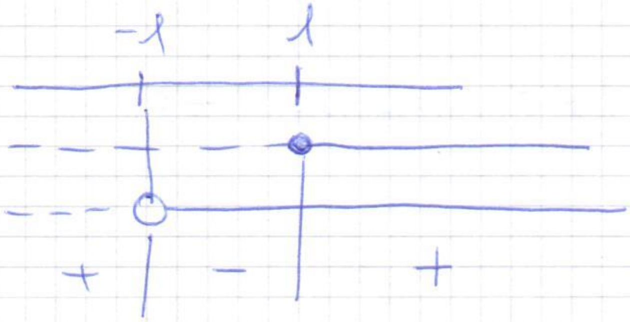
SVOLGIMENTI PROVA SCRITTA di ANALISI 1 del 7/10/2021

(1)

1) DOMINIO: $\dfrac{1-x}{1+x} \geq 0 \Rightarrow \dfrac{x-1}{x+1} \leq 0$



Mi interessano i valori non negativi

$\Rightarrow x \in (-1, 1] = D$

$f(x) > 0 \quad \forall x \in D$

$f(0) = e \qquad f(1) = e^0 = 1$

$$f'(x) = e^{\sqrt{\;}} \cdot \left(\sqrt{\frac{1-x}{1+x}}\right)' = e^{\sqrt{\;}} \frac{1}{2\sqrt{\frac{1-x}{1+x}}} \cdot \left[\frac{-(1+x)-(1-x)}{(1+x)^2}\right]$$

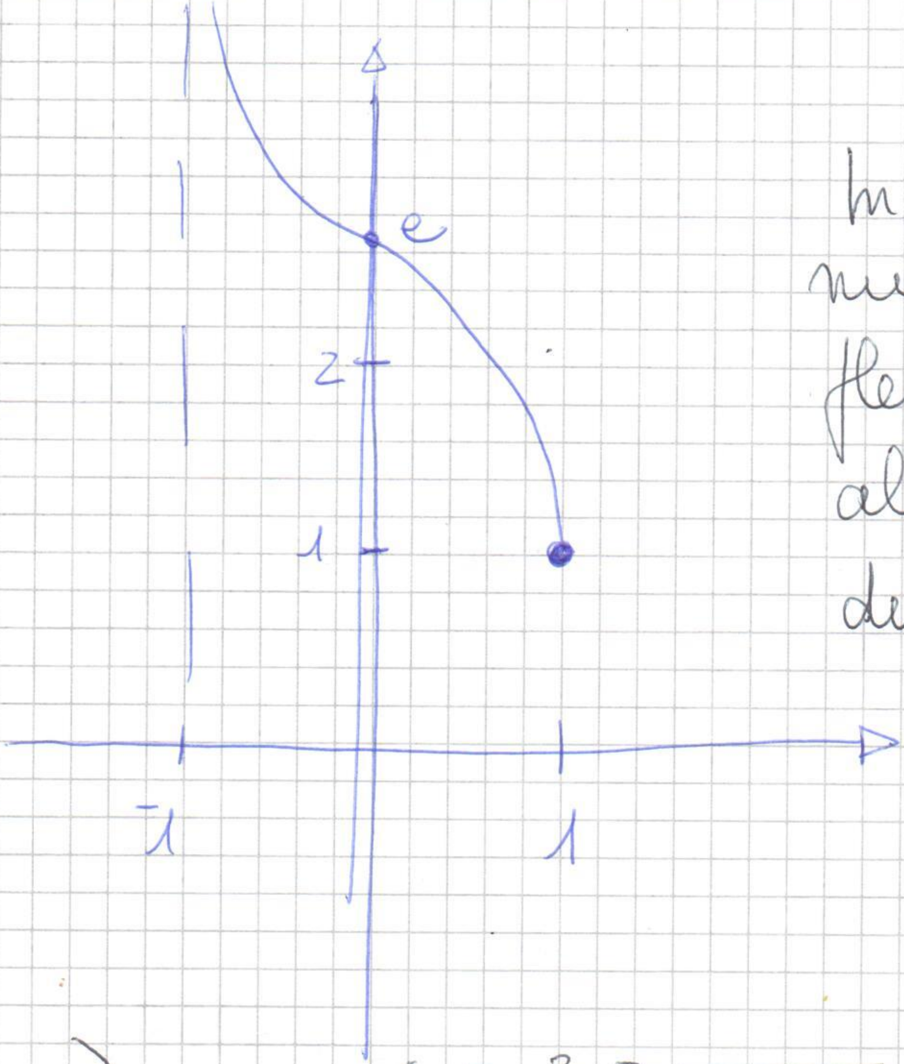
$$= \frac{e^{\sqrt{\;}}}{\cancel{2}} \cdot \sqrt{\frac{1+x}{1-x}} \left(\frac{-\cancel{2}}{(1+x)^2}\right) < 0 \quad \forall x \in (-1, 1).$$

f decrescente in D.

$$\lim_{x \to -1^+} f(x) = e^{\sqrt{\frac{2}{0^+}}} = e^{+\infty} = +\infty$$

$$\lim_{x \to 1^-} f'(x) = \frac{e^0}{2} \cdot (+\infty) \cdot \left(\frac{-2}{4}\right) = -\infty$$ NON DERIVABILE in $x=1$.

In ipotesi di numero ~~fl~~ minimo di flessi, un plausibile grafico è



In ipotesi di numero minimo di flessi, è necessario almeno un flesso discendente

2) $$\lim_{x\to+\infty} x^{\frac{4}{3}}\left[x^{\frac{2}{3}}\left[\sqrt[3]{1+\frac{1}{x^2}}-\sqrt[3]{1-\frac{1}{x^2}}\right]\right]$$

$$=\lim_{x\to+\infty} x^2\left[\left(1+\frac{1}{3x^2}+o\left(\frac{1}{x^2}\right)\right)-\left(1-\frac{1}{3x^2}+o\left(\frac{1}{x^2}\right)\right)\right]$$

$$=\lim_{x\to+\infty} x^2\,\frac{2}{3x^2}=\frac{2}{3}$$

3) $w = z - i$

$$\Rightarrow w^4 = -i = \cos\left(-\frac{\pi}{2}\right) + i \sin\left(-\frac{\pi}{2}\right)$$

$$\Rightarrow w_k = \cos\left(\frac{-\frac{\pi}{2} + 2k\pi}{4}\right) + i \sin\left(\frac{-\frac{\pi}{2} + 2k\pi}{4}\right)$$

$$k = 0, 1, 2, 3$$

$$w_0 = \cos\left(-\frac{\pi}{8}\right) + i \sin\left(-\frac{\pi}{8}\right)$$

$$w_1 = \cos\left(\frac{3}{8}\pi\right) + i \sin\left(\frac{3}{8}\pi\right)$$

$$w_2 = \cos\left(\frac{7}{8}\pi\right) + i \sin\left(\frac{7}{8}\pi\right)$$

$$w_3 = \cos\left(\frac{11}{8}\pi\right) + i \sin\left(\frac{11}{8}\pi\right)$$

$$\Rightarrow z_k = w_k + i, \quad k = 0, 1, 2, 3.$$

4) Criterio del rapporto.

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^{n+1}}{[(n+1)!]^2} \cdot \frac{(n!)^2}{n^n} = \frac{(n+1)^n \cancel{(n+1)}}{(n+1)^{\cancel{2}} n^n}$$

$$= \left(1 + \frac{1}{n}\right)^n \frac{1}{n+1} \longrightarrow \frac{e}{\infty} = 0.$$

$\Rightarrow$ la serie converge.

5) $f(x)$ è dispari:

$$f(-x) = (-x)^2 \sinh(-x)$$
$$= -x^2 \sinh(x) = -f(x)$$

$$\Rightarrow \int_{-\ln 2}^{\ln 2} f(x)\,dx = 0.$$

Verifichiamolo direttamente:

$$\int_{-\ln 2}^{\ln 2} x^2 \sinh(x)\,dx = \cosh x \cdot x^2 \Big|_{-\ln 2}^{\ln 2} - \int_{-\ln 2}^{\ln 2} 2x \cosh x\,dx$$

$$= \cosh(\ln 2) \cdot \ln^2 2 - \cosh(-\ln 2)(\ln 2)^2$$
$$-2\left[x \sinh x \Big|_{-\ln 2}^{\ln 2} - \int_{-\ln 2}^{\ln 2} \sinh x\,dx \right]$$

$\cosh x$ è pari $\Rightarrow$

$$= \cancel{\cosh(\ln 2) \cdot \ln^2 2} - \cancel{\cosh(\ln 2) \ln^2 2}$$
$$-2\left[\ln 2 \sinh(\ln 2) - (-\ln 2) \sinh(-\ln 2) - \cosh x \Big|_{-\ln 2}^{\ln 2} \right]$$

$$= -2\left[\cancel{\ln 2 \,\mathrm{senh}(\ln 2)} - \cancel{\ln 2\,\mathrm{senh}(\ln 2)} - \cosh(\ln 2) + \cosh(-\ln 2)\right]$$

$\cosh x$ è pari ; $\mathrm{senh}\, x$ è dispari

$$= -2\left[\cosh(\ln 2) - \cosh(\ln 2)\right] = 0 .$$