# SVOLGIMENTI PROVA SCRITTA di ANALISI 1 del 6/9/2018



1) $A(x) = \dfrac{1}{x(1+x^2)} \in C^0\big((-\infty,0)\cup(0,+\infty)\big)$

$B(y) = \dfrac{1+y^2}{y} = \dfrac{1}{y} + y$

$B'(y) = -\dfrac{1}{y^2} + 1 \implies B \in C^1\big((-\infty,0)\cup(0,+\infty)\big)$

Poichè $x_0 = 1$ e $y_0 = 1$

$\implies$ considero $A \in C^0\overbrace{(0,+\infty)}^{\ni 1}$; $B \in C^1\overbrace{(0,+\infty)}^{\ni 1}$.

Trattandosi di equazione a variabili separabili, allora $\exists!$ sol. $y \in C^1$, di tipo LOCALE.

$B(y) \neq 0 \implies$ NON ESISTONO SOL. SINGOLARI

$$\int \left[\frac{y}{1+y^2}\right] dy = \int \left[\frac{1}{x(1+x^2)}\right] dx$$

$$= \int \left[\frac{1}{x} - \frac{x}{1+x^2}\right] dx$$

$$\Rightarrow \quad \frac{1}{2}\log(1+y^2) = \log(|x|) - \frac{1}{2}\log(1+x^2) + \frac{1}{2}\log C$$

②

$(C>0)$

~~$x>0 \Rightarrow$~~

$$\Rightarrow \log(1+y^2) = \log\left(\frac{Cx^2}{1+x^2}\right)$$

$$\Rightarrow 1+y^2 = \frac{Cx^2}{1+x^2}$$

C.I.: $\quad 2 = \frac{C}{2} \quad \Rightarrow \quad C=4$

$$\Rightarrow \quad y^2 = \frac{4x^2}{1+x^2} - 1 = \frac{3x^2-1}{1+x^2}$$

$y>0 \quad \Rightarrow \quad y = +\sqrt{\frac{3x^2-1}{1+x^2}}$ ,

Si osservi che, benché $A \in C^0(0,+\infty)$ e $B \in C^1(0,+\infty)$, la soluzione è definita solo per $x > \frac{1}{\sqrt{3}}$, cioè nell'intervallo $\left(\frac{1}{\sqrt{3}}, +\infty\right)$;

proprio in virtù della natura LOCALE della soluzione.



2) $I_{def} = \mathbb{R}$

Studiamo $f$ in $[0, \pi]$

$f(x) = 0 \iff \cos x = 0 \iff x = \frac{\pi}{2}$

$f(x) > 0 \iff \cos x > 0 \iff x \in [0, \frac{\pi}{2})$

$\cos x < 0 \iff x \in (\frac{\pi}{2}, \pi]$

$f(0) = 1$; $f(\pi) = -e^{-\pi} \cong -0{,}04$

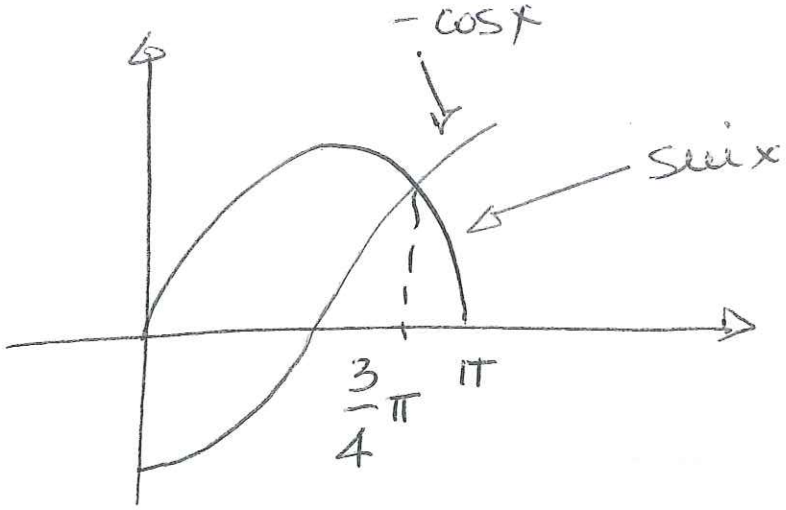
~~$f'(x) = -\sin x$~~ $f(x) = \cos x \cdot e^{-x}$

$\Rightarrow f'(x) = -\sin x \cdot e^{-x} - \cos x \cdot e^{-x}$

$= -e^{-x} [\sin x + \cos x]$

$f'(x) \geq 0 \iff$ ~~$\sin x \leq$~~ $\sin x + \cos x \leq 0$

$\iff \sin x \leq -\cos x$




$\Rightarrow$ $f$ crescente in $\left(\frac{3}{4}\pi, \pi\right]$

decrescente in $\left[0, \frac{3}{4}\pi\right)$

$x = \frac{3}{4}\pi$ PUNTO DI MIN. ASS.

$$f\left(\frac{3}{4}\pi\right) = \frac{\cos\left(\frac{3}{4}\pi\right)}{e^{\frac{3}{4}\pi}} = -\frac{\sqrt{2}}{2}\, e^{-\frac{3}{4}\pi}$$

$$\cong -0{,}07$$

$x = 0$ PUNTO DI MAX. REL.: $f(0) = 1$

$x = \pi$ " " " " : $f(\pi) = -e^{-\pi}$

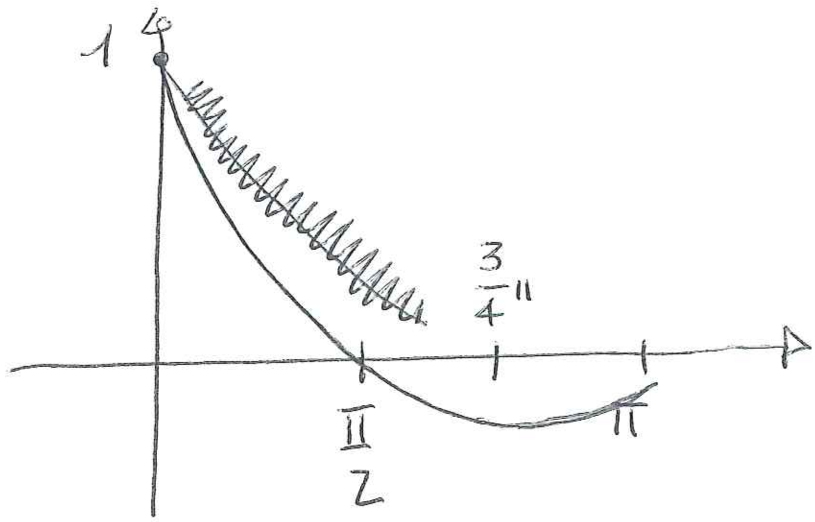
$\Rightarrow$ $x = 0$ PUNTO DI MAX. ASS.

$$f''(x) = e^{-x}(\sin x + \cos x) - e^{-x}(\cos x - \sin x)$$

$$= 2e^{-x}\sin x \geq 0 \qquad \forall x \in [0,\pi]$$

$\Rightarrow f$ sempre convessa.

Grafico:




3)

$$z^4 = -1 + \sqrt{3}i = 2\left[-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right]$$

$$= 2\left[\cos\left(\frac{2}{3}\pi\right) + i\sin\left(\frac{2}{3}\pi\right)\right]$$

$$= 2e^{i\frac{2}{3}\pi}$$

$$\Rightarrow z = \sqrt[4]{2e^{i\frac{2}{3}\pi}} \qquad 4 \text{ soluzioni}$$

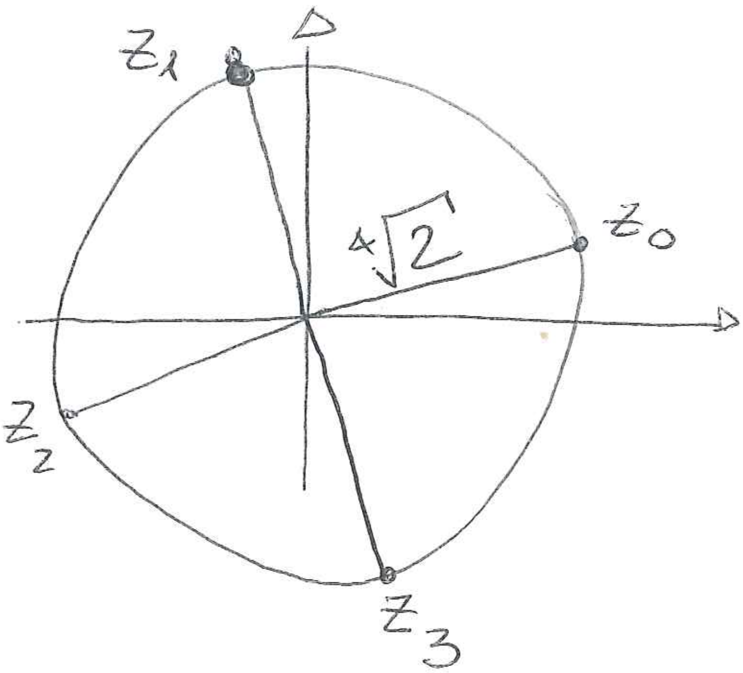
$$z_k = \sqrt[4]{2} \cdot e^{i\frac{1}{4}\left[\frac{2}{3}\pi + 2k\pi\right]}$$

$$z_0 = \sqrt[4]{2}\, e^{i\frac{\pi}{6}} = \sqrt[4]{2}\left[\frac{\sqrt{3}}{2} + i\frac{1}{2}\right]$$

$$z_1 = \sqrt[4]{2}\, e^{i\frac{2}{3}\pi} = \sqrt[4]{2}\left[-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$$

$$z_2 = \sqrt[4]{2}\, e^{i\frac{7}{6}\pi} = \sqrt[4]{2}\left[-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right]$$

$$z_3 = \sqrt[4]{2}\, e^{i\frac{5}{3}\pi} = \sqrt[4]{2}\left[\frac{1}{2} - i\frac{\sqrt{3}}{2}\right]$$




4) $$\ln\left(\frac{n+1}{n-1}\right) = \ln\left(1 + \frac{2}{n-1}\right) \underset{n\to\infty}{\sim} \frac{2}{n-1}$$

$$\Rightarrow \quad a_n \sim \frac{1}{\sqrt{n}} \cdot \frac{2}{n-1} \sim \frac{2}{n^{3/2}}$$

$$\Rightarrow \sum a_n \simeq 2 \sum \frac{1}{n^{3/2}}$$ convergente

5) ~~$f(x$~~ ~~$\operatorname{senh} x$~~

$$\operatorname{senh} x\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{\sqrt{x}} + \frac{1}{3!\,x^{3/2}} + \frac{1}{5!\,x^{5/2}} + o\left(\frac{1}{x^{5/2}}\right)$$

$$\operatorname{sen}\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{\sqrt{x}} - \frac{1}{3!\,x^{\frac{3}{2}}} + \frac{1}{5!\,x^{5/2}} + o\left(\frac{1}{x^{5/2}}\right)$$

$$f(x) = \left[\cancel{\frac{2}{\sqrt{x}}} - \cancel{\frac{2}{\sqrt{x}}} + \cancel{\frac{1}{3!\,x^{3/2}}} - \cancel{\frac{1}{3!\,x^{\frac{3}{2}}}} + \frac{2}{5!\,x^{\frac{5}{2}}} + o\left(\frac{1}{x^{\frac{5}{2}}}\right)\right] \cdot x$$

$$\Rightarrow f(x) \underset{x \to +\infty}{\sim} \cancel{2} \frac{1}{60\, x^{\frac{1}{2}}} = \frac{1}{60\sqrt{x}} > 0$$

NON integrabile in $[1, +\infty)$.

$$\Rightarrow \int_1^{+\infty} f(x)\,dx = +\infty$$