SVOLGIMENTI ANALISI 2 del 2/7/2020

1) $x \neq 0$. Si studia la EDO in $(-\infty, 0)$ oppure in $(0, +\infty)$.

$$y(x) = e^{-\int \left(\frac{1}{x^2} - \frac{2}{x}\right) dx} \left[\int e^{\int \left(\frac{1}{x^2} - \frac{2}{x}\right) dx} dx + C \right]$$

$$= e^{\frac{1}{x} + 2\ln|x|} \left[\int e^{-\frac{1}{x} - 2\ln|x|} dx + C \right]$$

$$= x^2 e^{\frac{1}{x}} \left[\int e^{-\frac{1}{x}} \frac{1}{x^2} dx + C \right]$$

$$= x^2 e^{\frac{1}{x}} \left[e^{-\frac{1}{x}} + C \right] = \underline{x^2 + C x^2 e^{\frac{1}{x}}}$$

In $(0, +\infty)$:

$$\lim_{x \to 0^+} y(x) = \lim_{x \to 0^+} C \frac{e^{\frac{1}{x}}}{\left(\frac{1}{x^2}\right)} \qquad \left(t = \frac{1}{x}\right)$$

$$= \lim_{t \to +\infty} C \frac{e^t}{t^2} = \begin{cases} C \cdot (+\infty) & \text{se } C \neq 0 \\ 0 & \text{se } C = 0 \end{cases}$$

$$\Rightarrow \quad \underline{y(x) = x^2}$$

In $(-\infty, 0)$:

$$\lim_{x\to 0^-} y(x) = \lim_{x\to 0^-} Cx^2 e^{+\frac{1}{x}} = 0 \qquad \forall C \in \mathbb{R}$$

$$\Rightarrow \boxed{y(x) = x^2 + Cx^2 e^{\frac{1}{x}}, \; C \in \mathbb{R}}$$

2) $I_{def} = \mathbb{R}$ né pari, né dispari

$f(x) = 0 \iff x = 0\,;\, x = 2$

$f(x) \geq 0 \quad \forall x \in \mathbb{R}$

$f(0) = 0$

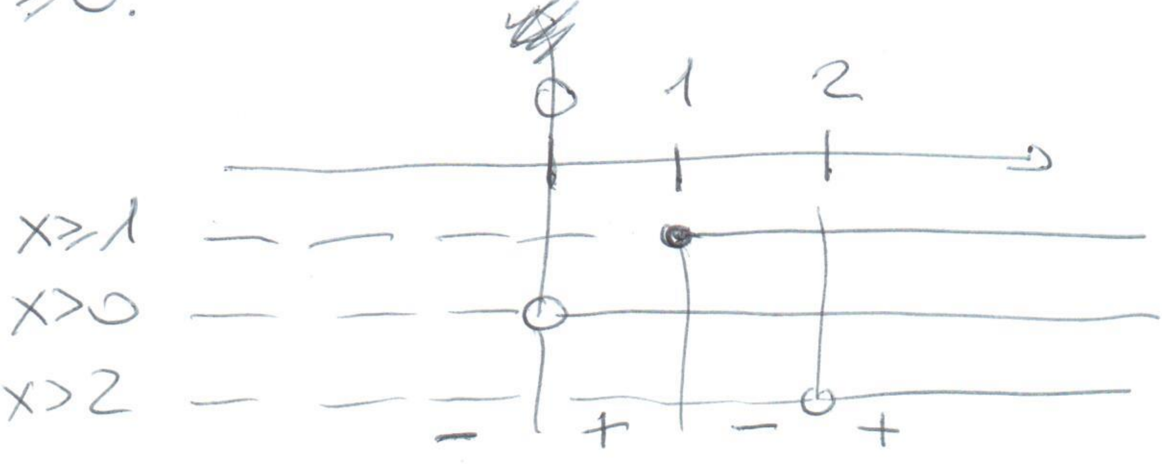
$$f(x) \underset{x\to\pm\infty}{\sim} x^{\frac{4}{3}} \Rightarrow f(x) \xrightarrow[x\to\pm\infty]{} +\infty$$

<u>superlineare</u> $\Rightarrow$ NO AS. OBLIQUO.

$$f'(x) = \frac{2}{3}(x^2-2x)^{-\frac{1}{3}}(2x-2) = \frac{4}{3}\frac{(x-1)}{[x(x-2)]^{\frac{1}{3}}}$$

$f'(x) = 0$ se $x = 1$

$f'(x) \geq 0$:



f decresce in $(-\infty, 0)$.

In $x=0$ e $x=2$ MIN. ASS.

f cresce in $(0,1)$

decresce in $(1,2)$

cresce in $(2,+\infty)$

2 bis

$x=1$ punto di MAX. REL.

$f(1)=1$

$\nexists$ MAX. ASS.

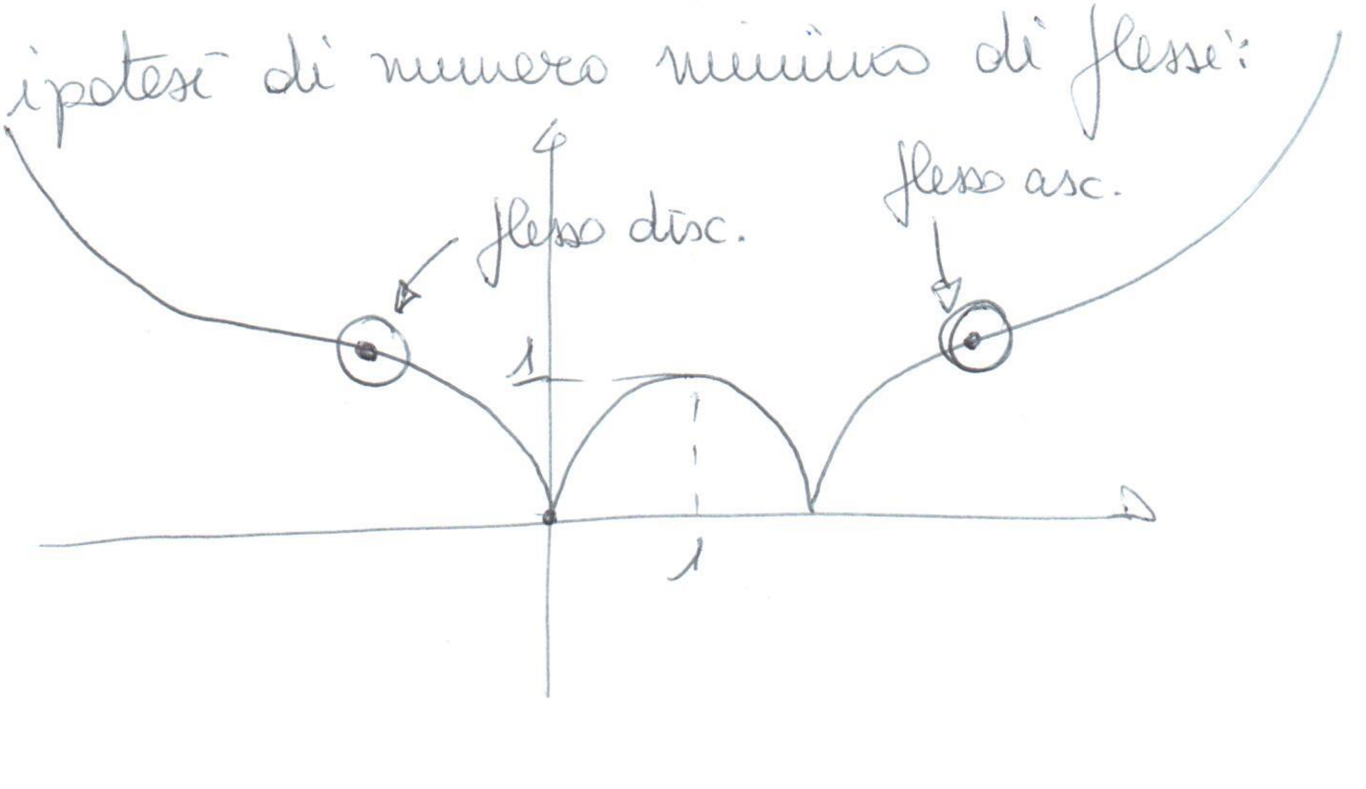
$$\lim_{x\to 2^\pm} f'(x) = \frac{4}{3}\cdot\frac{1}{[2\cdot 0^\pm]^{1/3}} = \pm\infty$$

CUSPIDE

$$\lim_{x\to 0^\pm} f'(x) = \frac{4}{3}\cdot\frac{(-1)}{[0^\pm\cdot(-2)]^{1/3}} = \frac{-1}{0^\pm\cdot(-2)} = \pm\infty$$

CUSPIDE.

In ipotesi di numero minimo di flessi:



In effetti,

$$f''(x) = \frac{4}{3}\left[\frac{[x(x-2)]^{\frac{1}{3}} - (x-1)\frac{1}{3}[x(x-2)]^{-\frac{2}{3}}2(x-1)}{[x(x-2)]^{2/3}}\right]$$

(3)

$$= \frac{4}{3}\left[\frac{x(x-2) - \frac{2}{3}(x-1)^2}{\underbrace{[x(x-2)]^{4/3}}_{\geq 0}}\right]$$

$$= \frac{4}{3}\left[\frac{x^2 - 2x - \frac{2}{3}x^2 + \frac{4}{3}x - \frac{2}{3}}{[x(x-2)]^{4/3}}\right]$$

$$= \frac{4}{9}\left[\frac{x^2-2x-2}{[x(x-2)]^{4/3}}\right] \geq 0$$

$$\Longleftrightarrow \quad x^2 - 2x - 2 \geq 0$$

$x_{1,2} = 1 \pm \sqrt{3} \quad \Rightarrow$ f convessa in $(-\infty, 1-\sqrt{3})$

concava in $(1-\sqrt{3}, 0)$

concava in $(0, 2)$

concava in $(2, 1+\sqrt{3})$

convessa in $(1+\sqrt{3}, +\infty)$.

3) $|z|^2 \arg z + i|z| \arg z - i|z|^2 + |z| = -|z|$

$\Rightarrow |z| \left[|z| \arg z + 2|z| + i(\arg z - |z|) \right] = 0$

4

$|z| = 0 \Rightarrow z = 0$ altrimenti, se $|z| \neq 0$

$$\begin{cases} \cancel{|z|}(\arg z + 2) = 0 \\ \arg = |z| \end{cases}$$

$$\Rightarrow \begin{cases} \arg z = |z| \\ |z| + 2 = 0 \end{cases}$$ impossibile.

Pertanto, se NON si assegna $\arg z$ all'origine, NESSUNA SOLUZIONE.

Se all'origine si assegna OGNI $\arg z$

$\Rightarrow$ unica sol.: $z = 0$.

4) Criterio della radice:

$$\sqrt[n]{a_n} = \frac{\left(1 + \frac{1}{n}\right)^{\alpha n}}{3} \longrightarrow \frac{e^\alpha}{3} \begin{cases} < 1 & \text{se } \alpha < \ln 3 \\ = 1 & \text{se } \alpha = \ln 3 \\ > 1 & \text{se } \alpha > \ln 3 \end{cases}$$

Per $\alpha = \ln 3$:

$$\frac{\left(1+\frac{1}{n}\right)^{n^2 \ln 3}}{3^n} = \frac{e^{n^2 \ln 3 \ln\left(1+\frac{1}{n}\right)}}{3^n}$$

$$= \frac{3^{n^2 \ln\left(1+\frac{1}{n}\right)}}{3^n} = 3^{n^2\left[\ln\left(1+\frac{1}{n}\right) - \frac{1}{n}\right]}$$

$$= 3^{n^2\left[\cancel{\frac{1}{n}} - \frac{1}{n^2} + o\left(\frac{1}{n^2}\right) \cancel{- \frac{1}{n}}\right]}$$

$$= 3^{-1+o(1)} \to \frac{1}{3} \neq 0$$

$\Rightarrow$ la serie DIVERGE per $\alpha = \ln 3$

$\Rightarrow$ convergente per $\alpha < \ln 3$

divergente per $\alpha \geq \ln 3$.

5) $$f(x) \underset{x\to+\infty}{\sim} \frac{x^4\left[\cancel{1}-\cancel{\frac{1}{x^2}}+\cancel{\frac{1}{2x^4}}-\frac{1}{6x^6}-\cancel{1}+\cancel{\frac{1}{x^2}}-\cancel{\frac{1}{2x^4}}+\frac{1}{3x^6}+o\left(\frac{1}{x^6}\right)\right]}{\ln x}$$

$$= \frac{\frac{1}{6x^2}+o\left(\frac{1}{x^2}\right)}{\ln x} \sim \frac{1}{6x^2\ln x}$$ 6

integrabile in $[a,+\infty)$

$\Rightarrow f$ è integrabile in $[a,+\infty)$. $a>1$

Per $x\to1^+$:

$$\lim_{x\to1^+} f(x) = \frac{e^{-1}-1+\ln 2}{0^+} \cong \frac{0{,}06}{0^+} = +\infty$$

Inoltre, detto $x = 1+\varepsilon$,

$$f(\varepsilon) \underset{\varepsilon\to0^+}{\sim} \frac{e^{-1}-1+\ln 2}{\ln(1+\varepsilon)} \underset{\varepsilon\to0^+}{\sim} \frac{e^{-1}-1+\ln 2}{\varepsilon}$$

che NON è INTEGRABILE

$\Rightarrow f(x)$ NON è INTEGRABILE in $(1,+\infty)$.