COMPITO B

B1

1) $f(x) \underset{x \to 0}{\sim} \frac{1}{\sin x} \sim \frac{1}{x}$ NON INTEGRABILE

$\Rightarrow$ ~~$\int$~~ $f(x) \geq 0$ in $\left(0, \frac{\pi}{2}\right]$

$$\Rightarrow \int_0^{\pi/2} f(x)\,dx = +\infty.$$

ANCHE SE NON NECESSARIO ~~RICHIESTO~~, calcoliamo l'integrale:

~~$t = \sin^2 x + 1$ $dt = 2\cos x\,dx$~~

$t = \sin x$ $\quad dt = \cos x\,dx$ $\quad t(0) = 0$ $\quad t\left(\frac{\pi}{2}\right) = 1$

$$\Rightarrow \int_0^1 \frac{1}{(t^2+1)t}\,dt$$

Fratti semplici:

$$\frac{1}{(t^2+1)t} = \frac{At+B}{t^2+1} + \frac{C}{t} = \frac{At^2+Bt+Ct^2+C}{(t^2+1)t}$$

$$\Rightarrow \begin{cases} A+C=0 \\ B=0 \\ C=1 \end{cases} \Rightarrow \begin{cases} A=-1 \\ B=0 \\ C=1 \end{cases}$$

$$\Rightarrow \int_0^1 \left[\frac{-t}{t^2+1} + \frac{1}{t}\right] dt = \cancel{\log\left[\frac{t}{t^2+1}\right]}$$

$$= \left[-\frac{1}{2}\log(t^2+1) + \log t\right]_0^1 = \log\left[\frac{t}{\sqrt{t^2+1}}\right]_0^1$$

$= +\infty$.

B_2

2) $$\lim_{x\to 0} \frac{\left[\cancel{x^2} - \frac{x^4}{2} + o(x^4)\right] + \left[\cancel{-x^2} - \frac{x^4}{2} + o(x^4)\right]}{\left[\cancel{1} - \frac{x^4}{2!} + o(x^4)\right] - \left[\cancel{1} + \frac{x^4}{2!} + o(x^4)\right]}$$

$$= \lim_{x\to 0} \frac{-x^4 + o(x^4)}{-x^4 + o(x^4)} = 1.$$

3) $I_{def} = \{x \neq 4\}$ $\qquad f(x) \geq 0 \quad \forall x \in I_{def}$

$f(0) = \frac{1}{4}$; $\nexists$ intersezioni con asse x.

$$f(x) = \begin{cases} \dfrac{x^2+1}{x-4} & \text{per } x > 4 \\[2ex] \dfrac{x^2+1}{4-x} & \text{per } x < 4 \end{cases}$$

$\lim_{x\to +\infty} f(x) = +\infty$

per $x > 4$
$$f(x) = \frac{x^2 - 4x + 4x - 16 + 17}{x-4}$$
$$= x + 4 + \frac{17}{x-4}$$

$\Rightarrow y = x+4$ ASINTOTO OBLIQUO per $x \to +\infty$

$y = 4 - x$ " " per $x \to -\infty$.

$\lim_{x\to 4^\pm} f(x) = +\infty$ AS. VERTICALE DX e SX: $x = 4$.

B_3

$$f'(x) = \begin{cases} \dfrac{2x(x-4)-(x^2+1)}{(x-4)^2} = \dfrac{x^2-8x-1}{(x-4)^2} & \text{per } x>4 \\ -\left[\dfrac{x^2-8x-1}{(x-4)^2}\right] & \text{per } x<4 \end{cases}$$

per $x>4$ $\quad f'(x)>0 \iff x<4-\sqrt{17}$, $\quad x>4+\sqrt{17}$

f decresce in $(-\infty, 4-\sqrt{17})$; cresce in $(4-\sqrt{17}, 4)$; decresce in $(4, 4+\sqrt{17})$; cresce in $(4+\sqrt{17}, +\infty)$.

$x_1 = 4-\sqrt{17}$ punto di MIN. REL.

$x_2 = 4+\sqrt{17}$ punto di MIN. REL.

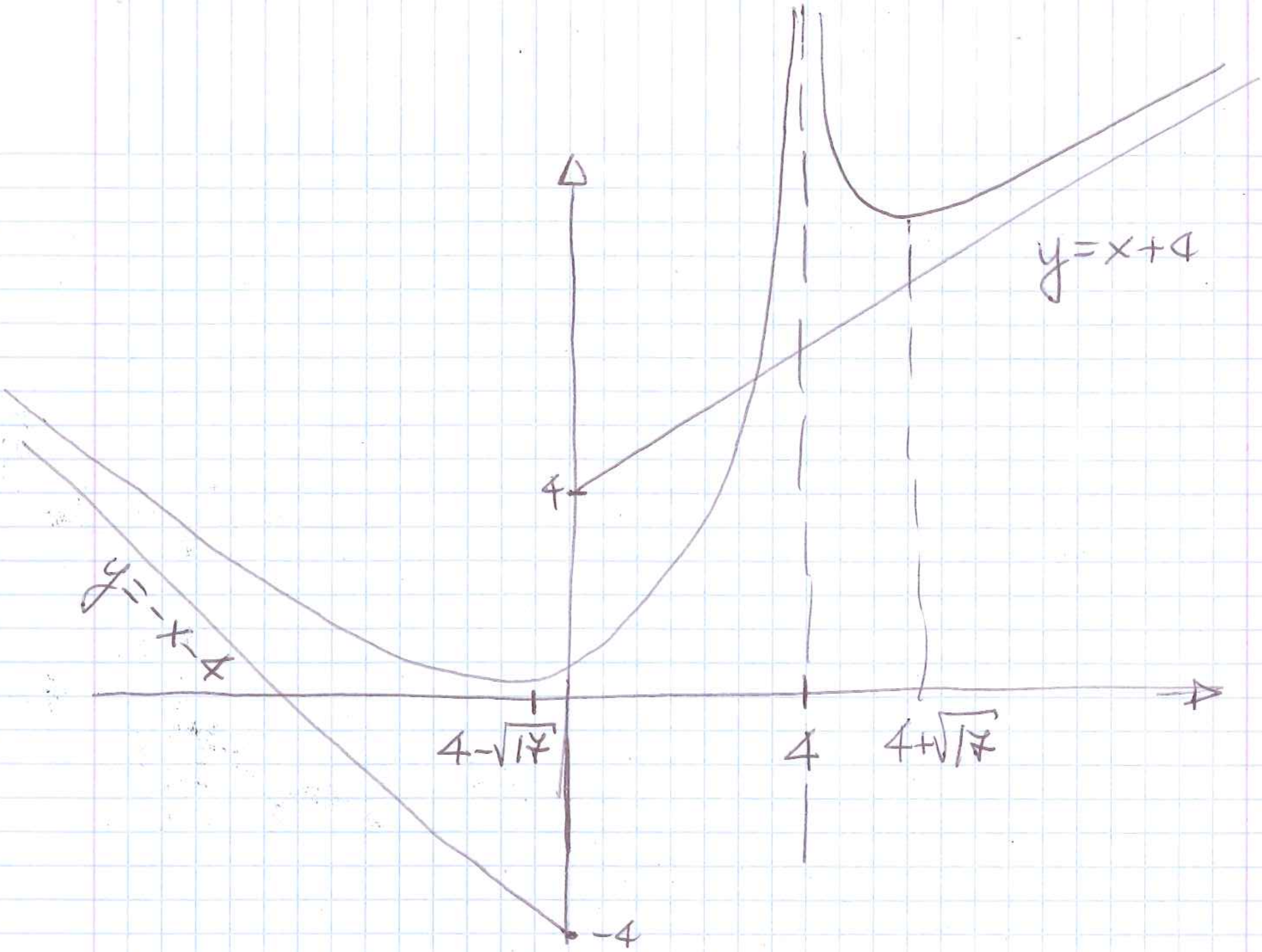
$$f(4-\sqrt{17}) = \left(\frac{34-8\sqrt{17}}{\sqrt{17}}\right) = 2\sqrt{17}-8 \gtrsim 0$$

$$f(4+\sqrt{17}) = \left(\frac{34+8\sqrt{17}}{\sqrt{17}}\right) = 2\sqrt{17}+8$$

MIN. ASS: $f(4-\sqrt{17}) = 2\sqrt{17}-8$.

$$f''(x) = \begin{cases} \dfrac{(2x-8)(x-4)^{\cancel{2}} - (x^2-8x-1)2\cancel{(x-4)}}{(x-4)^{3}} = \\ = \dfrac{\cancel{2x^2}-\cancel{16x}+32-\cancel{2x^2}\cancel{+16x}+2}{(x-4)^3} = \dfrac{34}{(x-4)^3} > 0 & \text{per } x>4 \\ \dfrac{-34}{(x-4)^3} & \text{per } x<4 \end{cases}$$

f convessa in $(-\infty, 4)$ e in $(4, +\infty)$.


$y=x+4$
$y=-x-4$
4
-4
$4-\sqrt{17}$
4
$4+\sqrt{17}$

4) $x^2 - \cancel{y^2} + 2ixy - \mathrm{Im}(-y+ix) = -x^2 - \cancel{y^2} - 2i - 1$

$2x^2 + 2ixy - x = -2i - 1$

$$\Rightarrow \begin{cases} 2x^2 - x + 1 = 0 & \Delta < 0 \\ 2xy = -2 \end{cases}$$

EQ. IMPOSSIBILE.

5) CONV. SEMPLICE: criterio di Leibniz

$a_n = \log\left(1 + \frac{1}{3n+2}\right) \xrightarrow[n \to +\infty]{} 0$

$a_{n+1} = \log\left(1 + \frac{1}{3(n+1)+2}\right) \leq a_n = \log\left(1 + \frac{1}{3n+2}\right)$

$\Leftrightarrow \cancel{1} + \frac{1}{3(n+1)+2} \leq \cancel{1} + \frac{1}{3n+2}$ (il logaritmo è monotono crescente)

$\Leftrightarrow 3(n+1)+2 \geq 3n+2$

ovviamente verificato $\forall n$.