SVOLGIMENTO PROVA SCRITTA di ANALISI 1 del 12/10/2022

1)

$$\frac{\ln(1+x)+\ln(1-x)+\sin(x^2)}{\operatorname{arctg}(x^4)} =$$

$$= \frac{\left[\cancel{x}-\cancel{\frac{x^2}{2}}+\cancel{\frac{x^3}{3}}-\frac{x^4}{4}+o(x^4)\right]+\left[-\cancel{x}-\cancel{\frac{x^2}{2}}-\cancel{\frac{x^3}{3}}-\frac{x^4}{4}+o(x^4)\right] + \left(\cancel{x^2}-\frac{x^6}{6}+o(x^6)\right)}{x^4}$$

$$= \frac{-\frac{x^4}{2}+o(x^4)}{x^4} \xrightarrow[x\to 0]{} -\frac{1}{2}$$

$\alpha \neq 0$:

$$\frac{x-\sin x}{\alpha x^3} = \frac{\cancel{x}-\left(\cancel{x}-\frac{x^3}{6}\right)+o(x^3)}{\alpha x^3} \xrightarrow[x\to 0]{} \frac{1}{6\alpha}$$

$\Rightarrow$ f è prolungabile in $x=0$ se

$$-\frac{1}{2}=\frac{1}{6\alpha} \Longrightarrow \alpha = -\frac{1}{3}$$

2) $|w| = \sqrt{3+1} = 2$

$w + \overline{w} = 2\sqrt{3}$

$\Rightarrow z^4 = 4\sqrt{3} = 4\sqrt{3}\left[\cos(0) + i\sin(0)\right]$

$\Rightarrow z_1 = \sqrt[4]{4\sqrt{3}}$ $\qquad z_3 = -\sqrt[4]{4\sqrt{3}}$

$z_2 = \sqrt[4]{4\sqrt{3}}\, i$ $\qquad z_4 = -\sqrt[4]{4\sqrt{3}}\, i.$

3) Criterio della radice:

$$\sqrt[n]{a_n} = \frac{e^{-n}}{\sqrt[n]{n}} \xrightarrow[n\to\infty]{} 0 < 1$$

$\Rightarrow$ convergente.

4) $\cancel{f(x)}$ $I_{def} = \{x \neq 0\}$ NO intersezione con asse y

$f(-x) = 2(-x)^2 - \ln(|-x|) = 2x^2 - \ln(|x|) = f(x)$

(f PARI).

$\lim_{x\to 0^\pm} f(x) = -\ln(0^+) = +\infty.$ AS. VERTICALE $x = 0$.

$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} x^2 = +\infty$

ANDAMENTO SUPERLINEARE

$\Rightarrow$ NO AS. OBLIQUO

$$f'(x) = 4x - \frac{1}{x} = \frac{4x^2-1}{x} \geq 0 \iff x \leq -\frac{1}{2} \quad \checkmark$$
$$x \geq \frac{1}{2}$$

f decresce in $\left(-\infty, -\frac{1}{2}\right)$; cresce in $\left(-\frac{1}{2}, 0\right)$; ~~e~~ decresce in $\left(0, \frac{1}{2}\right)$; cresce in $\left(\frac{1}{2}, +\infty\right)$.

$x = \pm\frac{1}{2}$ punti di MIN. ASSOLUTO.

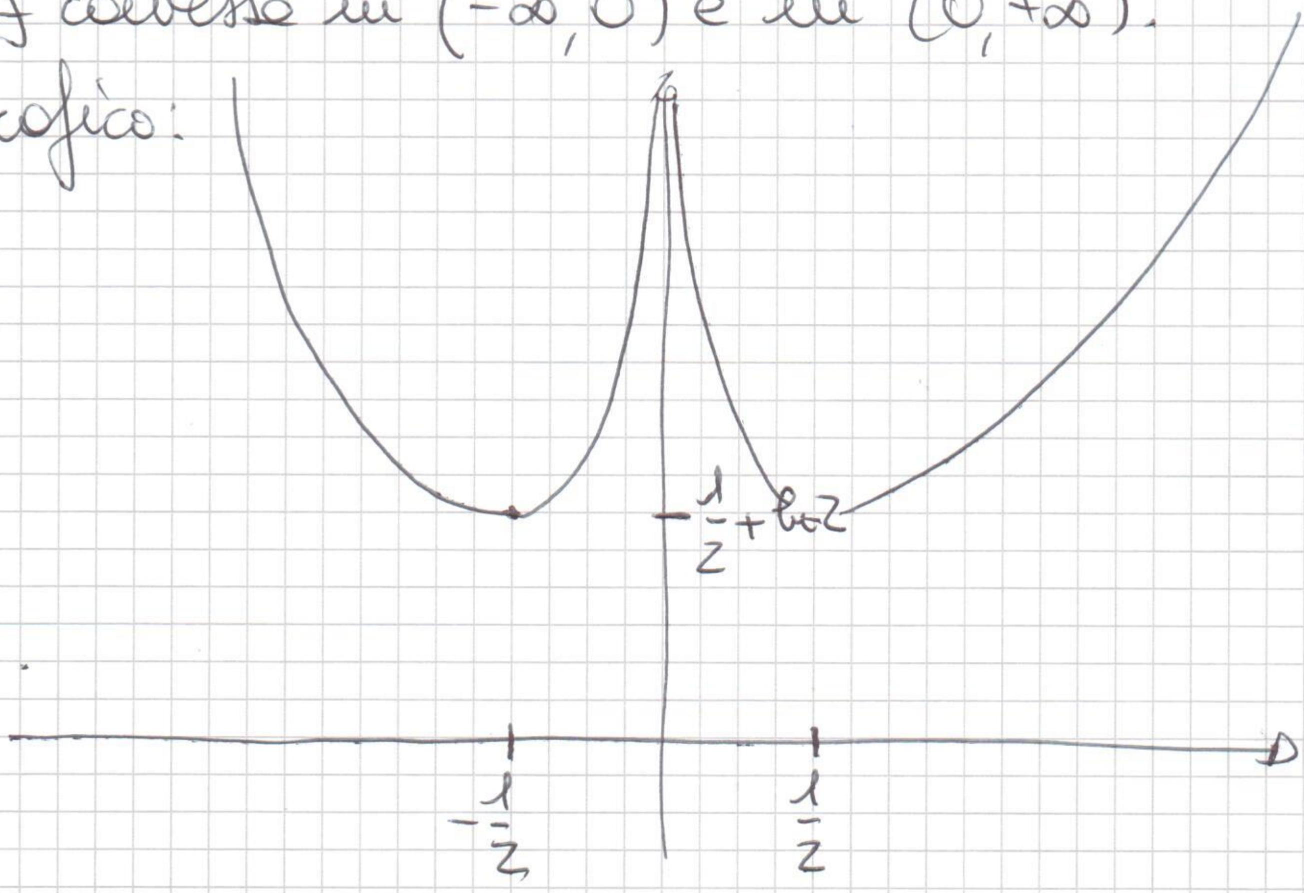
Poichè $f\left(\pm\frac{1}{2}\right) = \frac{1}{2} - \ln\left(\frac{1}{2}\right) = \frac{1}{2} + \ln 2 > 0$

$\Rightarrow f(x) > 0 \quad \forall x \in I_{def}$.

$$f''(x) = 4 + \frac{1}{x^2} > 0 \quad \forall x \in I_{def}$$

f convessa in $(-\infty, 0)$ e in $(0, +\infty)$.

Grafico:



5) $x \neq 0$

$$\Rightarrow \quad y' + \frac{y}{x} = x - \frac{2}{x}$$

~~Poiché~~ .

$$y(x) = e^{-\int \frac{1}{x}dx} \cdot \left[\int e^{\int \frac{1}{x}dx} \left(x - \frac{2}{x}\right) dx + C \right]$$

$$= e^{-\ln|x|} \left[\int e^{\ln|x|} \left(x - \frac{2}{x}\right) dx + C \right]$$

$$= \frac{1}{|x|} \left[\int |x| \left(x - \frac{2}{x}\right) dx + C \right]$$

$$= \begin{cases} \frac{1}{x} \left[\int (x^2 - 2) dx + C \right] & \text{se } x > 0 \\ -\frac{1}{x} \left[\int -(x^2 - 2) dx + C \right] & \text{se } x < 0 \end{cases}$$

$$= \frac{1}{x} \left[\int (x^2 - 2) dx + K \right]$$

$$= \frac{1}{x} \left[\frac{x^3}{3} - 2x + K \right] = \frac{x^2}{3} - 2 + \frac{K}{x}$$

INT. GENERALE

$\forall K \in \mathbb{R}$

$$\lim_{x \to +\infty} y(x) = +\infty$$

$\Rightarrow \nexists$ soluzioni al problema ai limiti proposto.

6) $\sqrt{x} = t$; $dt = \frac{1}{2\sqrt{x}} dx$; $dx = 2t\, dt$

$t(0) = 0$; $t(+\infty) = +\infty$

$$\int_0^{+\infty} e^{-\sqrt{x}}\, dx = \int_0^{+\infty} 2t e^{-t}\, dt$$

Sappiamo che ogni funzione $t^n e^{-t}$ è integrabile in $[0, +\infty)$.

Oltretutto, $\int_0^{+\infty} t^n e^{-t}\, dt = n!$

$$\Rightarrow \int_0^{+\infty} e^{-\sqrt{x}}\, dx = 2 \int_0^{+\infty} t e^{-t}\, dt = 2.$$