SVOLGIMENTO PROVA SCRITTA di ANALISI MAT. 1 del 15/7/2015 ①

1) $e^{\rho + i\vartheta} = 8e^{i\left(\frac{\pi}{2} + 2k\pi\right)}$

$$\Rightarrow \begin{cases} e^{\rho} = 8 \\ \vartheta = \frac{\pi}{2} + 2k\pi \end{cases} \Rightarrow \begin{cases} \rho = \log 8 \\ \vartheta = \frac{\pi}{2} \end{cases}$$

(si chiedeva l'argomento principale)

$$z = \log 8 \, e^{i\frac{\pi}{2}}$$
$$= i \log 8$$
$$= \log 8 \left[\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)\right]$$

2) $$a_n \sim -\log(n^2)\left[\cancel{1} - \cancel{\frac{1}{2n}} + \frac{1}{4!n^2}\right.$$
$$\left. - \left[\cancel{1} - \cancel{\frac{1}{2n}} + \frac{1}{2(2n)^2}\right] + o\left(\frac{1}{n^2}\right)\right]$$
$$= -2\log(n)\left[\left(\frac{1}{24} - \frac{1}{8}\right)\frac{1}{n^2} + o\left(\frac{1}{n^2}\right)\right]$$
$$\sim +\frac{1}{6}\frac{\log n}{n^2}$$ termine legato alla serie di Abel,
$$\sum \frac{1}{n^p (\log n)^q}$$, convergente per $p > 1$.

3) Eq. omogenea associata: ②

$\alpha^2+2\alpha+1=0 \Rightarrow (\alpha+1)^2=0$

$\Rightarrow \alpha_{1,2}=-1$

$\Rightarrow y_0(x)=C_1e^{-x}+C_2xe^{-x}$

Eq. non omogenea. Principio di sovrapposizione. Per

$y''+2y'+y=e^{-x}$ $\quad y_p^1=Ax^2e^{-x}$

$\Rightarrow {y_p^1}'=A(-x^2+2x)e^{-x}$; ${y_p^1}''=A(x^2-4x+2)e^{-x}$

$\Rightarrow A[\cancel{x^2}-\cancel{4x}+2-\cancel{2x^2}+\cancel{4x}+\cancel{x^2}]=1$

$\Rightarrow A=\frac{1}{2} \Rightarrow y_p^1(x)=\frac{1}{2}x^2e^{-x}$

Per $y''+2y'+y=x^2$ $\quad y_p^2(x)=Bx^2+Cx+D$

$(y_p^2)'=2Bx+C$; $(y_p^2)''=2B$

$\Rightarrow 2B+4Bx+2C+Bx^2+Cx+D=x^2$

$\begin{cases} B=1 \\ 4B+C=0 \\ 2B+2C+D=0 \end{cases}$ $\begin{cases} B=1 \\ C=-4 \\ D=6 \end{cases}$

$$\Rightarrow \quad y_p^2(x) = x^2 - 4x + 6$$

Quindi l'integrale generale è

$$y(x) = C_1 e^{-x} + C_2 x e^{-x} + \frac{1}{2} x^2 e^{-x} + x^2 - 4x + 6$$

$$y(0) = C_1 + 6 = \underline{1} \quad \Rightarrow \quad C_1 = -5$$

$$y'(x) = -C_1 e^{-x} + C_2 e^{-x} - C_2 x e^{-x} + x e^{-x} - \frac{1}{2} x^2 e^{-x} + 2x - 4$$

$$y'(0) = -C_1 + C_2 - 4 = 0$$

$$\Rightarrow \quad C_2 = -1$$

$$\Rightarrow \quad y(x) = -5 e^{-x} - x e^{-x} + \frac{1}{2} x^2 e^{-x} + x^2 - 4x + 6.$$

4) $G(x) = \int \left[e^{\sin(x)} + e^{\sin^2(x)} \right] 2 \sin x \cos x \, dx$ ④

$t = \sin x \quad \Rightarrow \quad dt = \cos x \, dx$

$$= \int \left[e^t + e^{t^2} \right] 2t \, dt \Big|_{t=\sin x}$$

$$= \left[\int e^t 2t \, dt + \int 2t e^{t^2} dt \right]_{t=\sin x}$$

per parti

$$= \left[2te^t - \int 2e^t dt + e^{t^2} \right]_{t=\sin x}$$

$$= \left[2te^t - 2e^t + e^{t^2} \right]_{t=\sin x} + c$$

$$= 2 \sin x \, e^{\sin x} - 2e^{\sin x} + e^{\sin^2 x} + c$$

$G(\pi) = -2 + 1 + c = 1 \quad \Rightarrow \quad c = 2$

$$\Rightarrow G(x) = 2 \sin x \, e^{\sin x} - 2e^{\sin x} + e^{\sin^2 x} + 2.$$

5)

$$f(x) = \left| \frac{(x-2)(x-4)}{x-3} \right|$$

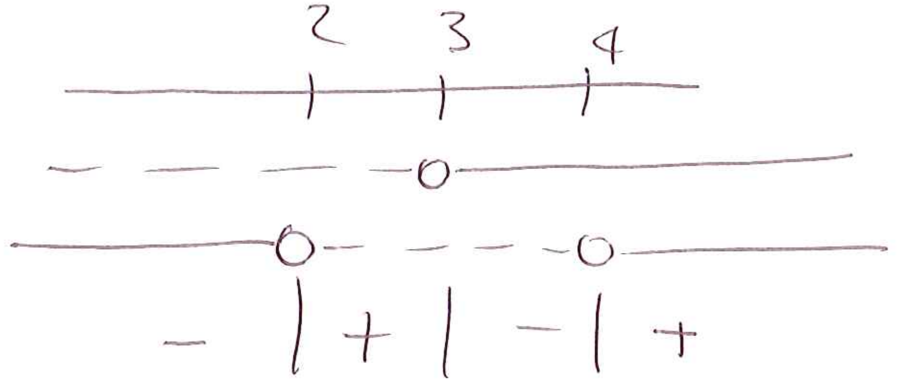
$$D = \mathbb{R} - \{3\} = (-\infty, 3) \cup (3, +\infty)$$

$$\lim_{x \to 3^{\pm}} f(x) = \left| \frac{1 \cdot (-1)}{0} \right| = +\infty$$

$x = 3$ ASINTOTO VERTICALE

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \left| \frac{x^2}{x} \right| = \lim_{x \to \pm\infty} |x| = +\infty$$

Segno di $\frac{(x-2)(x-4)}{x-3}$:



$$\Rightarrow \quad f(x) = \begin{cases} -\left[\frac{(x-2)(x-4)}{x-3}\right] & \text{se } x < 2; \; 3 < x < 4 \\ \left[\frac{(x-2)(x-4)}{(x-3)}\right] & \text{se } 2 \leq x < 3; \; x \geq 4. \end{cases}$$

$\Rightarrow$ ~~per $x < 2$~~ $\cancel{f(x) = -\left[\frac{x^2 - 3x - 3x + 9 - 1}{x - 3}\right]}$

Poichè $\frac{x^2-6x+8}{x-3} = \frac{(x-3)^2-1}{x-3}$ ⑥

$$= x-3 - \frac{1}{x-3}$$

allora

$$f(x) = \begin{cases} -(x-3) + \frac{1}{x-3} & \text{se } x<2;\ 3<x<4 \\ (x-3) - \frac{1}{x-3} & \text{se } 2\le x<3;\ x\ge 4 \end{cases}$$

Per $x \to +\infty$ $f(x) \sim x-3$

AS. OBLIQUO

Per $x \to -\infty$ $f(x) \sim -x+3$

AS. OBLIQUO

Poichè $\lim_{x\to\pm\infty} f(x) = +\infty$ NON ESISTE MAX. ASS.

$f(x) \ge 0$ e $f(2)=f(4)=0$

MIN. ASSOLUTO.