COMPITO B

B_1

1)
$$z = 2^5 \left[\frac{\cos(4\pi) + i \operatorname{sen}(4\pi)}{\left(\cos\left(\frac{10}{3}\pi\right) + i \operatorname{sen}\left(\frac{10}{3}\pi\right)\right) e^{i\pi}} \right]$$

$$= 2^5 \left[\frac{-1}{\left[\cos\left(-\frac{2}{3}\pi\right) + i \operatorname{sen}\left(-\frac{2}{3}\pi\right)\right]} \right]$$

$$= -2^5 \left[\cos\left(\frac{2}{3}\pi\right) + i \operatorname{sen}\left(\frac{2}{3}\pi\right)\right]$$

$$= 2^5 \left[-\cos\left(\frac{2}{3}\pi\right) - i \operatorname{sen}\left(\frac{2}{3}\pi\right)\right]$$

$$= 2^5 \left[\cos\left(-\frac{\pi}{3}\right) + i \operatorname{sen}\left(-\frac{\pi}{3}\right)\right]$$

$$= 2^5 \left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 2^4\left(1 - \sqrt{3}i\right)$$

2) OMOGENEA ASSOCIATA:

$$y'' + y' = 0 \Rightarrow \alpha^2 + \alpha = 0$$

$$\Rightarrow \alpha_1 = 0 \ ; \ \alpha_2 = -1$$

B2

$$\Rightarrow y_0(x) = C_0 + C_1 e^{-x}$$

NON OMOGENEA: poichè $\alpha = 0$ è radice del polinomio caratteristico, allora

$$y_P(x) = x(Ax+B)$$

$$y_P'(x) = 2Ax + B$$

$$y_P''(x) = 2A$$

$$\Rightarrow 2A + 2Ax + B = 2x + 4$$

$$\Rightarrow \begin{cases} A = 1 \\ B = 4 - 2A = 2 \end{cases}$$

$$\Rightarrow y(x) = C_0 + C_1 e^{-x} + x^2 + 2x$$

B 2bis

$$\lim_{x\to-\infty} y(x) = \lim_{x\to-\infty} C_1 e^{-x} = \pm\infty$$

se $C_1 \gtrless 0$

$\Rightarrow C_1 > 0$

Per $C_1 = 0$: $\quad y(x) = C_0 + x^2 + 2x$

$$\Rightarrow \lim_{x\to-\infty} y(x) = \lim_{x\to-\infty} x^2 = +\infty$$

$$\Rightarrow \boxed{\lim_{x\to-\infty} y(x) = +\infty \iff C_1 \geq 0,\ C_0 \in \mathbb{R}}$$

$$\lim_{x\to+\infty} y(x) = \lim_{x\to+\infty} x^2 = +\infty \quad \forall C_0, C_1 \in \mathbb{R}$$

* Chiavi e cassetto microfono

ore 12^{10}

$$\Rightarrow \lim_{x \to \pm\infty} y(x) = +\infty \iff \begin{cases} C_1 \geq 0 \\ C_0 \in \mathbb{R} \end{cases}$$ B3

3) La serie è definita per

$$\left| \frac{1+n}{3+2n} \right| \leq 1 \quad \text{cioè} \quad \frac{1+n}{3+2n} \leq 1$$

$$\iff 1+n \leq 3+2n \iff n \geq -2$$

vero $\forall n \in \mathbb{N}$

CONVERGENZA ASSOLUTA:

$$\sum |a_n| = \sum \left| \arcsin\left(\frac{1+n}{3+2n}\right) \right|^n$$

Criterio della radice

$$\sqrt[n]{|a_n|} = \left| \arcsin\left(\frac{1+n}{3+2n}\right) \right|$$

$$\xrightarrow[n \to \infty]{} \left| \arcsin\left(\frac{1}{2}\right) \right| = \frac{\pi}{6} < 1$$

$\Rightarrow$ CONV. ASSOLUTA e quindi SEMPLICE.

4) Verificiamo la continuità in $x=0$. (B4)

$$f(x)=\begin{cases}\dfrac{\left(\cancel{1}+\cancel{4x^2}+\dfrac{16x^4}{2}+o(x^4)\right)\cancel{-1}\cancel{-4x^2}}{6x^4} & x<0\\[2ex] \dfrac{\left(\cancel{2x}+\dfrac{8x^3}{6}+o(x^3)\right)\cancel{-2x}}{\cancel{4}x^b} & x>0\end{cases}$$

$$=\begin{cases}\dfrac{4}{3}+o(1) & x<0\\[2ex] \dfrac{\dfrac{4}{3}x^3+o(x^3)}{\cancel{4}x^b} & x>0\end{cases}$$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^-} f(x)=\frac{4}{3}$$

$$\iff b=3$$

$$f(0)=\lim_{x\to 0} f(x)=\frac{4}{3} \iff a=\frac{4}{3}$$

f CONTINUA in $x=0$

$$\iff a=\frac{4}{3}\ ;\ b=3.$$

B5

DERIVABILITÀ in $x=0$:

$$f'(x)=\begin{cases}\frac{1}{6}\left[\frac{(8xe^{4x^2}-8x)\cancel{x^4}-(e^{4x^2}-1-4x^2)4\cancel{x^3}}{x^{\cancel{8}5}}\right] & x<0\\ \frac{[2\cosh(2x)-2]\cancel{x^3}-(\sinh(2x)-2x)3\cancel{x^2}}{x^{\cancel{6}4}} & x>0\end{cases}$$

$$=\begin{cases}\frac{1}{6x^5}\left[8x^2(e^{4x^2}-1)-4(e^{4x^2}-1-4x^2)\right] & x<0\\ \frac{1}{x^4}\left[2(\cosh(2x)-1)x-3(\sinh(2x)-2x)\right] & x>0\end{cases}$$

$$=\begin{cases}\frac{1}{6x^5}\left[8x^2(\cancel{4x^2}+8x^4+o(x^4))-4\left(\cancel{8x^4}+\frac{64x^6}{3!}+o(x^6)\right)\right] & x<0\\ \frac{1}{x^4}\left[2x\left(\frac{(2x)^2}{2}+\frac{(2x)^4}{4!}+o(x^4)\right)-3\left(\frac{(2x)^3}{3!}+\frac{(2x)^5}{5!}+o(x^5)\right)\right] & x>0\end{cases}$$

$$=\begin{cases}\frac{1}{6x^5}\left[64\left(1-\frac{2}{3}\right)x^6+o(x^6)\right] & x<0\\ \frac{1}{x^4}\left[\cancel{4x^3}+\frac{\cancel{16}\,4}{\cancel{12}\,3}x^5-\cancel{4x^3}-\frac{\cancel{32}\,4}{\cancel{40}\,5}x^5+o(x^5)\right] & x>0\end{cases}$$

$$= \begin{cases} \frac{32}{9}x + o(x) & x<0 \\ \left(\frac{4}{3}-\frac{4}{5}\right)x + o(x) = \frac{8}{15}x + o(x) & x>0 \end{cases}$$

$$\lim_{x \to 0^\pm} f'(x) = 0 \Rightarrow f'(0) = 0$$

f è derivabile $\forall x \in \mathbb{R}$

In alternativa, sviluppando ulteriormente il polinomio di McLaurin di f:

$$f(x) = \begin{cases} \left[8x^4 + \frac{64x^6}{3!} + o(x^6)\right]\frac{1}{6x^4} & x<0 \\ \dfrac{\frac{4}{3}x^3 + \frac{32x^5}{5!} + o(x^5)}{x^3} & x>0 \end{cases}$$

$$= \begin{cases} \frac{4}{3} + \frac{16}{9}x^2 + o(x^2) & x<0 \\ \frac{4}{3} + \frac{4}{15}x^2 + o(x^2) & x>0 \end{cases}$$

Quindi

$$f'(x) \sim \begin{cases} \frac{32}{9}x & x<0 \\ \frac{8}{15}x & x>0 \end{cases}$$

da cui $\lim_{x \to 0^\pm} f'(x) = f'(0) = 0$. B_7

5) f pari

$$\Rightarrow \int_{-1}^{1} f(x)\,dx = 2\int_0^1 \left(x^4 + 4|x|\right)dx$$

$$= 2\int_0^1 \left(x^4 + 4x\right)dx \qquad \text{poiché } x \geq 0$$

$$= 2\left[\frac{x^5}{5} + 2x^2\right]_0^1 = 2\left(\frac{1}{5} + 2\right) = \frac{22}{5}$$