D1

COMPITO D

1) $A(x) = \dfrac{e^{-\operatorname{arctg} x}}{(1+x^2)} \in C^\infty(\mathbb{R})$

$B(y) = e^y \in C^\infty(\mathbb{R})$

$\Rightarrow \exists!$ sol. y^1 LOCALE.

$e^y > 0 \quad \forall y \quad \Rightarrow$ NO SOL. SINGOLARI.

Separiamo le variabili

$$\int dy\, e^{-y} = \int \frac{e^{-\operatorname{arctg} x}}{1+x^2} dx$$

$$-e^{-y} = -e^{-\operatorname{arctg} x} + C$$

$y(0)=0 \Rightarrow -1 = -1 + C \Rightarrow C = 0$

$\Rightarrow e^{-y} = e^{-\operatorname{arctg} x} \Rightarrow \boxed{y(x) = \operatorname{arctg} x}$

2) $\left|e^{i(x+iy)}\right|^2 |z|^2 = e^{-2y}$

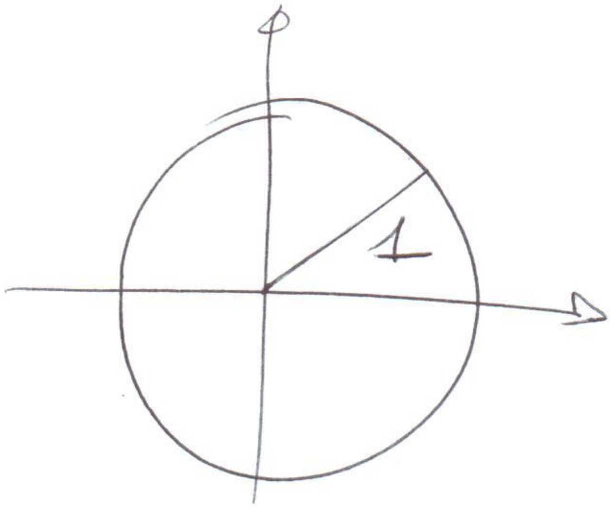
$|e^{ix}|^2 |e^{-y}|^2 |z|^2 = e^{-2y}$

$e^{-2y} |z|^2 = e^{-2y} \Rightarrow |z|^2 = 1$

$\Rightarrow x^2 + y^2 = 1$

Graficamente

D2



3) $$a_n = \frac{n\sqrt{n+2} + \sqrt{n^3+n}}{\left(n\sqrt{n+2} - \sqrt{n^3+n}\right)\left(n\sqrt{n+2} + \sqrt{n^3+n}\right)}$$

$$= \frac{n\sqrt{n+2} + \sqrt{n^3+n}}{n^2(n+2) - (n^3+n)} = \frac{n\sqrt{n+2} + \sqrt{n^3+n}}{2n^2 - n}$$

~~$2n^2 - n \neq 0 \iff n = 0$ oppure $n = 1$~~

$2x^2 - x \neq 0 \iff x = 0 \; ; \; x = \frac{1}{2}$

$\Rightarrow$ la successione è sempre definita.

$$a_n \sim \frac{2\sqrt{n^3}}{2n^2} = \frac{1}{\sqrt{n}} \xrightarrow[n \to \infty]{} 0$$

Poiché $a_n \sim \frac{1}{\sqrt{n}}$

$$\Rightarrow \sum a_n \approx \sum \frac{1}{\sqrt{n}} \text{ che diverge.}$$

4) $f(x) = \dfrac{1}{x^2(x^2-2)}$ $\qquad$ f pari

P3

$I_{def} = \{x \neq 0;\ x \neq \pm\sqrt{2}\} =$

$(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, 0) \cup (0, \sqrt{2}) \cup (\sqrt{2}, +\infty)$.

$f(x) \neq 0 \quad \forall x \in I_{def}$ $\qquad$ $\nexists$ intersezione con asse y.

$f(x) > 0 \iff x^2-2>0 \iff x<-\sqrt{2};\ x>\sqrt{2}$.

$\lim_{x \to \pm\infty} f(x) = 0$ $\qquad$ AS. ORIZZONTALE $y=0$ per $x \to \pm\infty$.

$$\lim_{x \to \sqrt{2}^\pm} f(x) = \lim_{x \to \sqrt{2}^\pm} \frac{1}{x^2(x-\sqrt{2})(x+\sqrt{2})} = \frac{1}{2 \cdot 0^\pm \cdot 2\sqrt{2}}$$

$$= \pm\infty$$

AS. VERTICALE $\quad x = \sqrt{2}$ e, per parità, $x = -\sqrt{2}$.

$$f'(x) = \frac{-1}{[x^2(x^2-2)]^2}\left[4x^3-4x\right]$$

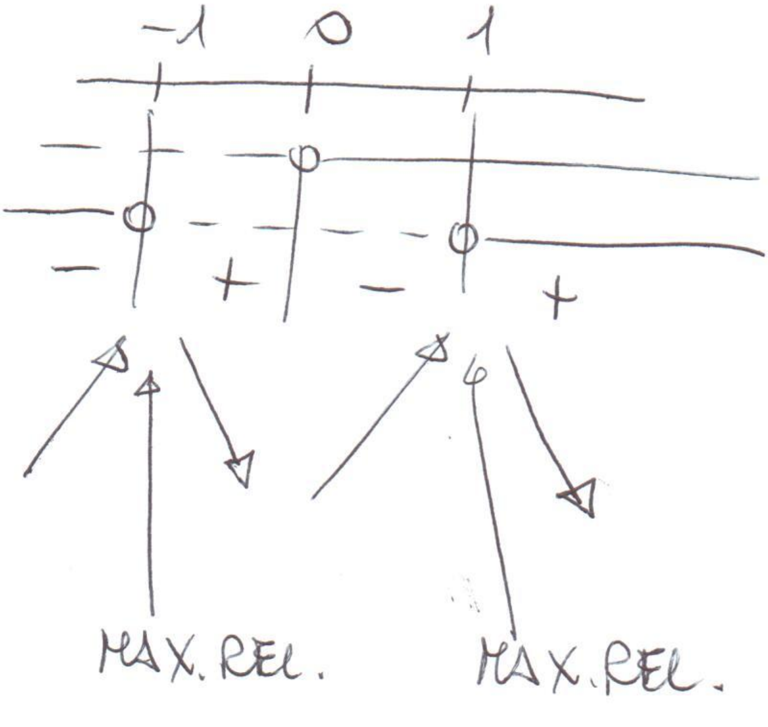
$$= \frac{-4x(x^2-1)}{x^4(x^2-2)^2}$$

$f'(x)=0 \iff x(x^2-1)=0$

$\iff x=0$ $\cancel{\in} I_{def}$; $x=\pm 1$

$f(\pm 1) = -1.$

$f'(x)>0 \iff x(x^2-1)<0$



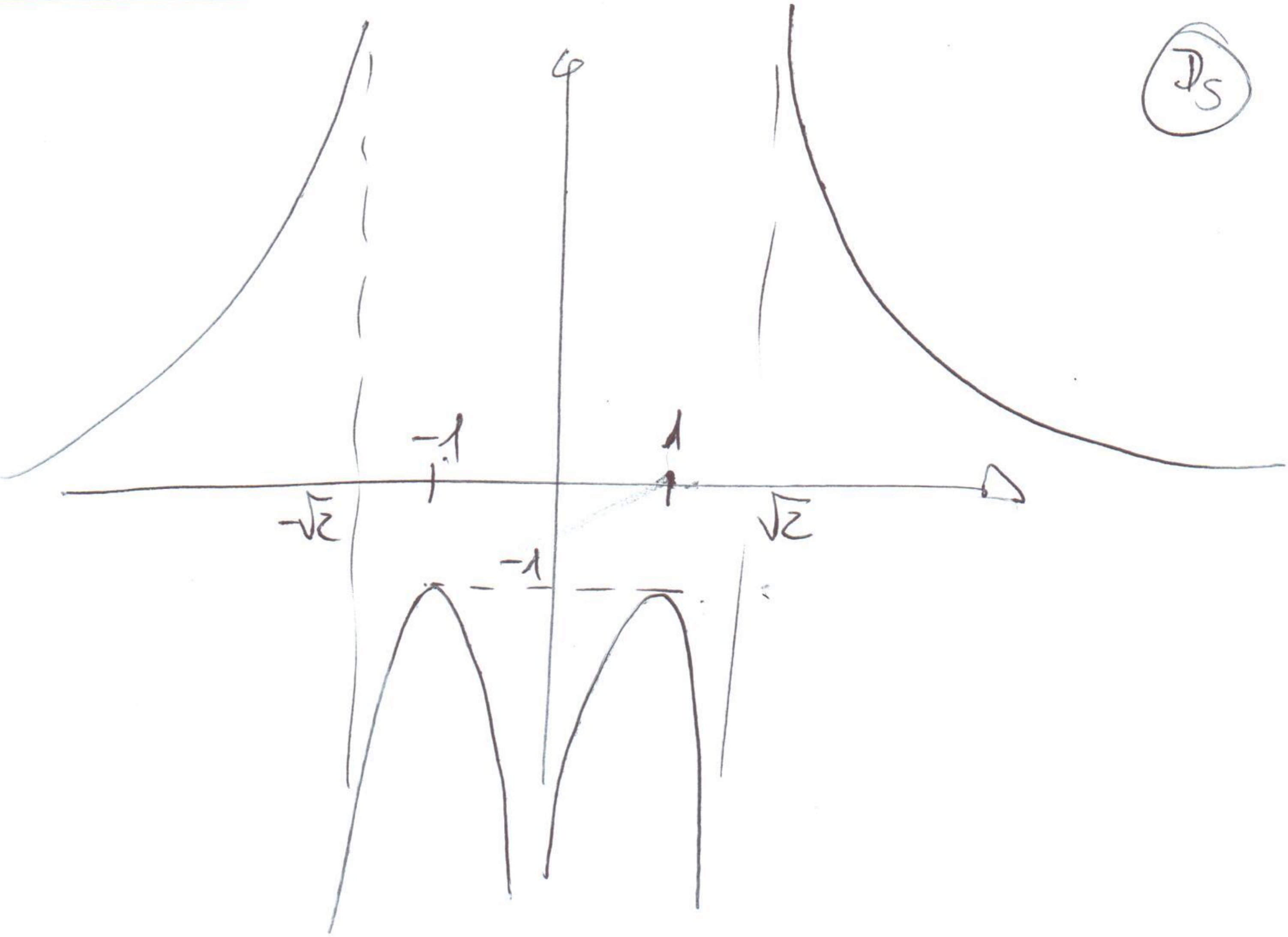
f cresce in $(-\infty,-\sqrt{2})$,
in $(-\sqrt{2},-1)$;
decresce in $(-1,0)$;
cresce in $(0,1)$;
decresce in $(1,\sqrt{2})$;
decresce in $(\sqrt{2},+\infty)$.

∄ MAX. e MIN. ASSOLUTI, perché

$$\lim_{x\to\sqrt{2}^\pm} f(x) = \pm\infty.$$

In ipotesi di numero minimo di flessi il grafico è

D5



In effetti,

$$f''(x) = -4\left[\frac{(3x^2-1)\left(x^{\cancel{4}}(x^2-2)^{\cancel{2}}\right) - (x^3-x)\left[4x^{\cancel{3}}(x^2-2)^{\cancel{2}} + x^{\cancel{4}}2\cancel{(x^2-2)}2x\right]}{x^{\cancel{8}5}(x^2-2)^{3}}\right]$$

$$= -4\cancel{x}\left[\frac{(3x^2-1)(x^2-2) - (x^2-1)\left[4x^2-8+4x^2\right]}{x^4(x^2-2)^3}\right]$$

$$= -4\left[\frac{3x^4-6x^2-x^2+2-8(x^4-2x^2+1)}{x^4(x^2-2)^3}\right]$$

$$= -4\left[\frac{-5x^4+9x^2-6}{x^4(x^2-2)^3}\right] = 4\left[\frac{5x^4-9x^2+6}{x^4(x^2-2)^3}\right]$$

D_6

$$5x^4-9x^2+6>0 \quad \forall x$$

$$\Rightarrow f''(x)>0 \iff \frac{1}{(x^2-2)^3} \geqslant 0$$

$$\iff \frac{1}{x^2-2} \geqslant 0 \quad \iff x<-\sqrt{2}\ ;\ x>\sqrt{2}.$$

5)

$$f(x) \underset{x\to\infty}{\sim} \frac{e^{\frac{1}{x}}}{\cos^2\left(\frac{1}{x}\right)x^2} \underset{x\to\infty}{\sim} \frac{1}{x^2}$$

INTEGRABILE

$$\int_{\frac{4}{\pi}}^{+\infty} \frac{e^{tg\left(\frac{1}{x}\right)}}{\cos^2\left(\frac{1}{x}\right)x^2}\,dx =$$

$$\left[\begin{array}{l} tg\left(\frac{1}{x}\right)=t \\ dt=\frac{1}{\cos^2\left(\frac{1}{x}\right)}\cdot\left(-\frac{1}{x^2}\right)dx \end{array}\right.$$

$$t\left(\frac{4}{\pi}\right)=tg\left(\frac{\pi}{4}\right)=1$$

$$t(\infty)=tg(0)=0$$

$$= \int_1^0 -e^t\,dt = \left[-e^t\right]_1^0 = e-1.$$