COMPITO C

C1

1)

$$f(x)=\frac{1}{x^2(2x^2-1)}$$

I_{def}: $\begin{cases} x^2\neq 0 \\ 2x^2-1\neq 0 \end{cases} \Rightarrow I_{def}=\left(-\infty,-\frac{1}{\sqrt2}\right)\cup\left(-\frac{1}{\sqrt2},0\right)\cup\left(0,\frac{1}{\sqrt2}\right)\cup\left(\frac{1}{\sqrt2},+\infty\right)$

$=\mathbb{R}-\left\{0,\pm\frac{1}{\sqrt2}\right\}.$

NO intersezione asse y

$f(-x)=f(x)$ f PARI

Segno: $f(x)\neq 0$ sempre

$f(x)>0 \iff 2x^2-1>0$

$\iff x<-\frac{1}{\sqrt2};\ x>\frac{1}{\sqrt2}$

$$\lim_{x\to 0} f(x)=\frac{1}{0^+\cdot(-1)}=-\infty$$ AS. VERTICALE $x=0$

$$\lim_{x\to\frac{1}{\sqrt2}^\pm} f(x)=\lim_{x\to\frac{1}{\sqrt2}^\pm}\frac{1}{x^2(\sqrt2 x-1)(\sqrt2 x+1)}=\frac{1}{\frac12 0^\pm\cdot 2}$$

$=\pm\infty.$

AS. VERTICALE $x=\frac{1}{\sqrt2}$ e, per simmetria, $x=-\frac{1}{\sqrt2}$

C_2

$\lim_{x \to \pm\infty} f(x) = 0$ AS. ORIZZONTALE $y = 0$ a $\pm\infty$.

$$f'(x) = \frac{-1}{[x^2(2x^2-1)]^2}\,[8x^3 - 2x] \cancel{= 0}$$

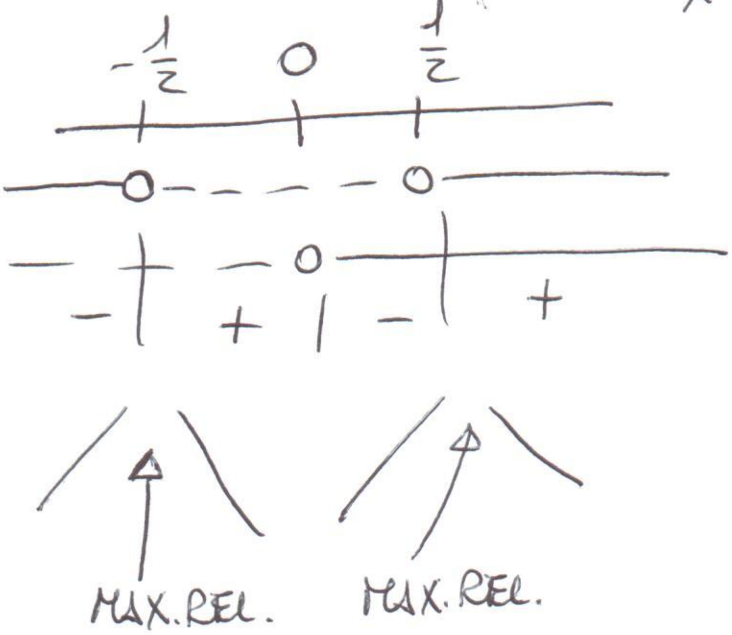
$$\cancel{\Leftrightarrow 2x(4x^2-1) = 0 \Leftrightarrow x = 0 \notin I_{def}}$$

$$= \frac{2x(1-4x^2)}{x^4(2x^2-1)^2} = \frac{2(1-4x^2)}{x^3(2x^2-1)^2}$$

$$f'(x) = 0 \Leftrightarrow 1 - 4x^2 = 0 \Leftrightarrow x = \pm\frac{1}{2}$$

$$f\left(\pm\frac{1}{2}\right) = \frac{1}{2\left(\frac{1}{2}\right)^4 - \frac{1}{4}} = -8$$

$$f'(x) > 0 \Leftrightarrow \frac{1-4x^2}{x^3} > 0 \Leftrightarrow \frac{4x^2-1}{x} < 0$$

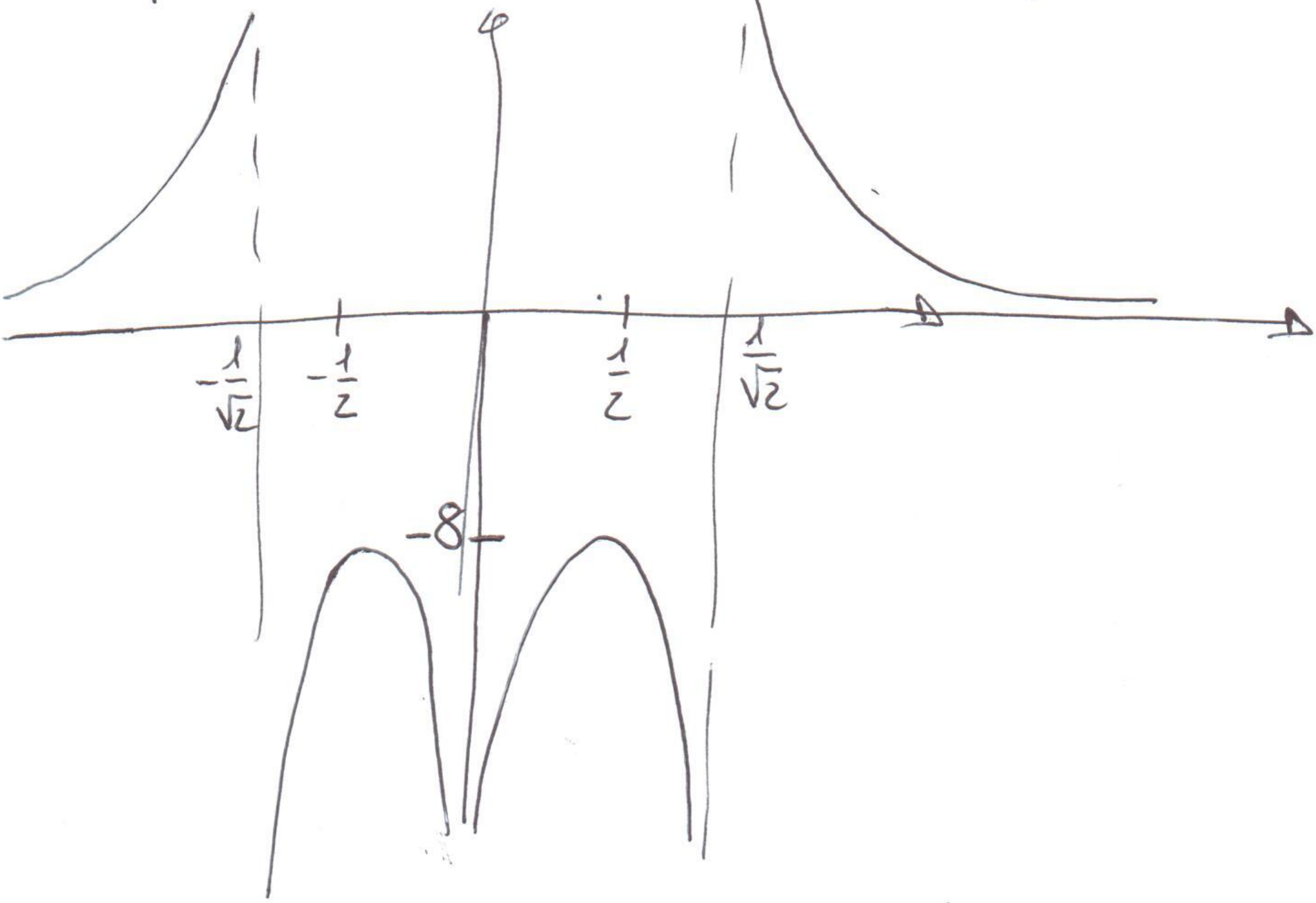


f cresce in $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$;
cresce in $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{2}\right)$;
decresce in $\left(-\frac{1}{2}, 0\right)$;
cresce in $\left(0, \frac{1}{2}\right)$;
decresce in $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$;
decresce in $\left(\frac{1}{\sqrt{2}}, +\infty\right)$

$x = \pm \frac{1}{2}$ punti di MAX. REL.

$\nexists$ MAX. ASS. perché $\lim_{x \to \frac{1}{\sqrt{2}}^{\pm}} f(x) = \pm\infty$.
o MIN. ASS.

In ipotesi di numero minimo di flessi:



f convessa in $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$ e in $\left(\frac{1}{\sqrt{2}}, +\infty\right)$

concava in $\left(-\frac{1}{\sqrt{2}}, 0\right)$ e in $\left(0, \frac{1}{\sqrt{2}}\right)$.

~~In effetti~~

$\cancel{f''(x) = 2\left[-8x \cdot x^3(2x^2-1)^2 - (1-4x^2)\right[}$

IN EFFETTI

C_4

$$f''(x) = 2\,\frac{\left[-8x\cdot x^{\cancel{3}}(2x^2-1)^{\cancel{2}} - (1-4x^2)\left[3\cancel{x^{2}}(2x^2-1)^{\cancel{2}} + x^{\cancel{3}}\,2\,\cancel{(2x^2-1)}\,4x\right]\right]}{x^{4}(2x^2-1)^{\cancel{4}\,3}}$$

$$= \frac{2}{x^4(2x^2-1)^3}\left[-8x^2(2x^2-1) - (1-4x^2)\cdot(\underbrace{6x^2-3+8x^2})\right]$$

$$= \frac{2}{x^4(2x^2-1)^3}\left[-16x^4+8x^2-14x^2+56x^4+3-12x^2\right]$$

$$= \frac{2}{x^4(2x^2-1)^3}\left(\underbrace{40x^4-18x^2+3}_{>0}\right)$$

$$f''(x) \neq 0$$

$$f''(x) > 0 \iff 2x^2-1>0 \iff x<-\frac{1}{\sqrt{2}};$$

$$x > \frac{1}{\sqrt{2}}.$$

2) $$\frac{e^{\sin\left(\frac{1}{x}\right)}\cos\left(\frac{1}{x}\right)}{x^2} \underset{x\to\infty}{\sim} \frac{e^{\frac{1}{x}}}{x^2} \underset{x\to\infty}{\sim} \frac{1}{x^2}$$

C_5

INTEGRABILE a $+\infty$.

$$\int_{\frac{2}{\pi}}^{+\infty} \frac{e^{\sin\left(\frac{1}{x}\right)} \cdot \cos\left(\frac{1}{x}\right)}{x^2}\,dx = \left[\begin{array}{l} t = \sin\left(\frac{1}{x}\right) \\ \Rightarrow dt = \cos\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right) \\ t\left(\frac{2}{\pi}\right) = \sin\left(\frac{\pi}{2}\right) = 1 \\ t(+\infty) = \sin(0) = 0 \end{array}\right]$$

$$= -\int_1^0 e^t\,dt = \left[-e^t\right]_1^0 = \underline{-1+e.}$$

3) Equazione a variabili separabili.

$$A(x) = \frac{e^{-\operatorname{arctg} x}}{1+x^2} \in C^\infty(\mathbb{R})$$

$$B(y) = e^y \in C^\infty(\mathbb{R})$$

$\Rightarrow \exists!$ sol. $y \in C^1$ LOCALE.

$B(y) = e^y > 0 \Rightarrow$ NO SOL. SINGOLARI.

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$$\Rightarrow \int e^{-y}\,dy = \int \frac{e^{-\operatorname{arctg} x}}{1+x^2}\,dx$$

$$-e^{-y} = -e^{-\operatorname{arctg} x} + C$$

$$y(0)=0 \Rightarrow -1 = -1 + C \Rightarrow C = 0$$

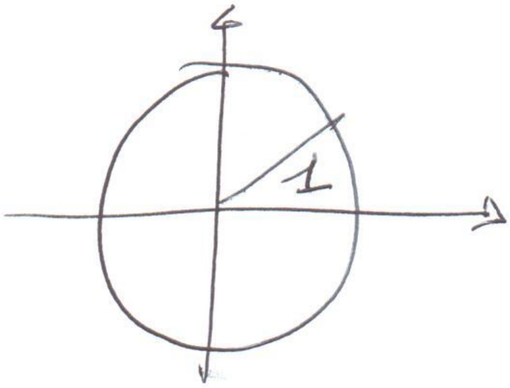
$$\Rightarrow e^{-y} = e^{-\operatorname{arctg} x} \Rightarrow \underline{y = \operatorname{arctg} x}$$

4) $$\left|e^{-i(x+iy)}\right|^2 \cdot \left|\bar{z}\right|^2 = e^{2y}$$

$$\underbrace{\left|e^{-ix}\right|^2}_{=1} \cancel{\left|e^{+y}\right|^2} \cdot \left|\bar{z}\right|^2 = \cancel{e^{2y}}$$

$$\Rightarrow \left|\bar{z}\right|^2 = 1$$

$$\Rightarrow x^2 + y^2 = 1$$



5) $$a_n = \frac{(n+2)\sqrt{n} + \sqrt{n^3+1}}{(n+2)^2 n - n^3 - 1} = \frac{(n+2)\sqrt{n} + \sqrt{n^3+1}}{\cancel{n^3} + 4n^2 + 4n - \cancel{n^3} - 1}$$

$$\sim \frac{2\sqrt{n^3}}{4n^2} = \frac{1}{2\sqrt{n}} \longrightarrow 0$$

Poichè $a_n \sim \dfrac{1}{2\sqrt{n}}$ 7

$\Rightarrow \sum a_n \approx \frac{1}{2} \sum \frac{1}{\sqrt{n}}$ divergente.