SVOLGIMENTI PROVA DI ESONERO DI ANALISI MAT. 1 DEL 21/11/23 - TURNO 11,30

$\boxed{\text{COMPITO B}}$

$B_1$

1)
$$z_{1,2} = \frac{3(i-1) + \sqrt{9(i-1)^2 + 36i}}{2}$$

$$= \frac{3}{2}\left[(i-1) + \sqrt{(i-1)^2 + 4i}\right] = \frac{3}{2}\left[i-1 + \sqrt{(i+1)^2}\right]$$

$$= \frac{3}{2}\left[i-1 \pm (i+1)\right] = \begin{array}{l} 3i \\ -3 \end{array}$$

In alternativa:

$$z_{1,2} = \frac{3(i-1) + 3\sqrt{\cancel{-1}\cancel{+1} - 2i + 4i}}{2} =$$

$$= \frac{3}{2}\left[i-1 + \sqrt{2i}\right] = \frac{3}{2}\left(i-1 + \sqrt{2}\sqrt{e^{i\pi/2}}\right)$$

$$\Rightarrow z_1 = \frac{3}{2}\left(i-1+\sqrt{2}\, e^{i\frac{\pi}{4}}\right) = \frac{3}{2}\left(i-1+\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)\right)$$

$$= \frac{3}{2}\left(i-1+1+i\right) = 3i$$

$$z_2 = \frac{3}{2}\left(i-1-\sqrt{2}\, e^{i\frac{\pi}{4}}\right) = \frac{3}{2}\left[i-1-\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)\right]$$

$$= \frac{3}{2}\left(i-1-1-i\right) = -3.$$

Metodo ancor più rapido (proposto dal Prof. Cafra):

$$z^2+3z-3iz-9i=0$$

$$z(z+3)-3i(z+3)=0$$

$$(z-3i)(z+3)=0$$

$$\Rightarrow z_1=3i \quad ; \quad z_2=-3$$

Il risultato non contraddice il Teo. Fond. dell'Algebra, perchè siamo in presenza di un'equazione algebrica di 2° grado, che ammette 2 soluzioni.

2) $$\frac{\binom{n+1}{n-1}}{\binom{n+2}{n}} = \frac{\left[\frac{(n+1)!}{(n-1)!\,2!}\right]}{\left[\frac{(n+2)!}{n!\,2!}\right]} = \frac{(n+1)!}{(n-1)!\,\cancel{2!}} \cdot \frac{n!\,\cancel{2!}}{(n+2)!}$$

B3

$$= \frac{n}{n+2}$$

$$\Rightarrow \sum a_n = \sum \frac{n}{(n+2)\,n^\alpha} \simeq \sum \frac{1}{n^\alpha}$$

convergente per $\alpha > 1$
divergente per $\alpha \leq 1$.

3) $$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{x^3}{x}\left[\frac{\sqrt{x+1}-\sqrt{x-2}}{\sqrt{x-2}\,\sqrt{x+1}}\right]$$

$$= \lim_{x\to+\infty} \frac{x^2}{x}\left[\frac{\left(\sqrt{x+1}-\sqrt{x-2}\right)\left(\sqrt{x+1}+\sqrt{x-2}\right)}{\sqrt{x+1}+\sqrt{x-2}}\right]$$

$$= \lim_{x\to+\infty} x\left[\frac{\cancel{x}+1-\cancel{x}+2}{\sqrt{x+1}+\sqrt{x-2}}\right] = 3\lim_{x\to+\infty}\frac{x}{2\sqrt{x}}$$

$$= \frac{3}{2}\lim_{x\to+\infty}\sqrt{x} = +\infty.$$