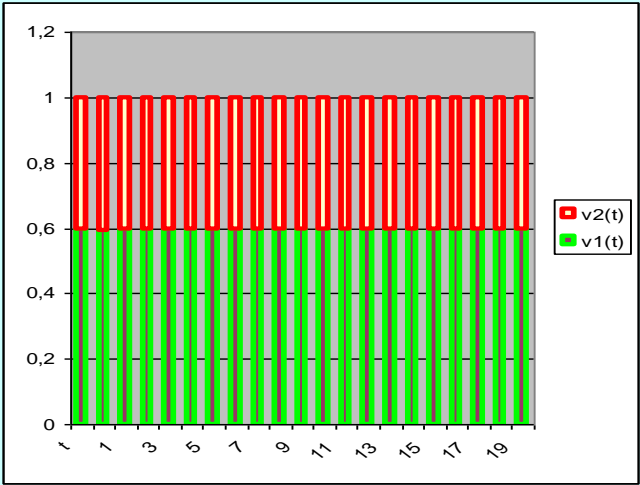
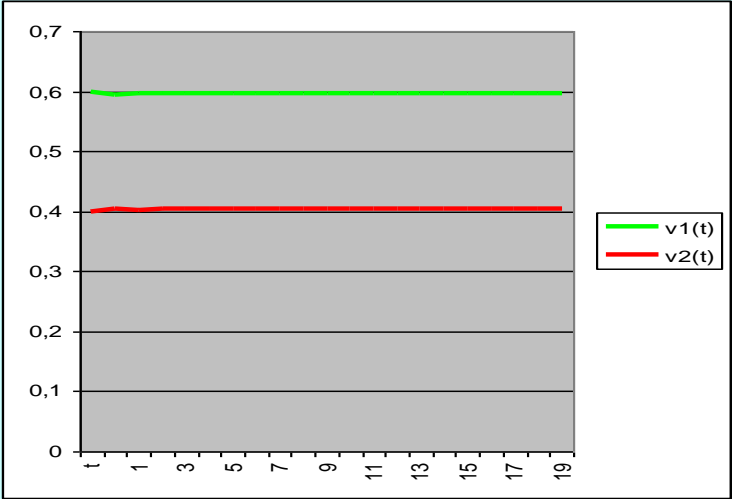
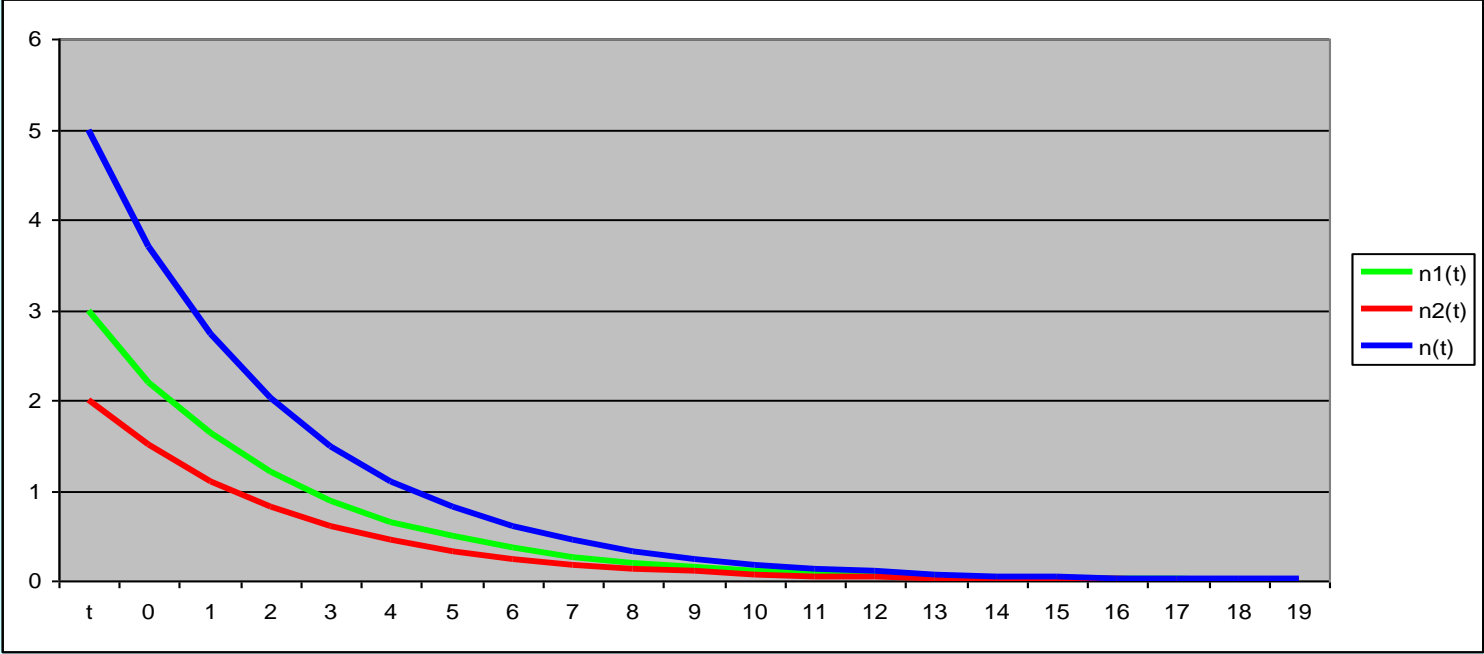
Popolazione oscillante - profilo oscillante

Popolazione stabile - profilo stabile

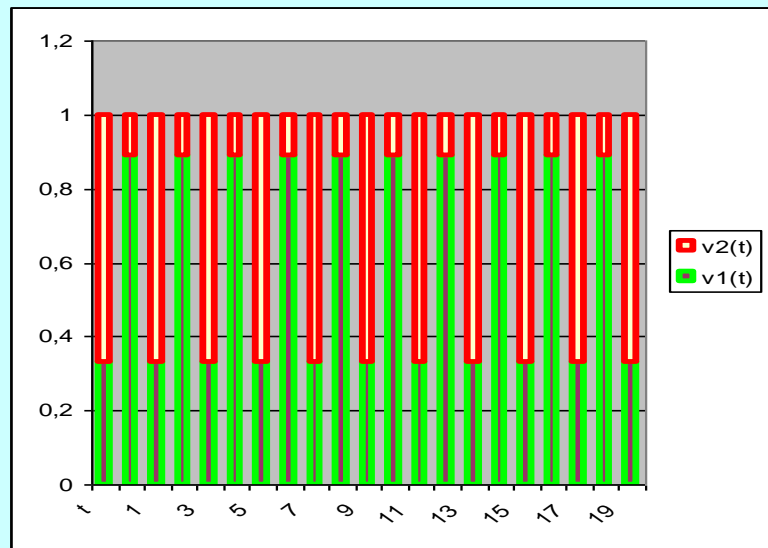
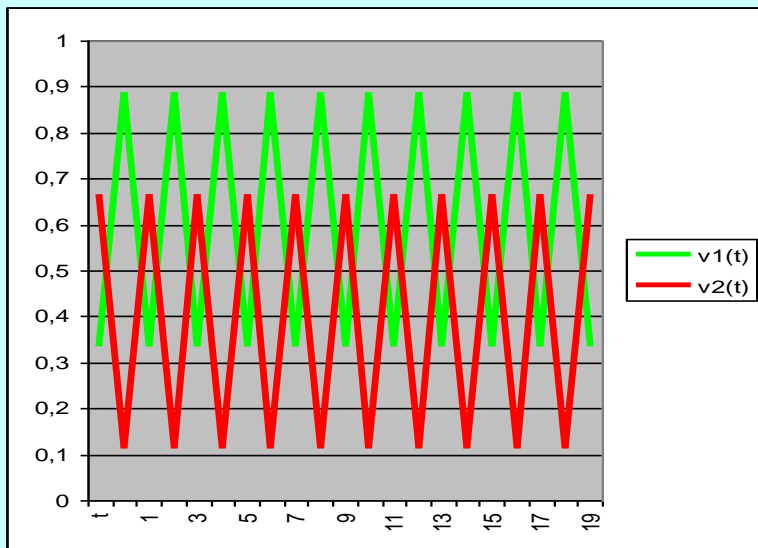
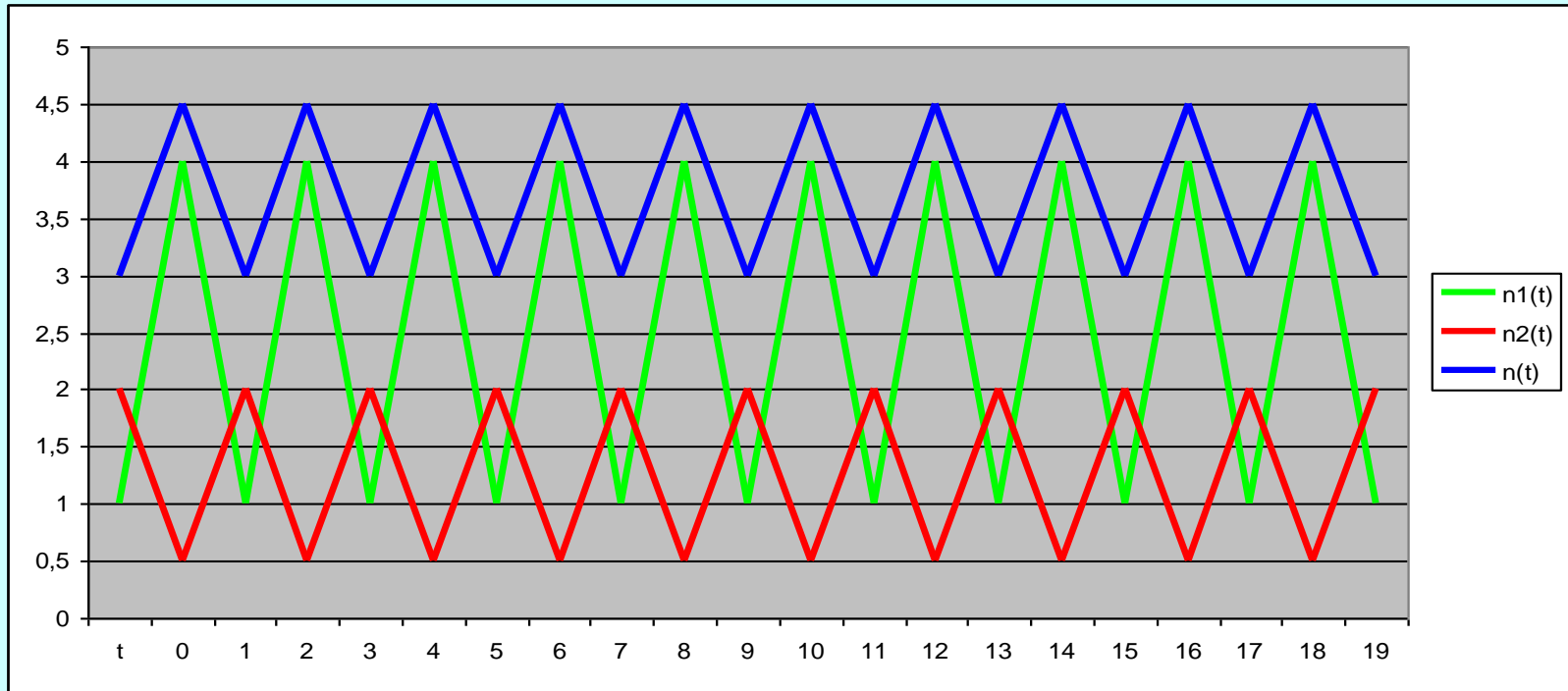
Popolazione crescente - profilo convergente



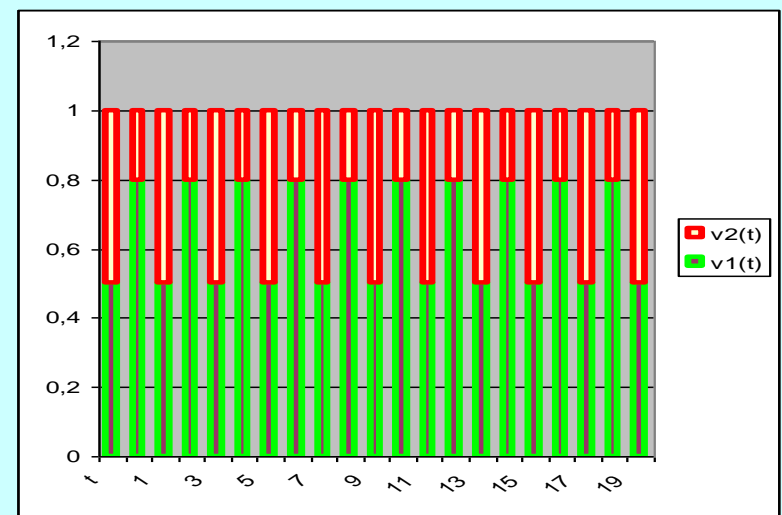
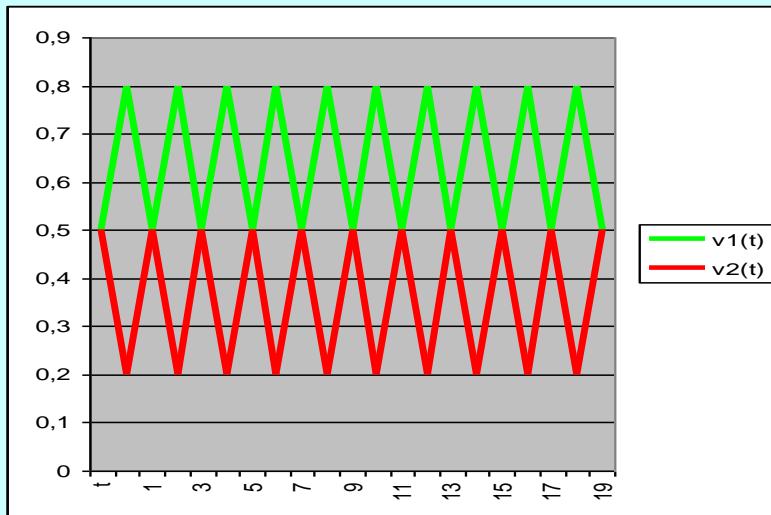
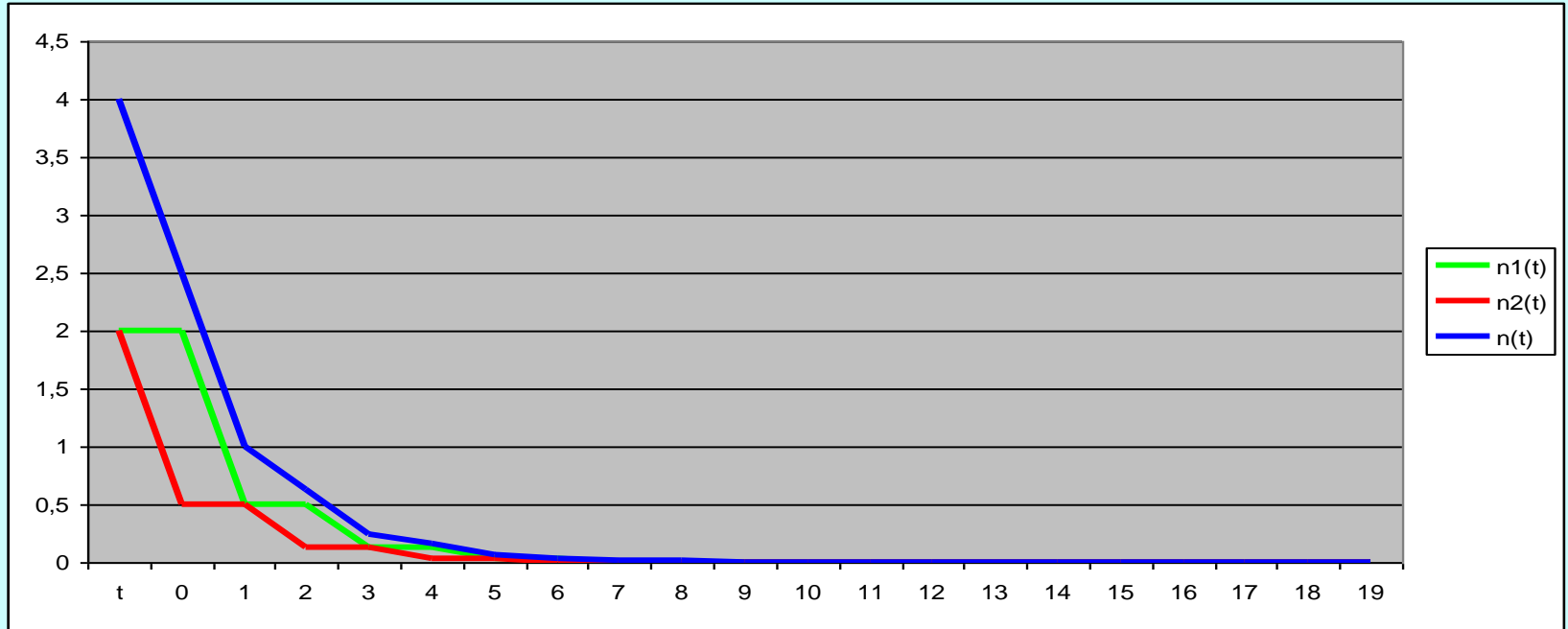
Popolazione decrescente - profilo convergente



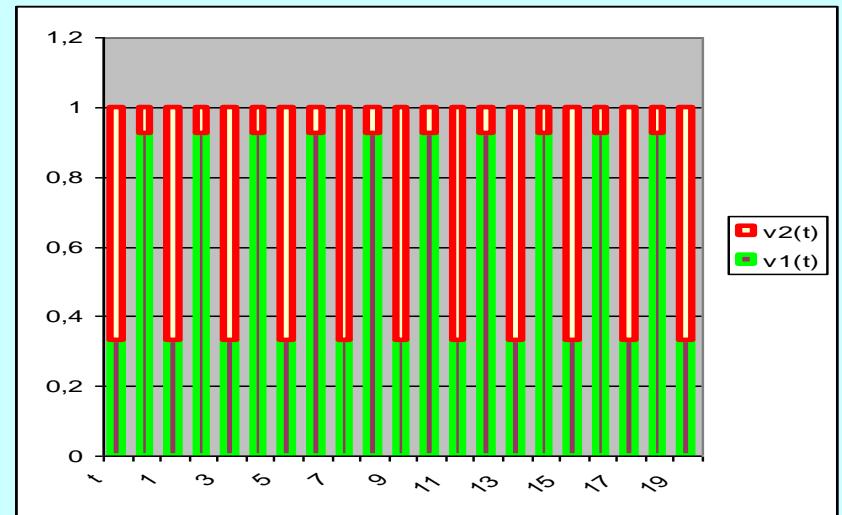
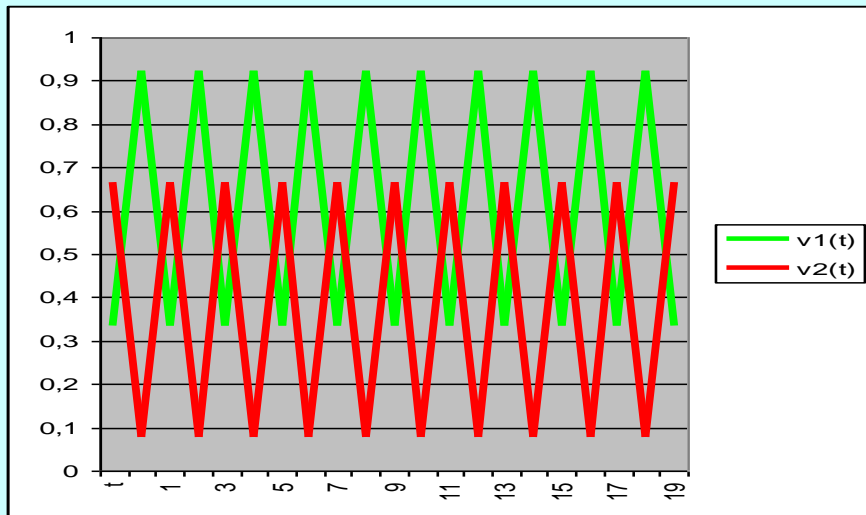
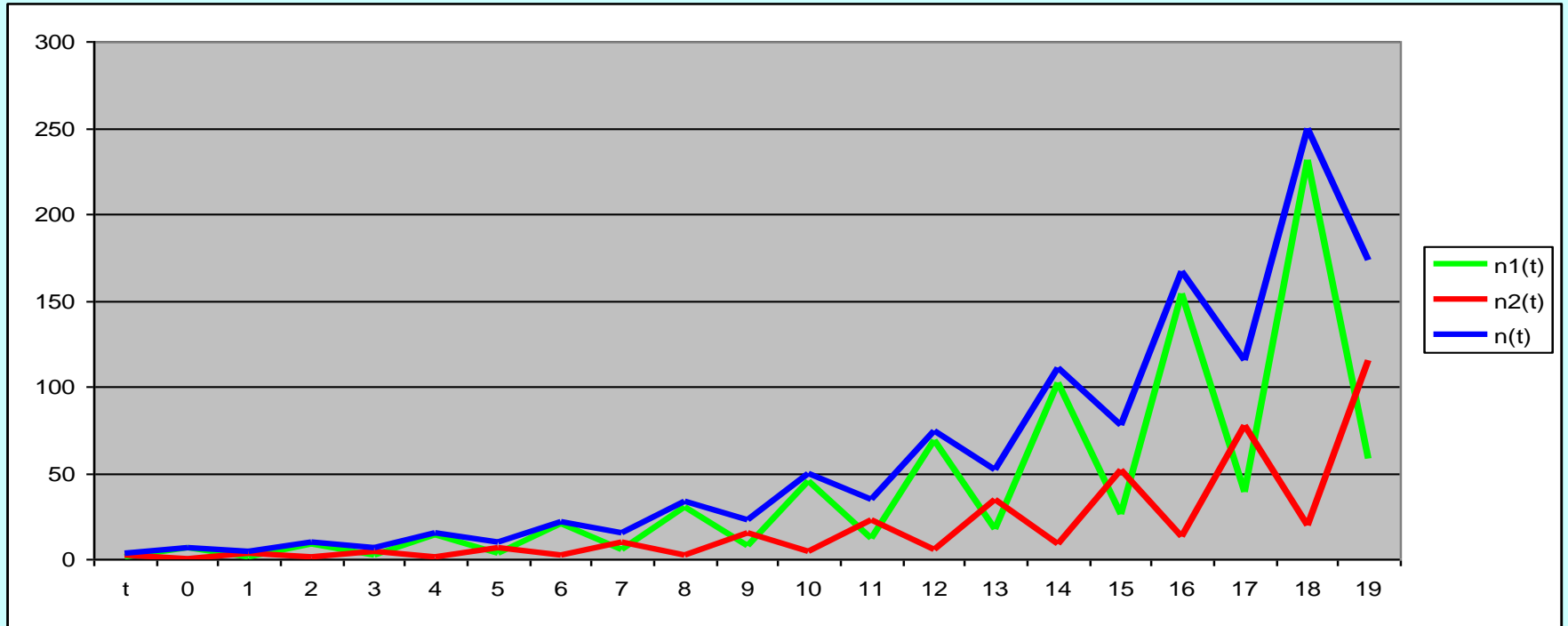
Popolazione oscillante - profilo oscillante



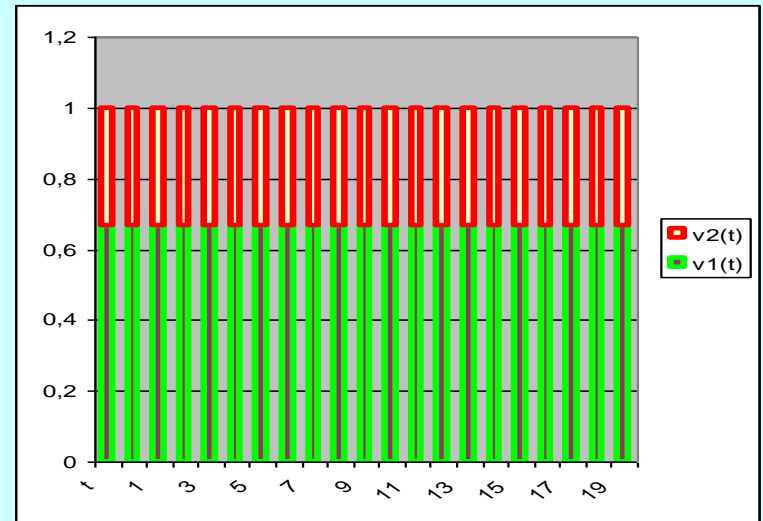
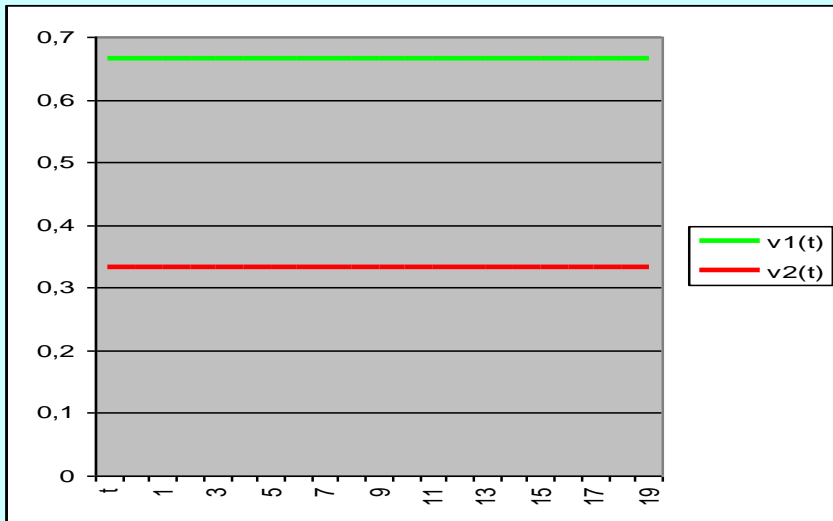
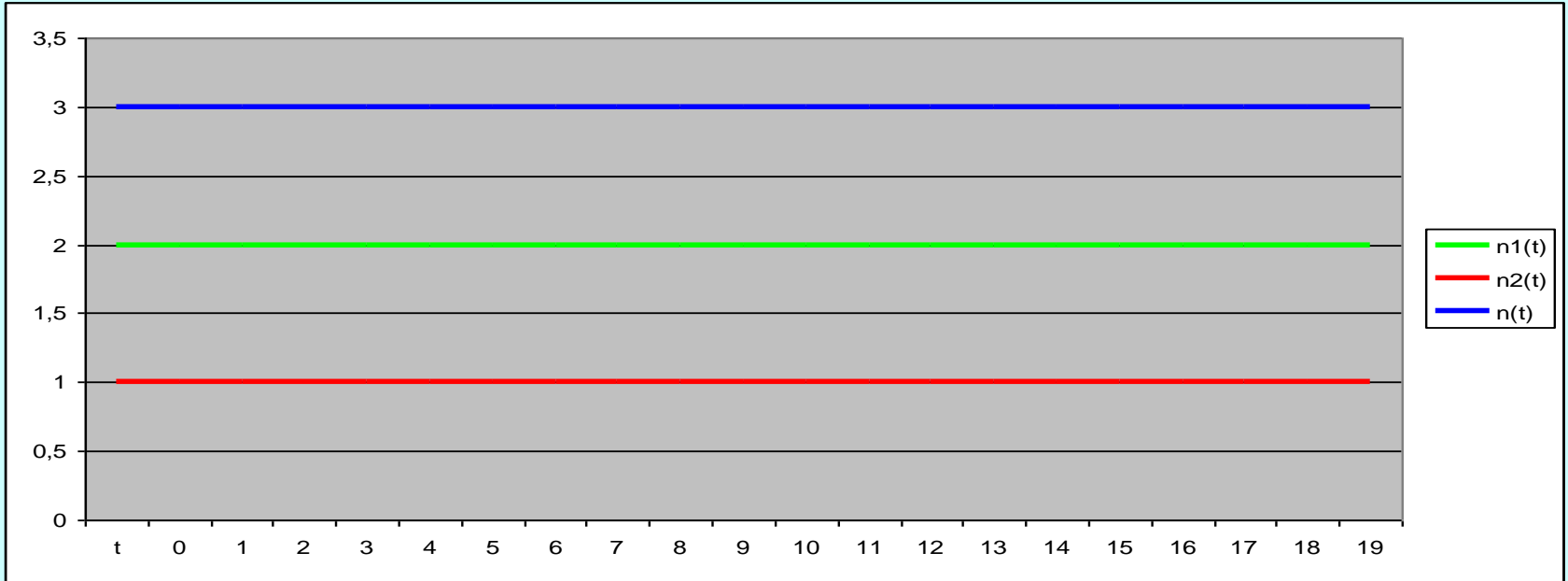
Popolazione pseudodecrescente - profilo oscillante



Popolazione pseudocrescente - profilo oscillante



Popolazione stabile - profilo stabile

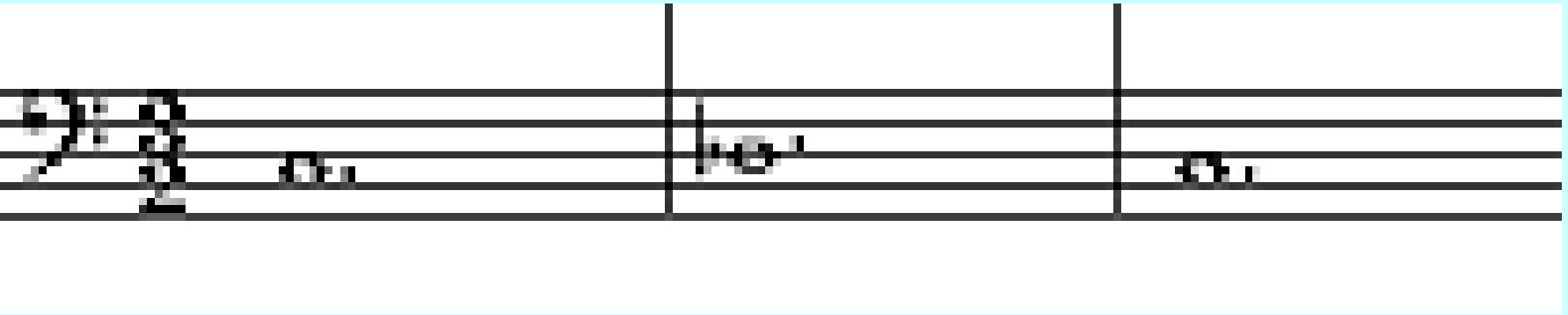


Processi iterativi

Do

Reb

Do

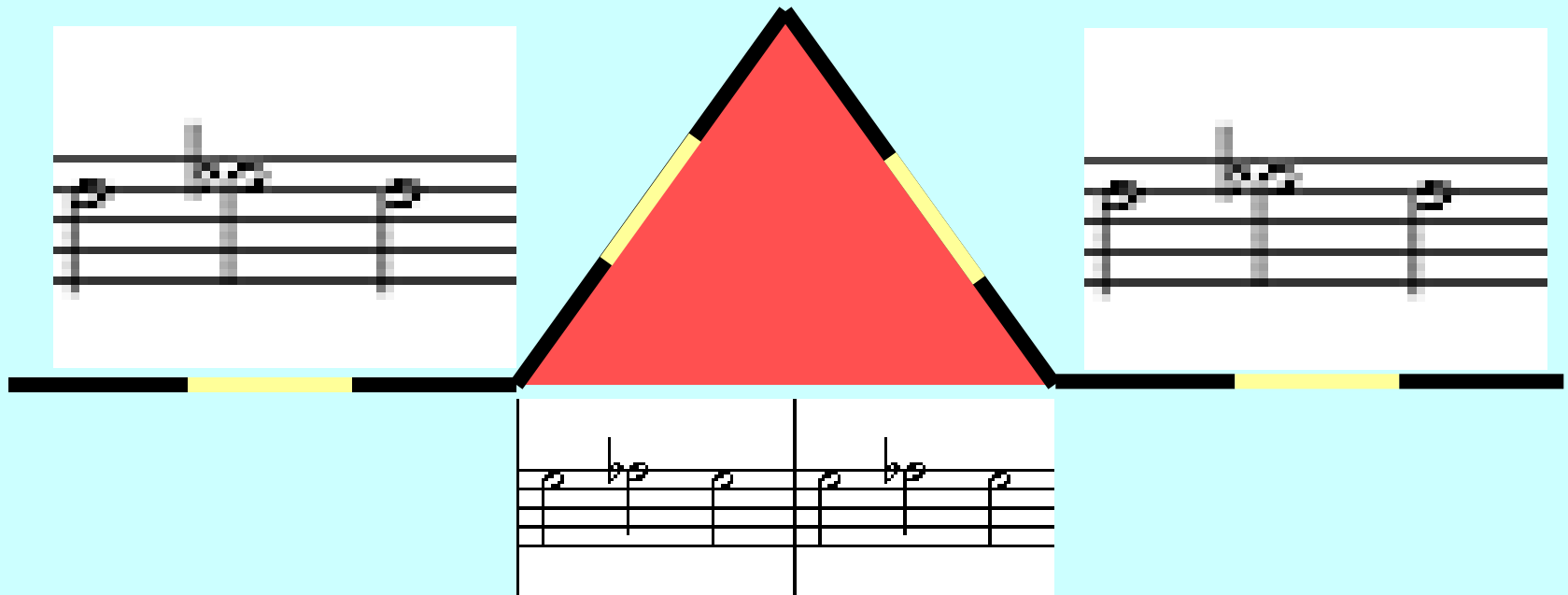


Livello 0

$$L_0 = 3u$$

Processi iterativi

Fa Solb Fa Sol Lab Sol Fa Solb Fa

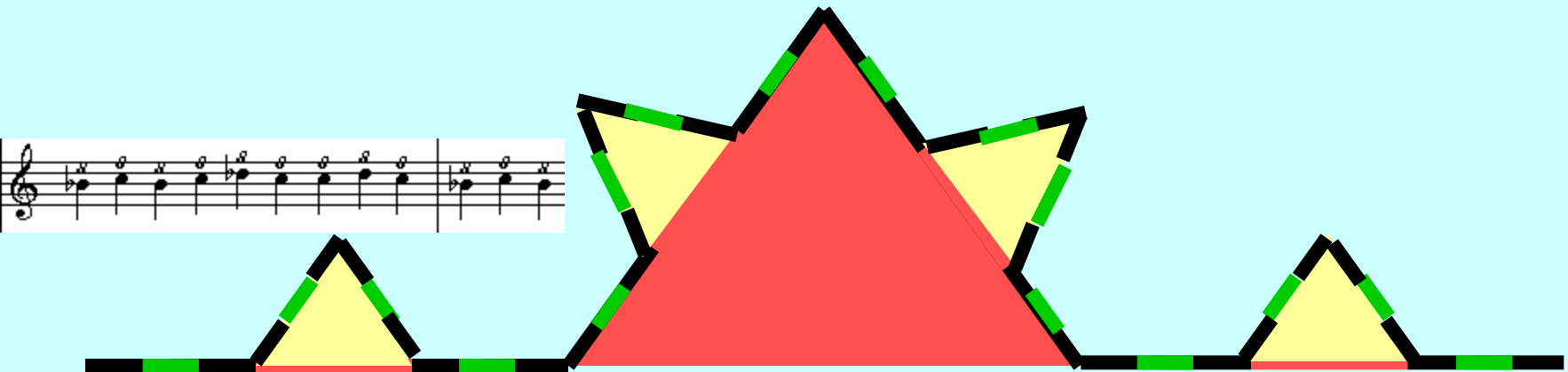


Livello 1

$$L_1 = 4u$$

Rapporto $L_1/L_0=4/3$ - intervallo di quarta

Processi iterativi

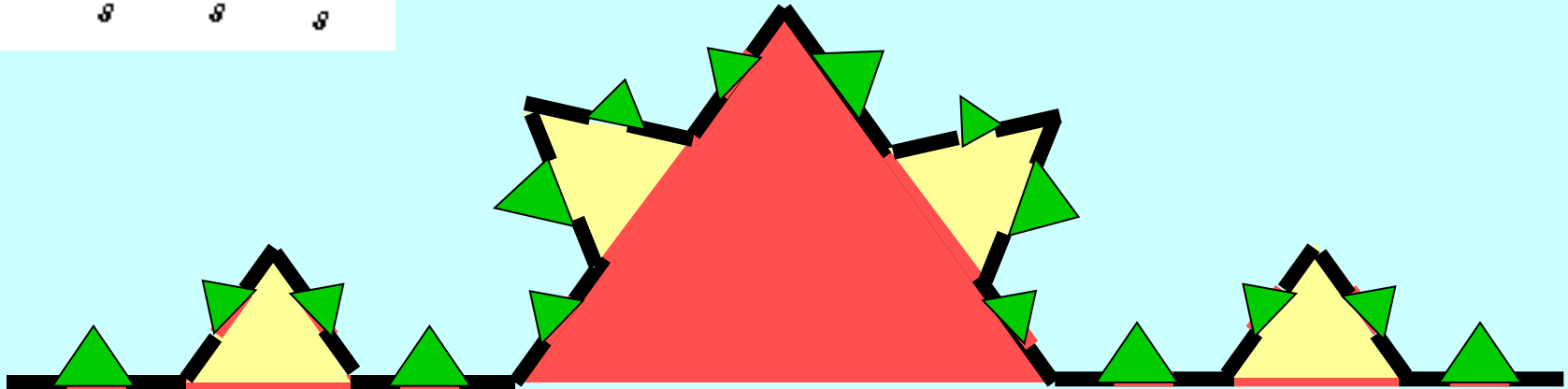


Livello 2

$$L_2 = 16/3u$$

Rapporto $L_2/L_1=4/3$ - intervallo di quarta

Processi iterativi

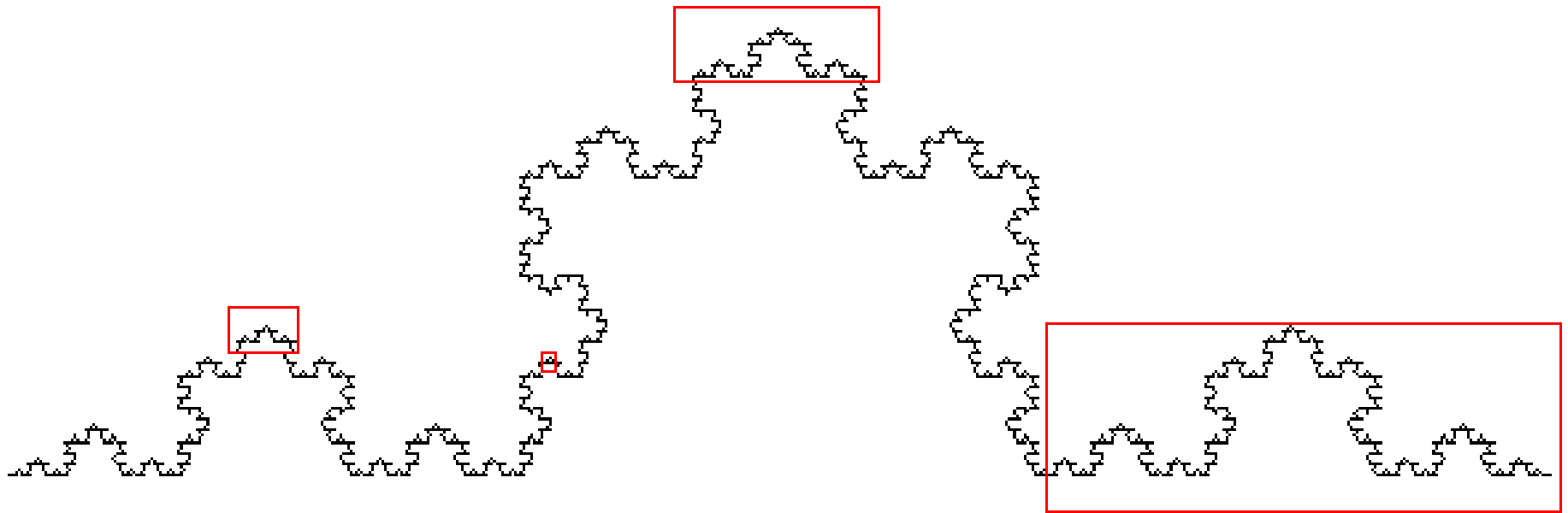


Livello 3

$$L_3 = 64/9u$$

Rapporto $L_3/L_2=4/3$ - intervallo di quarta

Processi iterativi



NORMALIZZAZIONI MUSICALI

The screenshot displays the Encore 4.0 software interface. The title bar reads "Encore 4.0 - [D:\MUSICA~2\SETTIM~1\FRATTALE.ENC]". The menu bar includes "File", "Edit", "Notes", "Measures", "Score", "View", "Windows", and "Setup". The toolbar contains various icons for editing and playback, including a "Voice" dropdown, a mouse cursor, a color palette, a pencil, a red circle, a green play button, a black square, a "Thru A-" button, an "M1" button, a magnifying glass, and a set of numbered buttons (1-7) with arrows.

The main workspace is titled "Koch" and contains a system of musical notation. The system consists of six staves. The first three staves are empty. The fourth staff is a bass clef staff containing a sequence of guitar chords: B^{\flat} , B^{\flat} , B^{\flat} , B^{\flat} , B^{\flat} , B^{\flat} , B^{\flat} . The fifth staff is a bass clef staff containing a sequence of notes: B^{\flat} , B^{\flat} , B^{\flat} , B^{\flat} , B^{\flat} , B^{\flat} , B^{\flat} . The sixth staff is a treble clef staff and is empty.

In the bottom right corner, a yellow tooltip box displays the text "Connessione ADSL" and "Nessun hardware". The Windows taskbar at the bottom shows the Start button, several application icons, and the system tray with the time "22.41".

NORMALIZZAZIONI MUSICALI

The image shows a screenshot of the Encore 4.0 software interface. The title bar reads "Encore 4.0 - [D:\MUSICA~2\SETTIM~1\FRATTALE.ENC]". The menu bar includes "File", "Edit", "Notes", "Measures", "Score", "View", "Windows", and "Setup". The toolbar contains various icons for editing and playback, including a "Voice" dropdown, a mouse cursor, a color palette, a pencil, a red circle, a green play button, a black square, a "Thru A-" button, an "M1" button, a magnifying glass, and a set of numbered buttons (1-7) with arrows. The main workspace displays two systems of musical notation. Each system consists of three staves: a top staff with a treble clef, a middle staff with a bass clef, and a bottom staff with a bass clef. The notation includes notes, rests, and dynamic markings like "s". The bottom of the screen shows the Windows taskbar with the Start button, several application icons, and the system tray displaying the time "22.41".

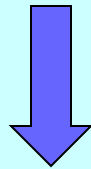
NORMALIZZAZIONI MUSICALI

The screenshot displays the Encore 4.0 software interface. The title bar reads "Encore 4.0 - [D:\MUSICA~2\SETTIM~1\FRATTALE.ENC]". The menu bar includes "File", "Edit", "Notes", "Measures", "Score", "View", "Windows", and "Setup". The toolbar contains a "Voice -" dropdown, a mouse cursor, a color selection tool, a play button, a stop button, a "Thru A-" button, an "M1" button, a zoom tool, and a navigation bar with buttons for first, previous, next, last, and a set of numbered buttons (1-7). The main window shows a musical score with multiple staves. The top two staves are bass clefs, the middle two are treble clefs, and the bottom two are bass clefs. The notation includes various notes, rests, and dynamic markings. The bottom of the screen shows the Windows taskbar with the Start button and several application icons, including a clock showing 22.42.

NORMALIZZAZIONI MUSICALI

I valori del *profilo* sono numeri compresi tra 0 e 1

$$0 \leq x \leq 1$$



$$1 \leq 2^x \leq 2$$

Il rapporto 2 : 1 è 1' **OTTAVA**

NORMALIZZAZIONI MUSICALI

f_1	f_2	p_1					
0	2	0,2					
t	$n_1(t)$	$n_2(t)$	$n(t)$	$v_1(t)$	$v_2(t)$	f1	f2
0	5	2	7	0,714285714	0,285714286	1,640671	1,219014
1	4	1	5	0,8	0,2	1,741101	1,148698
2	2	0,8	2,8	0,714285714	0,285714286	1,640671	1,219014
3	1,6	0,4	2	0,8	0,2	1,741101	1,148698
4	0,8	0,32	1,12	0,714285714	0,285714286	1,640671	1,219014
5	0,64	0,16	0,8	0,8	0,2	1,741101	1,148698
6	0,32	0,128	0,448	0,714285714	0,285714286	1,640671	1,219014
7	0,256	0,064	0,32	0,8	0,2	1,741101	1,148698
8	0,128	0,0512	0,1792	0,714285714	0,285714286	1,640671	1,219014
9	0,1024	0,0256	0,128	0,8	0,2	1,741101	1,148698
10	0,0512	0,02048	0,07168	0,714285714	0,285714286	1,640671	1,219014
11	0,04096	0,01024	0,0512	0,8	0,2	1,741101	1,148698
12	0,02048	0,008192	0,028672	0,714285714	0,285714286	1,640671	1,219014
13	0,016384	0,004096	0,02048	0,8	0,2	1,741101	1,148698
14	0,008192	0,0032768	0,0114688	0,714285714	0,285714286	1,640671	1,219014
15	0,0065536	0,0016384	0,008192	0,8	0,2	1,741101	1,148698
16	0,0032768	0,00131072	0,00458752	0,714285714	0,285714286	1,640671	1,219014
17	0,00262144	0,00065536	0,0032768	0,8	0,2	1,741101	1,148698
18	0,00131072	0,000524288	0,001835008	0,714285714	0,285714286	1,640671	1,219014
19	0,001048576	0,000262144	0,00131072	0,8	0,2	1,741101	1,148698
20	0,000524288	0,000209715	0,000734003	0,714285714	0,285714286	1,640671	1,219014

NORMALIZZAZIONI MUSICALI

Scala temperata

do	1
do#	1,059463
re	1,122462
re#	1,189207
mi	1,259921
fa	1,33484
fa#	1,414214
sol	1,498307
sol#	1,587401
la	1,681793
la#	1,781797
si	1,887749
do	2

Processo iterativo



**Progressione geometrica
di ragione di 1,059463....**

NORMALIZZAZIONI MUSICALI

$v_1(t)$	$v_2(t)$	$v_3(t)$	$s1$	$s2$	$s3$	voce 1	voce 2	voce3
0,333333333	0,333333333	0,333333333	1,25992105	1,25992105	1,25992105	re#	re#	re#
0,827586207	0,103448276	0,068965517	1,774713579	1,074338239	1,048964255	la	do#	do
0,409090909	0,545454545	0,045454545	1,327848828	1,459480106	1,03200828	mi	fa#	do
0,638297872	0,191489362	0,170212766	1,556491688	1,141941988	1,125224418	sol	re	re
0,647058824	0,294117647	0,058823529	1,565972441	1,226134843	1,041616011	sol	re#	do
0,527472527	0,362637363	0,10989011	1,441401775	1,285774249	1,079146035	fa#	mi	do#
0,642857143	0,244897959	0,112244898	1,561418364	1,185008953	1,080908871	sol	re	do#
0,590673575	0,32642487	0,082901554	1,505949691	1,253902237	1,059146059	sol	re#	do
0,587301587	0,301587302	0,111111111	1,50243397	1,232499705	1,080059739	sol	re#	do#
0,616966581	0,285347044	0,097686375	1,533647133	1,21870339	1,070056056	sol	re#	do#
0,589005236	0,314136126	0,096858639	1,50420921	1,243266972	1,069442294	sol	re#	do#
0,602346806	0,293350717	0,104302477	1,518184161	1,225483209	1,07497453	sol	re#	do#
0,60311284	0,299610895	0,097276265	1,518990493	1,23081241	1,069751917	sol	re#	do#
0,595689092	0,303723057	0,100587851	1,511194223	1,23432564	1,072210264	sol	re#	do#
0,602730819	0,296488947	0,100780234	1,518588322	1,228151849	1,072353253	sol	re#	do#
0,599414253	0,301659616	0,098926131	1,515101297	1,232561485	1,070975985	sol	re#	do#
0,599217986	0,300097752	0,100684262	1,514895194	1,231227834	1,072281919	sol	re#	do#
0,601073345	0,29907302	0,099853635	1,516844658	1,230353616	1,071664734	sol	re#	do#

Codificazione : approssimazione per difetto

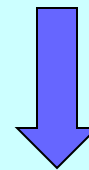
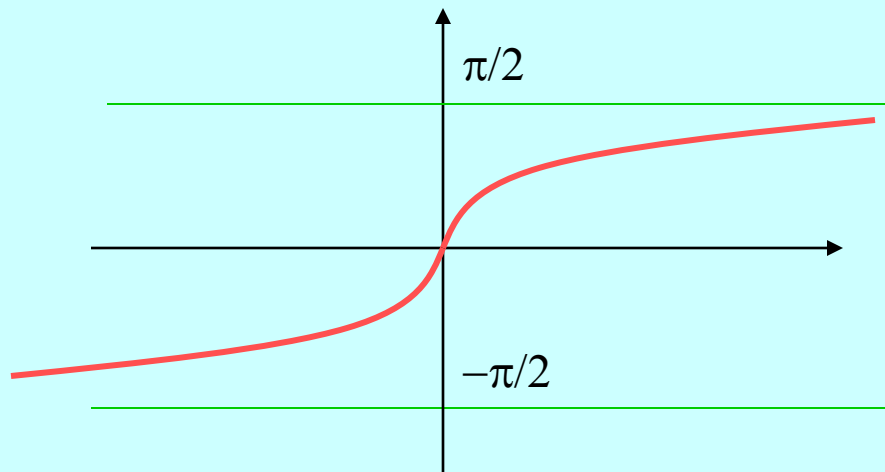
NORMALIZZAZIONI MUSICALI

I valori della popolazione sono numeri compresi tra 0 e ∞

$$0 \leq x \leq \infty$$



$$0 \leq (2/\pi)\arctg(x) \leq 1$$



$$1 \leq 2^t \leq 2$$

Il rapporto 2 : 1 è 1 ' OTTAVA

NORMALIZZAZIONI MUSICALI

t	n1(t)	n2(t)	n3(t)	n(t)	p1	p2	p3	p	parte1	parte2	parte3	superiore
0	2	2	2	6	1,63036	1,63036	1,63036	1,86002	sol#	sol#	sol#	la#
1	12	1	0,8	13,8	1,92860	1,41446	1,34702	1,93782	si	fa#	fa	si
2	5,2	6	0,4	11,6	1,83973	1,86002	1,18293	1,92617	la#	la#	re	si
3	13,6	2,6	2,4	18,6	1,93691	1,70130	1,68068	1,95382	si	la	sol#	si
4	14,8	6,8	1,04	22,64	1,94199	1,87580	1,42676	1,96209	si	la#	fa#	si
5	17,76	7,4	2,72	27,88	1,95163	1,88551	1,71250	1,96928	si	la#	la	si
6	25,68	8,88	2,96	37,52	1,96662	1,90406	1,73266	1,97730	si	si	la	si
7	29,6	12,84	3,552	45,992	1,97109	1,93321	1,77241	1,98159	si	si	la	si
8	39,888	14,8	5,136	59,824	1,97869	1,94199	1,83786	1,98599	si	si	la#	si
9	50,144	19,944	5,92	76,008	1,98317	1,95694	1,85822	1,98912	si	si	la#	si
10	63,568	25,072	7,9776	96,6176	1,98686	1,96580	1,89353	1,99158	si	si	si	si
11	82,0544	31,784	10,0288	123,8672	1,98997	1,97311	1,91482	1,99359	si	si	si	si
12	103,6832	41,0272	12,7136	157,424	1,99220	1,97929	1,93256	1,99510	si	si	si	si
13	132,9088	51,8416	16,41088	201,16128	1,99407	1,98374	1,94766	1,99632	si	si	si	si
14	169,32672	66,4544	20,73664	256,51776	1,99549	1,98746	1,95859	1,99726	si	si	si	si
15	215,85536	84,66336	26,58176	327,10048	1,99662	1,99030	1,96776	1,99800	si	si	si	si
16	275,65376	107,92768	33,865344	417,446784	1,99750	1,99254	1,97480	1,99859	si	si	si	si
17	351,316736	137,82688	43,171072	532,314688	1,99819	1,99430	1,98035	1,99904	si	si	si	si
18	448,338048	175,658368	55,130752	679,127168	1,99873	1,99568	1,98475	1,99940	si	si	si	si
19	571,839744	224,169024	70,2633472	866,2721152	1,99916	1,99677	1,98817	1,99968	si	si	si	si
20	729,3914368	285,919872	89,6676096	1104,978918	1,99949	1,99762	1,99088	1,99990	si	si	si	si

Codificazione : approssimazione per difetto

NORMALIZZAZIONI MUSICALI

				$v_1(t)$	$v_2(t)$							
				1,414213562	1,414213562	fa	fa					
				1,811447329	1,104089514	la#	do#					
f_1	f_2	p_1										
0	3	0,5										
t	$n_1(t)$	$n_2(t)$	$n(t)$	$v_1(t)$	$v_2(t)$	p1	p2	p	parte1	parte2	superiore	
0	3	0,5	3,5	0,8571	0,1429	1,2272	1,7694	1,3673	re#	la	fa	
1	1,5	1,5	3	0,5000	0,5000	1,5433	1,7358	1,2272	sol	la	re#	
2	4,5	0,75	5,25	0,8571	0,1429	1,3286	1,8412	1,3673	mi	la#	fa	
3	2,25	2,25	4,5	0,5000	0,5000	1,6634	1,8166	1,2272	sol#	la#	re#	
4	6,75	1,125	7,875	0,8571	0,1429	1,4516	1,8922	1,3673	fa#	si	fa	
5	3,375	3,375	6,75	0,5000	0,5000	1,7618	1,8749	1,2272	la	la#	re#	
6	10,125	1,6875	11,8125	0,8571	0,1429	1,5798	1,9275	1,3673	sol	si	fa	
7	5,0625	5,0625	10,125	0,5000	0,5000	1,8356	1,9156	1,2272	la#	si	re#	
8	15,1875	2,53125	17,71875	0,8571	0,1429	1,6945	1,9515	1,3673	la	si	fa	
9	7,59375	7,59375	15,1875	0,5000	0,5000	1,8883	1,9435	1,2272	si	si	re#	
10	22,78125	3,796875	26,578125	0,8571	0,1429	1,7857	1,9678	1,3673	la#	si	fa	
11	11,390625	11,390625	22,78125	0,5000	0,5000	1,9248	1,9623	1,2272	si	si	re#	
12	34,171875	5,6953125	39,8671875	0,8571	0,1429	1,8529	1,9787	1,3673	la#	si	fa	
13	17,0859375	17,0859375	34,171875	0,5000	0,5000	1,9497	1,9750	1,2272	si	si	re#	
14	51,2578125	8,54296875	59,80078125	0,8571	0,1429	1,9004	1,9860	1,3673	si	si	fa	
15	25,62890625	25,62890625	51,2578125	0,5000	0,5000	1,9666	1,9835	1,2272	si	si	re#	
16	76,88671875	12,81445313	89,70117188	0,8571	0,1429	1,9331	1,9909	1,3673	si	si	fa	
17	38,44335938	38,44335938	76,88671875	0,5000	0,5000	1,9779	1,9892	1,2272	si	si	re#	
18	115,3300781	19,22167969	134,5517578	0,8571	0,1429	1,9553	1,9941	1,3673	si	si	fa	
19	57,66503906	57,66503906	115,3300781	0,5000	0,5000	1,9854	1,9931	1,2272	si	si	re#	
20	172,9951172	28,83251953	201,8276367	0,8571	0,1429	1,9703	1,9963	1,3673	si	si	fa	

NORMALIZZAZIONI MUSICALI

Contrappunto

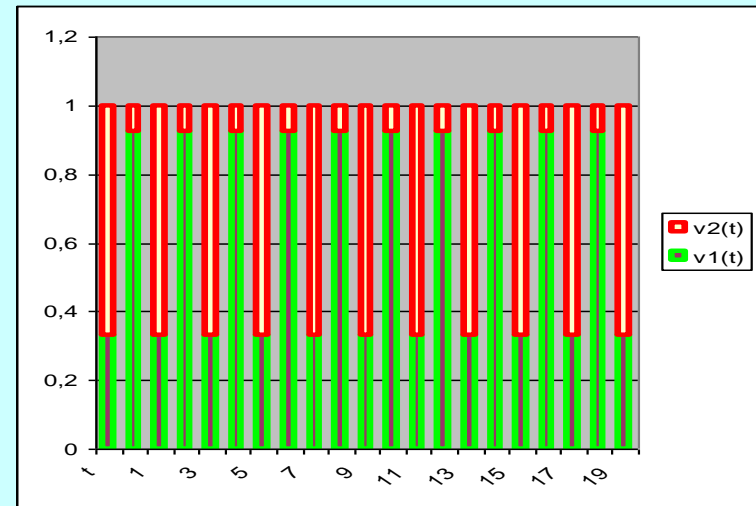
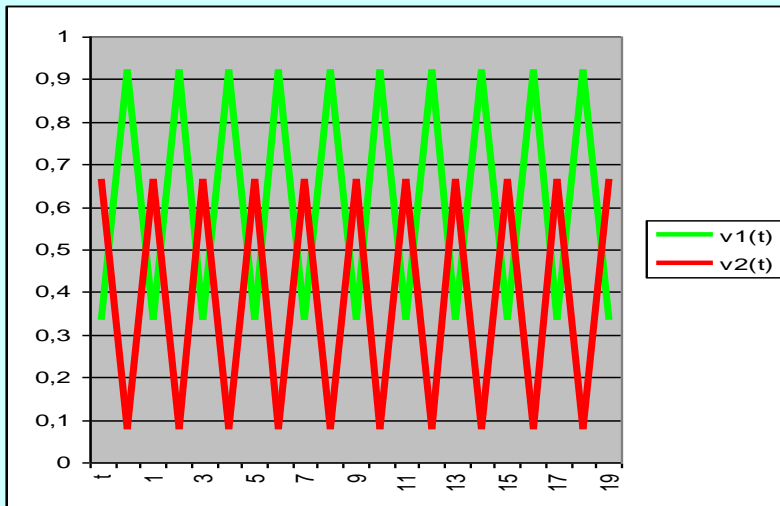
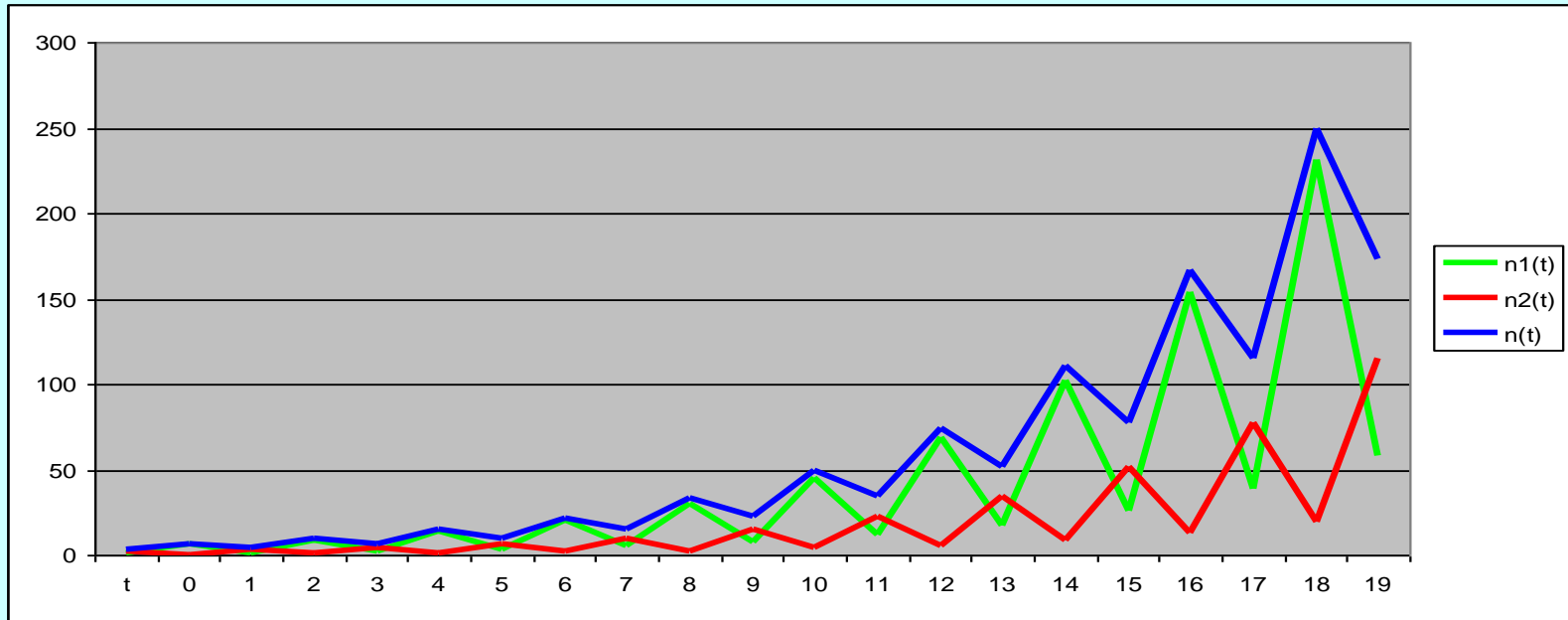
A —————> B

Popolazione pseudocrescente- profilo oscillante

A Temi della popolazione

B Temi della popolazione e profilo

Popolazione pseudocrescente - profilo oscillante



NORMALIZZAZIONI MUSICALI

Encore 4.0 - [D:\MUSICA~2\SETTIM~1\2FASCE.ENC]

File Edit Notes Measures Score View Windows Setup

Voice - [Mouse] [Color] [Eraser] [Red] [Green] [Black] Thru A- M5 [Mute] [Zoom] [Page 1] [Page 2] [Page 3] [Next]

Contrappunto I

B. Cifra

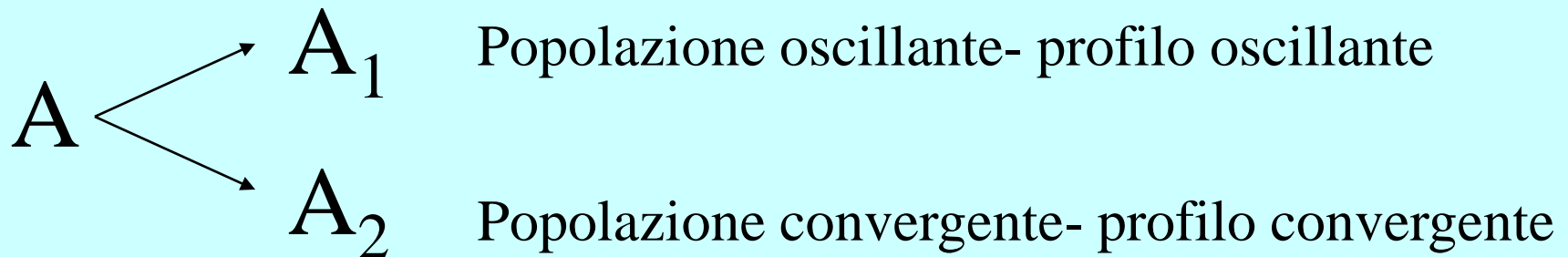
The screenshot displays the Encore 4.0 software interface. The main window shows a musical score titled "Contrappunto I" with a subtitle "B. Cifra". The score is presented in two systems of staves. The first system consists of five staves: two treble clefs and three bass clefs, all in 4/4 time. The second system also consists of five staves: one treble clef and four bass clefs, all in 4/4 time. The notation includes various musical symbols such as notes, rests, and accidentals. The software's menu bar and toolbar are visible at the top, and the Windows taskbar is at the bottom.

NORMALIZZAZIONI MUSICALI

The image shows a screenshot of the Encore 4.0 software interface. The title bar reads "Encore 4.0 - [D:\MUSICA~2\SETTIM~1\2FASCE.ENC]". The menu bar includes "File", "Edit", "Notes", "Measures", "Score", "View", "Windows", and "Setup". The toolbar contains various icons for editing and playback, including a "Voice" dropdown, a mouse cursor, a color palette, a pencil, a red circle, a green play button, a black square, a "Thru A-" button, "M5", a red circle with a slash, a magnifying glass, and navigation arrows. The main window displays a musical score with multiple staves. The score includes a vocal line with a long slur, a piano accompaniment with dense sixteenth-note patterns, and a bass line with a steady eighth-note accompaniment. The Windows taskbar is visible at the bottom, showing the Start button and various application icons, with the system clock displaying "22.32".

NORMALIZZAZIONI MUSICALI

TREND



B Popolazione pseudocrescente- profilo convergente

NORMALIZZAZIONI MUSICALI

Encore 4.0 - [D:\MUSICA~2\SETTIM~1\TRENDS.ENC]

File Edit Notes Measures Score View Windows Setup

Voice - [Mouse] [Color] [Erase] [Play] [Stop] Thru A- M1 [Mute] [Zoom] [Page 1] [Page 2] [Page 3]

TREND

B. Cifra

The image shows a musical score for a piece titled "TREND" in 3/4 time. The score is displayed in a software window titled "Encore 4.0 - [D:\MUSICA~2\SETTIM~1\TRENDS.ENC]". The score is written for guitar and bass. The guitar part is in the upper staves, and the bass part is in the lower staves. The score is in 3/4 time and features a key signature of one sharp (F#). The guitar part consists of a series of chords and single notes, while the bass part provides a rhythmic accompaniment. The score is titled "TREND" and is labeled "B. Cifra" (B. Cifra). The software interface includes a menu bar (File, Edit, Notes, Measures, Score, View, Windows, Setup) and a toolbar with various editing and playback tools. The Windows taskbar at the bottom shows the Start button and several application icons, with the system clock displaying 22.35.

NORMALIZZAZIONI MUSICALI

The image shows a screenshot of the Encore 4.0 music notation software interface. The title bar reads "Encore 4.0 - [D:\MUSICA~2\SETTIM~1\TRENDS.ENC]". The menu bar includes "File", "Edit", "Notes", "Measures", "Score", "View", "Windows", and "Setup". The toolbar contains various icons for editing, playback, and navigation, including a "Voice" dropdown menu, a mouse cursor, color selection tools, a red circle, a green play button, a black square, a "Thru A-" button, "M1", a red circle with a slash, a magnifying glass, and navigation arrows. The main workspace displays a musical score with multiple staves. The top staff is a bass clef staff with a 3/4 time signature. Below it are several treble clef staves, some with large slurs. The bottom staff is a bass clef staff with a large slur. The Windows taskbar is visible at the bottom, showing the Start button and various application icons, with the system clock displaying "22.37".

NORMALIZZAZIONI MUSICALI

The image shows a screenshot of the Encore 4.0 software interface. The title bar reads "Encore 4.0 - [D:\MUSICA~2\SETTIM~1\TRENDS.ENC]". The menu bar includes "File", "Edit", "Notes", "Measures", "Score", "View", "Windows", and "Setup". The control panel features a "Voice -" dropdown, a mouse cursor, a color palette, a yellow arrow, a red circle, a green play button, a black square, a "Thru A-" button, an "M1" button, a red circle with a diagonal line, a magnifying glass, and navigation arrows. The main area displays a musical score with seven staves. The top staff is a treble clef with a melodic line. The second and third staves are also treble clefs with similar melodic lines. The fourth staff is a treble clef with a more complex melodic line. The fifth staff is a bass clef with a melodic line. The sixth and seventh staves are bass clefs with a rhythmic accompaniment. The score is divided into measures by vertical bar lines. The Windows taskbar at the bottom shows the Start button and various application icons, with the system clock displaying "21.59".

NORMALIZZAZIONI MUSICALI



The main area of the software displays a musical score with eight staves. The top two staves are treble clefs, and the bottom two are bass clefs. The score is divided into measures by vertical bar lines. The first staff (top) contains a melodic line starting with a sharp sign (F#) and a slur over a series of notes. The second staff contains a similar melodic line with a sharp sign (F#) and a slur. The third and fourth staves are empty. The fifth staff (bass clef) contains a melodic line starting with a flat sign (Bb) and a slur over a series of notes. The sixth and seventh staves are empty. The eighth staff (bass clef) is empty.

NORMALIZZAZIONI MUSICALI

The image shows a screenshot of the Encore 4.0 software interface. The title bar reads "Encore 4.0 - [D:\MUSICA~2\SETTIM~1\TRENDS.ENC]". The menu bar includes "File", "Edit", "Notes", "Measures", "Score", "View", "Windows", and "Setup". The control panel features a "Voice -" dropdown, a mouse cursor, a color palette, a pencil icon, a red circle, a green play button, a black square, a "Thru A-" button, an "M1" button, a red circle with a slash, a magnifying glass, and navigation arrows. The main workspace displays a musical score with eight staves. The top staff is a bass clef staff. The second and eighth staves are treble clef staves. The third, fourth, fifth, sixth, and seventh staves are bass clef staves. The score contains various musical notations, including notes, rests, and slurs. The Windows taskbar is visible at the bottom, showing the Start button and several application icons. The system clock in the bottom right corner displays "22.39".

NORMALIZZAZIONI MUSICALI

The image shows a screenshot of the Encore 4.0 music notation software interface. The window title is "Encore 4.0 - [D:\MUSICA~2\SETTIM~1\TRENDS.ENC]". The menu bar includes "File", "Edit", "Notes", "Measures", "Score", "View", "Windows", and "Setup". The toolbar contains various icons for editing and playback, including a "Voice" dropdown menu, a mouse cursor, a color palette, a pencil, a red circle, a green play button, a black square, a "Thru A-" button, an "M1" button, a magnifying glass, and navigation arrows. The main workspace displays a musical score with multiple staves. The top staff is a bass clef staff with a key signature of one sharp (F#) and a common time signature (C). The subsequent staves are treble clef staves. The score consists of several measures of music, with notes and rests connected by curved lines, likely representing a melodic line or a specific rhythmic pattern. The interface is set against a light blue background.

YANNIS XENAKIS (1922-2001)

ha realizzato composizioni attraverso vere e proprie
Catene di Markov

“la musica è l’espressione delle visioni dell’universo, delle sue onde, dei suoi alberi, dei suoi uomini, allo stesso titolo delle teorie fondamentali della fisica teorica, della logica astratta, dell’algebra moderna”

GRAZIE!



Bruno