SVOLGIMENTI PROVA SCRITTA di ANALISI 1 del 4/7/2022

1) L'equazione è definita per $x > 0$.

Poniamo $z = y'$

$$\Rightarrow z' - \frac{2}{x} z = (x+1)\sqrt{x}$$

$$\Rightarrow z(x) = e^{\int \frac{2}{x} dx} \left[C_1 + \int e^{-\int \frac{2}{x} dx} (x+1)\sqrt{x}\, dx \right]$$

$$= e^{2\ln(|x|)} \left[C_1 + \int e^{-2\ln(|x|)} (x+1)\sqrt{x}\, dx \right]$$

$(x>0)$

$$= e^{2\ln x} \left[C_1 + \int e^{-2\ln x} (x+1)\sqrt{x}\, dx \right]$$

$$= x^2 \left[C_1 + \int \frac{(x+1)\sqrt{x}}{x^2} dx \right] = x^2 \left[C_1 + \int \left[\frac{1}{\sqrt{x}} + \frac{1}{x^{3/2}} \right] dx \right]$$

$$= x^2 \left[C_1 + 2\sqrt{x} - 2\frac{1}{\sqrt{x}} \right]$$

$$y'(1) = z(1) = 0 = C_1 + 2 - 2 = C_1 \Rightarrow C_1 = 0$$

$$\Rightarrow \quad y'(x) = z(x) = 2x^{\frac{5}{2}} - 2x^{\frac{3}{2}}$$

$$\Rightarrow y(x) = \frac{4}{7} x^{\frac{7}{2}} - \frac{4}{5} x^{\frac{5}{2}} + C_2$$

2

$$y(1)=1=\frac{4}{7}-\frac{4}{5}+C_2$$

$$\Rightarrow C_2=1+\frac{4}{5}-\frac{4}{7}=\frac{35+28-20}{35}=\frac{43}{35}$$

$$\Rightarrow \underline{y(x)=\frac{4}{7}x^{7/2}-\frac{4}{5}x^{5/2}+\frac{43}{35}}$$

2) $$\lim_{x\to 0^-}\frac{e^{x^2}-\cos x-\frac{3}{2}x^2}{x^4}=$$

$$=\lim_{x\to 0^-}\frac{\cancel{1}+\cancel{x^2}+\frac{x^4}{2}-\left[\cancel{1}-\cancel{\frac{x^2}{2}}+\frac{x^4}{24}\right]+o(x^4)-\cancel{\frac{3}{2}x^2}}{x^4}$$

$$=\lim_{x\to 0^-}\frac{\frac{11}{24}x^4}{x^4}=\underline{\frac{11}{24}=a}$$

$$\lim_{x\to 0^+}\frac{x^2-\sin(x^2)}{bx^6}=\lim_{x\to 0^+}\frac{\cancel{x^2}-\left[\cancel{x^2}-\frac{x^6}{6}+o(x^6)\right]}{bx^6}$$

$$=+\frac{1}{6b}=a \iff \frac{1}{6b}=\frac{11}{24}\iff \underline{b=\frac{4}{11}}$$

3) $(z^4+1)(\bar{z}-i)^2=0$ (3)

$\Rightarrow \bar{z}=i$ oppure $z^4=-1$

$\Rightarrow z=-i$ con $m_a=2$

Inoltre

$z=\sqrt[4]{-1}=\sqrt[4]{e^{i\pi}}$

$\Rightarrow z_k=e^{i\frac{\pi+2k\pi}{4}}$ $k=0,1,2,3$

$z_0=e^{i\frac{\pi}{4}}=\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}$

$z_1=e^{i\frac{3}{4}\pi}=-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}$

$z_2=e^{i\frac{5}{4}\pi}=-\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}$

$z_3=e^{i\frac{7}{4}\pi}=\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}$.

4) $\ln\left(1+\frac{1}{n}\right)\underset{n\to+\infty}{\sim}\frac{1}{n}$

la serie è a termini positivi

$\Rightarrow$ CRITERIO del CONFRONTO ASINTOTICO

$$a_n \sim \frac{1}{n^{1-\alpha}}$$

$$\Rightarrow \sum a_n \begin{cases} \text{converge per } 1-\alpha>1 \text{ cioè } \alpha<0 \\ \text{diverge per } \alpha \geq 0 \end{cases}$$

5) $D = \mathbb{R}$

$$f(-x) = -x - 2\operatorname{arctg}(-x) = -x + 2\operatorname{arctg} x$$
$$= -(x - 2\operatorname{arctg} x) = -f(x)$$

f DISPARI

$f(0) = 0$.

$$\lim_{x \to \pm\infty} f(x) = \pm\infty.$$

$$\lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \left[1 - \frac{2\operatorname{arctg} x}{x}\right] = 1$$

$$\lim_{x \to \pm\infty} \left[f(x) - x\right] = \lim_{x \to \pm\infty} (-2\operatorname{arctg} x) = \mp\pi$$

$\Rightarrow$ $y = x - \pi$ asintoto obliquo a $+\infty$

$y = x + \pi$ asintoto obliquo a $-\infty$.

$$f'(x) = 1 - \frac{2}{1+x^2} = \frac{x^2-1}{1+x^2} \geq 0$$

$$\Longleftrightarrow \quad x \leq -1 \quad \vee \quad x \geq 1$$

$x=-1$ punto di MAX. REL. : $f(-1) = -1 + \frac{\pi}{2} > 0$

$x=1$ " " MIN. REL. : $f(1) = 1 - \frac{\pi}{2} < 0$

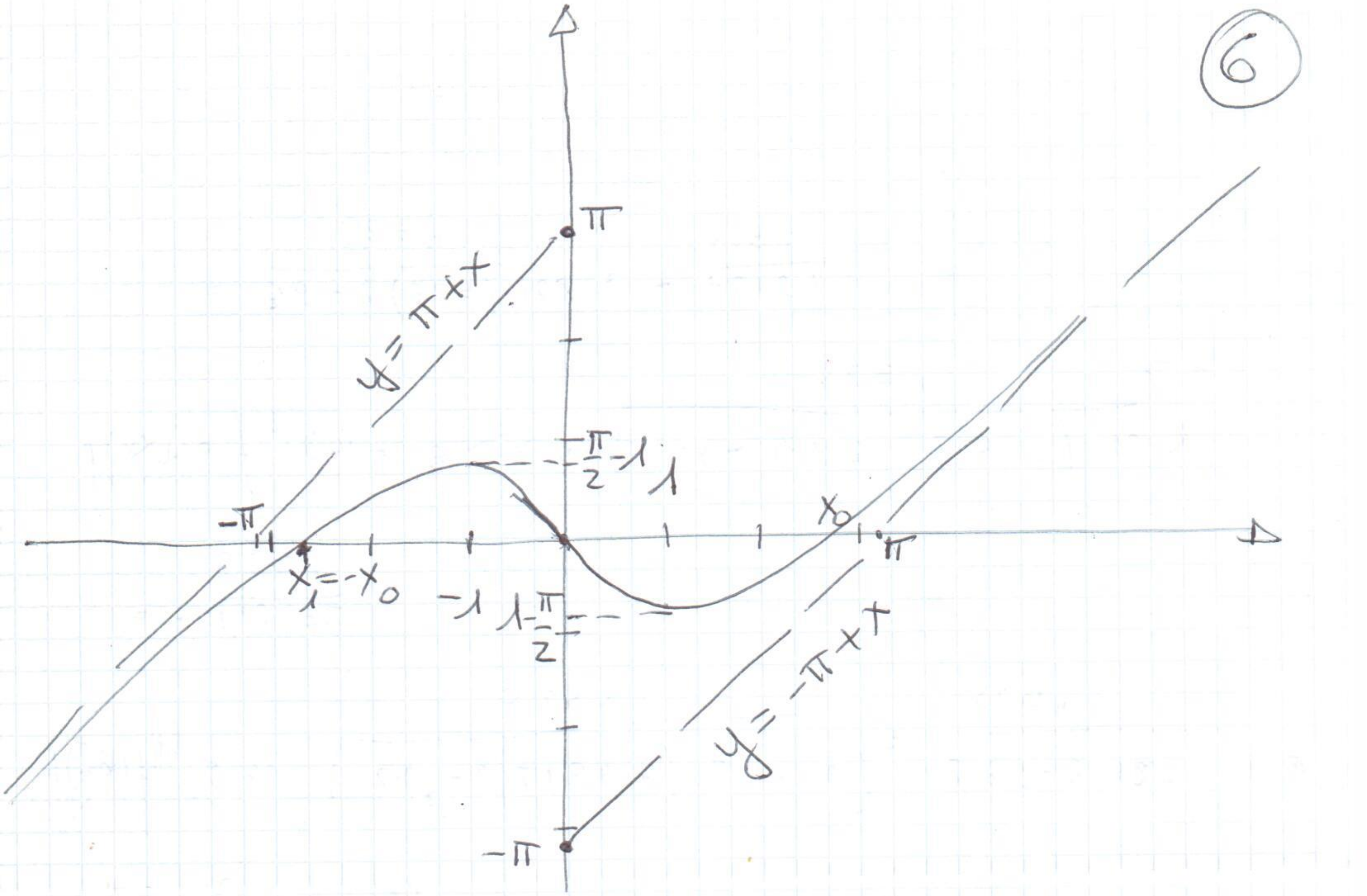
Poiché $\lim_{x \to \pm\infty} f(x) = \pm\infty \Rightarrow \nexists$ MAX-MIN ASSOLUTI

$$f''(x) = \frac{2x(1+x^2) - 2x(x^2-1)}{(1+x^2)^2} = \frac{4x}{(1+x^2)^2} \geq 0$$

$\Longleftrightarrow x \geq 0$

f concava in $(-\infty, 0)$; convessa in $(0, +\infty)$

FLESSO in $x=0$ (obliquo perché $f'(0) = -1$)
ASCENDENTE



Poiché per $x>0$ il grafico di f giace al di sopra dell'asintoto $y=-\pi+x$ (in quanto f è convessa); poiché $f(1)=1-\frac{\pi}{2}<0$; poiché $f(\pi)>0$ $\Longrightarrow$ f si annulla in $x_0\in(1,\pi)$. Per disparità, f si annulla anche in $x_1\in(-\pi,-1)$.

Dunque $f<0$ in $(-\infty,x_1)$ e in $(0,x_0)$

$f>0$ in $(x_1,0)$ e in $(x_0,+\infty)$.